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The analysis of Titan’s physical surface using multifractal geometry methods

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The analysis of Titan's physical surface using multifractal geometry methods

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Abstract. Titan makes up 95% of the mass of all 82 satellites of Saturn. Titan's diameter is 5152 km, which means that it is larger than the Moon by 50%, and it is also significantly larger than Mercury. On the satellite, a subsurface ocean is possible, the theory of the presence of which has already been advanced earlier by some scientists. It is located under a layer of ice and consists of 10% ammonia, which is a natural antifreeze for it and does not allow the ocean to freeze. On the one hand, the ocean contains a huge amount of salt, which makes the likelihood of life in it hardly possible. But on the other hand, since chemical processes constantly occur on Titan, forming molecules of complex hydrocarbon substances, this can lead to the emergence of the simplest forms of life. There are limitations on the probabilistic and statistical approaches, since not every process and not every result (form and structure of the system) is probabilistic in nature. In contrast to this, fractal analysis allows one to study the structure of complex objects, taking into account their qualitative specifics, for example, the relationship between the structure and the processes of its formation. When constructing a harmonic model of Titan, the method of decomposition of topographic information into spherical functions was used. As a result, based on the harmonic analysis of the Cassini mission data, a topographic model of Titan was created. In the final form, the model describing Titan's surface includes the expansion of the height parameter depending on the spherical coordinates into a slowly converging regression series of spherical harmonics. For modeling surface details of the surface on a scale of 1 degree requires analysis of the $(180 + 1)^2$ harmonic expansion coefficients. An over determined topographic information system was solved to meet the regression modelling conditions. In this case, a number of qualitative stochastic data, such as external measures, were used together with the standard postulation of the harmonic system of the Titan model. As a result of a sampling of self-similar regions (with close values of the self-similarity coefficients) on the surface of Titan, coinciding with the SRGB parameter (characterizes the color fractal dimension), the elements of the satellite's surface were determined, which with a high degree of probability were evolutionarily formed under the action of the same selenochemical processes.

1. Introduction

The purpose of this work is to build a digital harmonic regression model of the Titan surface based on Cassini space mission data and analyze it using fractal geometry methods to detect self-similar regions and structural parameters.

The objectives solved in our study are as follows: 1) Creation of a method for constructing a digital harmonic regression structural model of Titan (TDGRSM); 2) Development of automated software package for the analysis of elevation points (the height of which above the geoid of Titan corresponds to a certain color on the elevation scale) on the surface of Titan; 3) Construction of TDGRSM by expansion of the height function in terms of spherical functions; 4) Development of a method for analyzing physical surfaces using the parameter of color fractal dimension; 5) Building the SRGB 3D model of Titan; 6) Creation of SRGB 2D model of Titan; 7) Building the self-similarity coefficient 3D model of Titan; 8) Creation of the self-similarity coefficient 2D model of Titan; 9) Analysis and comparison of the results obtained.

The novelty of this work is as follows: 1) For the first time, satellite altimetry observations of Titan were used to construct 2D and 3D fractal models of Saturn's moon; 2) The author's method for analyzing physical surfaces using the parameter of color fractal dimension has been developed; 3) New results were obtained on the presence of self-similar regions on the surface of Titan. That is, those areas that with a high degree of probability were formed under the same evolutionary processes.

Based on data collected by NASA's Cassini space probe, a global topographic database of Titan's surface was created [1]. To compose this base, the Cassini probe made 120 flights around Titan in the period from 2004 to 2017 [2]. Currently, the main process analysis methods in complex systems are statistical methods [3]. Such multi-parameter systems should be studied by the methods of physics of complex systems, and fractal analysis is one of them [4]. The significant differences between the fractal dimension of the Titan surface model and its real physical parameters proves that there is a certain complex structural distribution of the Titan model in the phase space. Firstly, complex physical systems cannot be described as a single fractal and are multifractals consisting of a set of interconnected fractals with their own dimensions [5]. Secondly, in a fractal model, each part repeats the entire structure model and does not change when the scale changes, i.e. is recursive [6]. The use of multifractal analysis makes it possible to study the system as a spectrum of individual fractal dimensions. This method provides high accuracy in describing complex fractal structures by examining local areas. In this study, the author's mathematical algorithm was used to determine and analyze fractal parameters, which ensured high reliability and accuracy [7]. Further development applied method of comparative fractal analysis will allow astronomers to investigate a larger number of local chemical and physical parameters and anomalies of Titan.

2. Method of multifractal analysis of Titan macrofigure

Analysis of the map of Titan surface (Figure 1) shows that there were many processes of external influence on it by other small celestial bodies. This is confirmed by the presence of a large number of surface formations of impact character: hills and craters.

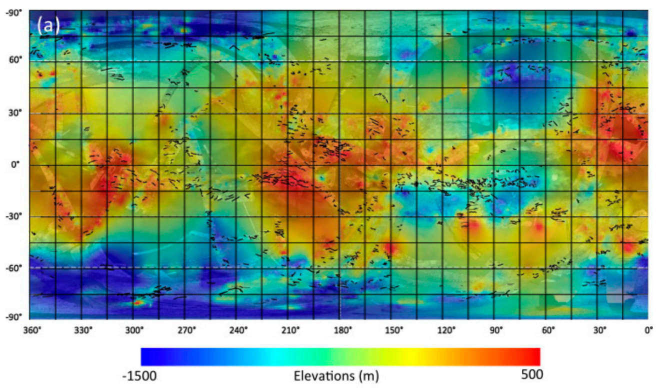

Figure 1. Map of Titan's surface.

When analyzing the altitude data of Titan, the entire surface of the satellite was divided into pixels, and its average altitude value was determined in each pixel using an approximation. This served as the basis for building a 3D model of Titan (Figure 2).

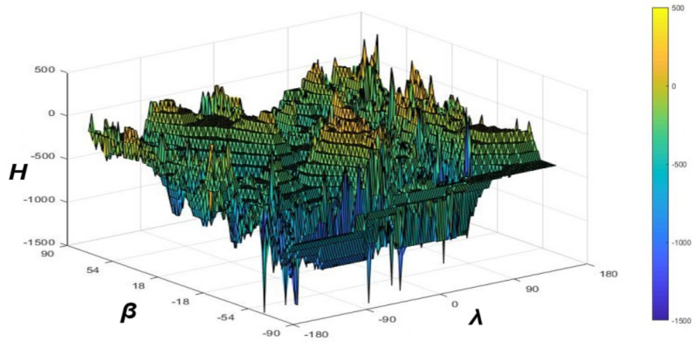

Figure 2. 3D model of the distribution of heights of Titan's surface.

Next, a fractal analysis was performed. Titan's structure was presented as a partially ordered set $A(N^2)$, where N^2– number of components a_{ij} of set $a_{ij} \in A(N^2)$, where $i, j = 1 \dots N$. As a result, for the set $A(N^2)$ fractal dimension D_ξ according to grade $H_\xi(a)$ is determined by the logarithmic slope $\log \Gamma_\xi(n^2)$, where $\Gamma_\xi(n^2)$ – number of discrete cubes - measures [8], S_ξ - the area occupied by the elements of the set *n*, in this way:

$$D_\xi = \sum_\gamma \frac{\log \Gamma_\xi\left(n_{\gamma+1}^2\right) - \log \Gamma_\xi(n_\gamma^2)}{abs(\log S_\xi(n_{\gamma+1}^2)) - abs(\log S_\xi(n_\gamma^2))} * \left(\frac{\alpha_{\gamma+1} - \alpha_\gamma}{N-1}\right) \quad (1)$$

As a characteristic of grade $H_\xi(a)$ three pixel colors are accepted: red (ξ=R), green (ξ=G) and blue (ξ=B), which are described by three fractal dimensions D_R, D_G, D_B. Fractal color dimensions form in the coordinate system of qualities the area of the SRGB triangle, described in detail in [8]:

$$\text{SRGB} = \frac{1}{2}\begin{vmatrix} x_1 & D_R & 1 \\ x_2 & D_G & 1 \\ x_3 & D_B & 1 \end{vmatrix} = \frac{1}{2} M[-2(D_R + D_B) + (D_G + D_B) + (D_R + D_G)], \quad (2)$$

where M is the scale factor.

3. Results of the analysis of Titan's macro-figure

As a result, the following models were obtained.

For the self-similarity coefficient K_ξ [9]:

$$K_\xi = \frac{D_\xi^0}{D_\xi}, \quad (3)$$

where D_ξ^0– fractal dimension of self-similar set:

$$D_\xi^0 = \frac{\log \Gamma_\xi(N^2) - \log \Gamma_\xi(1)}{abs(\log S_\xi(N^2)) - abs(\log S_\xi(1))}. \quad (4)$$

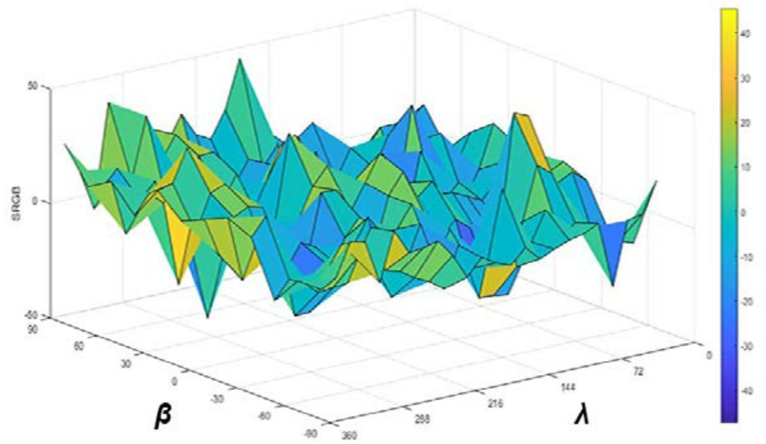

Figure 3. SRGB 3D model of Titan.

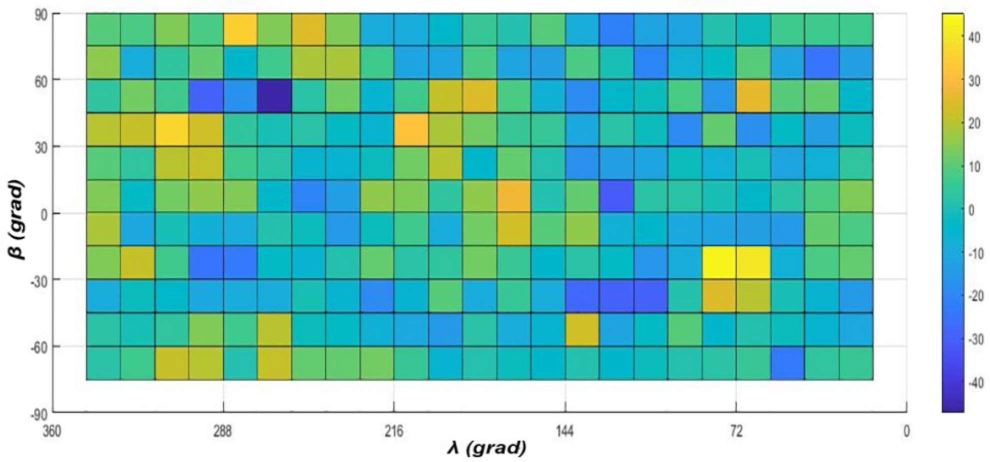

Figure 4. SRGB 2D model of Titan.

Figure 3 shows the 3D model of Titan and Figure 4 shows the 2D model, respectively. They show hills and lowlands according to the color altitude scale.

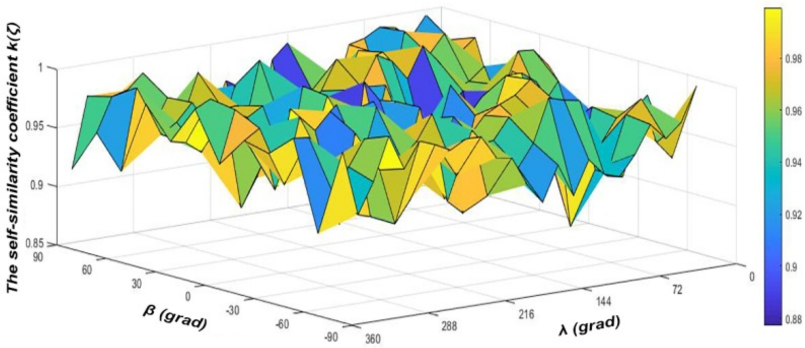

Figure 5. 3D model of the distribution K_ξ of Titan.

Figures 5 and 6 show a 3D model and a 2D model of the distribution K_ξ of Titan's surface. Self-similar regions of the satellite surface are visible.

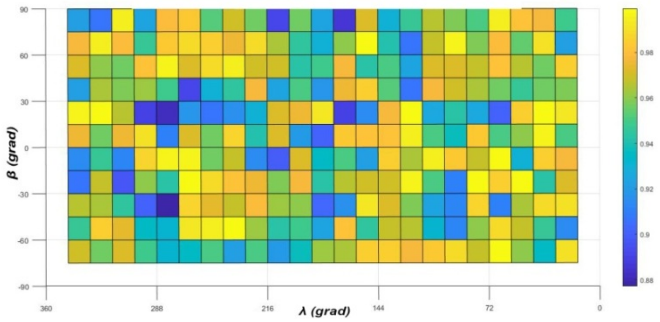

Figure 6. 2D model of the distribution K_ξ of Titan.

It should be emphasized that the constructed models are approximated, since the average values of heights were taken in the calculations.

4. Summary and conclusions

Currently, interest in the study of satellites of the solar system planets is an actual task [10]. This is due to the prospect of their robotic study [11]. The second reason is the development of the theory of the evolution of the solar system [12]. Recent studies have shown the discrepancy between classical theories and observations of exoplanetary systems [13]. There is also a lot of work to be done to determine the chemical composition of moons of planets [14].

When performing this work, an analysis of the surface of Saturn's moon Titan was carried out by the multifractal method [15]. It has been determined that Titan, due to the large number of interactions with various kinds of space bodies, has a large number of shock formations on its surface [16]. Titan models built using satellite data are structured by pixels, within the boundaries of which the average values of the surface heights were determined [16]. These 2D and 3D surface models of Titan contain chromatic parameters and a K_ξ coefficient distribution, according to which self-similar structures can be found [17].

The results of this work may be of certain interest in organizations that are engaged in the study of satellites of the planets of the solar system [18].

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