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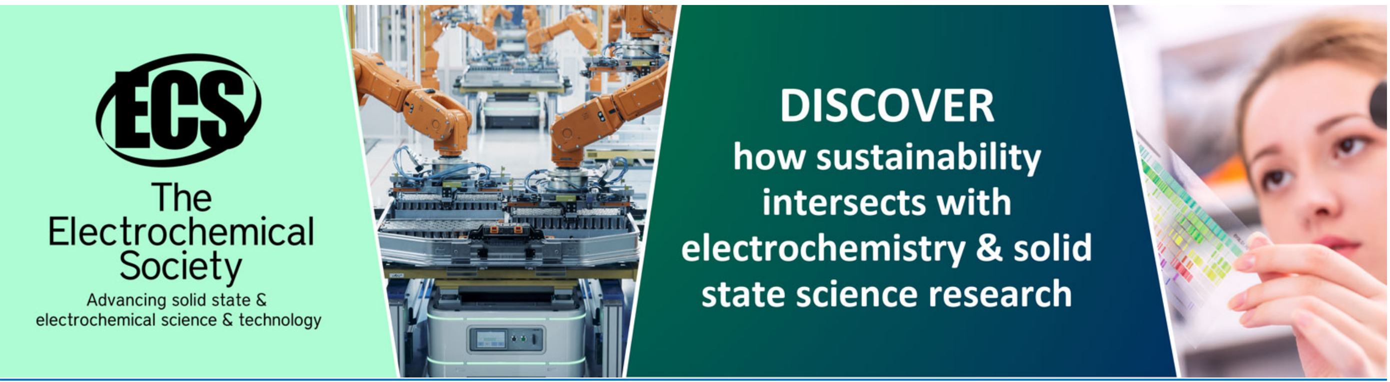

# Analysis of subharmonic oscillations in multi-phase ferroresonance circuits using a mathematical model

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**Abstract.** The article about solution of system of nonlinear differential equations that are almost impossible to solve by analytical methods by constructing a mathematical model of nonlinear oscillations occurring in three-phase ferroresonance circuits.A system of nonlinear differential equations was formed by approximating the volt-ampere characteristics of a ferromagnetic element in a ferroresonance circuit.Mathematical models for solving technical problems characterizing subharmonic oscillation processes in three-phase ferroresonance circuits and systems using the finite-difference method are expressed in the form of differential equations without appropriate initial conditions.Amathematical model of a system of equations representing subharmonic oscillations in ferroresonance connections depending on the value of the selected parameters in the field of change of any variable is considered.

## 1. Introduction
Mathematical models for solving technical issues describing vibrational processes in three-phase ferroresonance circuits and systems are represented in the form of differential equations that do not have appropriate initial conditions. Autoparametric vibration processes in the three-phase ferroresonance circuits under consideration are described using systems of nonlinear differential equations that form a mathematical model.Using a structured mathematical model, this is achieved by taking into account the main factors and conditions that occur in nonlinear electrical circuits.As the level of accuracy in solving a mathematical model increases, its appearance on a computer and the algorithmization of tasks become more complex [1,2]. However, the reliability of the mathematical model and the efficiency of the algorithm performed in the computer holds a special place.

## 2. Method
Model reliability issues are closely related to the ability to draw important conclusions and recommendations in practice.It should be noted that if the mathematical model is adequate, it allows not only to determine the quantitative characteristics of the studied processes, but also to identify qualitatively new phenomena of the considered processes.

One of the main ways to determine the change processes of any variable is to approach the finite-difference view of the differential expression using a mathematical model of the process.

The widespread use of finite-differentiated methods to solve energy-related process problems will require a comprehensive study of them over time, both in terms of predicting the condition of the object under consideration and their impact through a set of technical and mathematical parameters.

Solutions of complex problems using finite-difference methods, i.e., nonlinear connections in energy, usually consist of a combination of problems and methods of constructing analogues of differences. Therefore, the constructive theory of finite-difference methods is associated with the mutually consistent development of the studied process in accordance with the field of study. All available methods of studying problems should be used in the development of numerical methods: analytical, precise solutions, asymptotic evaluation, dimensional analysis, and experimental data.

In terms of computational experience, digital methods performed on a computer must be cost-effective, comprehensive, convergent, and stable against rounding errors.In this context, the ferroresonance processes observed in nonlinear circuits in energy are much more complicated in the traditional way, especially when considering the phase relationships in three-phase circuits.

In solving complicated problems, based on the law of conservation of energy, special attention should be paid to the conservatism of finite-difference analogues of the processes under consideration.

To write a diagram of the differences that approximate the differential equation given by the given initial conditions, the following steps must be performed:

- it is necessary to select the stage of integration over the time layer, which provides the conservatism of the finite-difference model;
- pay attention to the separate change area of the constant change limit of the argument (time);
- replacing the differential operators with some difference operator and constructing a difference analog for the initial data.

As a result, a mathematical problem can be solved using a computer based on the above sequence.

Attention should be paid to the recycling process that can be used to solve the energy problem.

Recently, due to new information technologies, more attention is paid to the development of research in all areas, including energy, as a method of scientific research.

It plays a key role in the study of processes occurring, especially in ferroresonant circuits, and includes: problem statement, problem solving algorithm, programming in algorithmic language, computer computation, and analysis of numerical calculations. The simplest scheme of three-phase ferroresonance circuits connected in parallel by a ferromagnetic element is shown [3,4,5,6,7] (figure 1).

There are two types of three-phase ferroresonance circuits parallel connection with zero-wire and without zero-wire,wederive the structure of the differential equations that mathematically substantiatethe autoparametric oscillations at the subharmonic frequency generated in each of them.

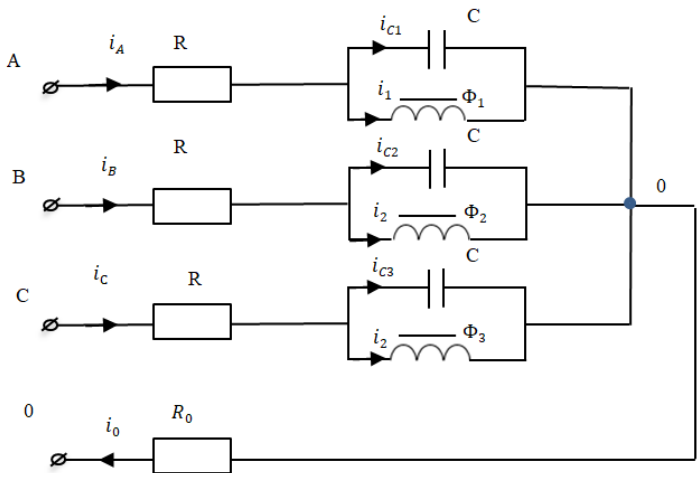


**Figure 1.** A symmetrical three-phase ferroresonance circuit in which the zero wire elements are connected in parallel.

We construct a system of differential equations for a three-phase circuit connected in parallel with zero wire (figure 1). According to Kirchhoff's 2nd law:

$$u_\nu = R'i + \mathrm{A}\dot{\Phi} + R_0 \cdot i_0 \tag{1}$$

Such

- $R'=\begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix}$, $\mathrm{A}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $R_0=\begin{bmatrix} R_0 & 0 & 0 \\ 0 & R_0 & 0 \\ 0 & 0 & R_0 \end{bmatrix}$ – special square matrices;
- $\nu$ -number of phases;
- $u_\nu, i,, \dot{\Phi}$ −symmetrical three-phase voltage source is the product of the instantaneous values of the linear voltages of the matrix columns, all the mains currents and the magnetic field current of each core of the three-phase ferromagnetic element;
- $R_1, R_{2,} R_3$ $and$ $R_0$ −in accordance with the phases and the active resistances of the zero wire.

According to Kirchhoff's 1st law:

$$i_A + i_B + i_C = i_0 \tag{2}$$

Here

$$i_A = i_1 + i_{C_\mathrm{A}}; i_B = i_2 + i_{C_\mathrm{B}}; i_C = i_3 + i_{C_C} \tag{3}$$

Since the current in the existing network of the capacitive element is as follows

$$i_{C_\mathrm{A}} = C\frac{dU_{C_\mathrm{A}}}{dt};\ i_{C_\mathrm{B}} = C\frac{dU_{C_\mathrm{B}}}{dt};\ i_{C_C} = C\frac{dU_{C_C}}{dt}. \tag{4}$$

From the equality of voltage fluctuations in a parallel network, that is to say

$$\frac{dU_{\mathrm{cA}}}{dt} = \frac{d^2\Phi_1}{dt^2}; \frac{dU_{\mathrm{cB}}}{dt} = \frac{d^2\Phi_2}{dt^2}; \frac{dU_{\mathrm{cC}}}{dt} = \frac{d^2\Phi_3}{dt^2}. \tag{5}$$

or

$$U_{C_\mathrm{A}} = \frac{d\Phi_1}{dt}; U_{C_\mathrm{B}} = \frac{d\Phi_2}{dt}; U_{C_C} = \frac{d\Phi_3}{dt}. \tag{6}$$

Here (4), (5) and 6 The current in the existing network of the capacitive element can be expressed as follows, taking into account the formulas:

$$i_{C_\mathrm{A}} = C\frac{d^2\Phi_1}{dt^2}; i_{C_\mathrm{B}} = C\frac{d^2\Phi_2}{dt^2}; i_{C_C} = C\frac{d^2\Phi_3}{dt^2}. \tag{7}$$

Substituting these (6) formulas into expression (3), we generate phase currents;

$$i_A = i_1 + C\frac{d^2\Phi_1}{dt^2}; i_B = i_2 + C\frac{d^2\Phi_2}{dt^2}; i_C = i_3 + C\frac{d^2\Phi_3}{dt^2}. \tag{8}$$

Substituting expression (7) into Equation (2) above, we can find the current in the zero wire.

$$i_0 = i_1 + i_2 + i_3 + C\sum_{n=1}^{3}\frac{d^2\Phi_n}{dt^2} \tag{9}$$

Substituting (7) and (8) into (1), we obtain the voltages in the phases:

$$U_\mathrm{v} = \mathrm{R}\left(i_\mathrm{v} + C\frac{d^2\Phi_\mathrm{v}}{dt}\right) + \frac{d\Phi_\mathrm{v}}{dt} + R_0\left(\sum_{n=1}^{3} i_n + \sum_{n=1}^{3}\frac{d^2\Phi_n}{dt^2}\right) \tag{10}$$

or by generalizing a system of equations by revealing the products:

$$U_{\Phi\nu} = \mathrm{R}C\frac{d^2\Phi_\nu}{dt} + \frac{d\Phi_\nu}{dt} + R_0\sum_{n=1}^{3} i_n + R_0C\sum_{n=1}^{3}\frac{d^2\Phi_n}{dt^2} + \mathrm{R}i_\nu \tag{11}$$

$$\text{here } U_{\Phi\nu} = U_{m\nu}\cos\left[wt + \varphi - 2\pi\frac{(\nu-1)}{3}\right] \tag{12}$$

If we approximate the VAX of a ferromagnetic element as$i = \alpha\Phi + \beta\Phi^3$cubic polynomial and put it in the system of equations (10), we get a system of second-order nonlinear differential equations:

$$U_\nu = \mathrm{R}\alpha\Phi_\nu + \mathrm{R}\beta{\Phi_\nu}^3 + \mathrm{R}C\frac{d^2\Phi_\nu}{dt^2} + \frac{d\Phi_\nu}{dt} + R_0\alpha\sum_{n=1}^{3}\Phi_\mathrm{n} + R_0\beta\Phi_n^3 + R_0C\sum_{n=1}^{3}\frac{d^2\Phi_n}{dt^2} \tag{13}$$

We divide the left and right sides of this system of differential equations by RC and form a second-order differential equation.

$$\frac{d^2\Phi_\mathrm{v}}{dt^2} + \frac{1}{\mathrm{R}C}\frac{d\Phi_\mathrm{v}}{dt} + \frac{R_0\alpha}{\mathrm{R}C}\sum_{n=1}^{3}\Phi_\mathrm{n} + \frac{R_0\boldsymbol{\beta}}{\mathrm{R}C}\sum_{n=1}^{3}\Phi_n^3 + \frac{R_0}{\mathrm{R}}\sum_{n=1}^{3}\frac{d^2\Phi_n}{dt^2} + \frac{\alpha}{\mathrm{C}}\Phi_\mathrm{v} + \frac{\boldsymbol{\beta}}{C}{\Phi_\nu}^3 = \frac{U_\mathrm{v}}{\mathrm{R}C} \tag{14}$$

$$\text{In this } U_\nu = U_{m\nu}\cos\left[\omega t + \varphi - 2\pi\frac{(\nu-1)}{3}\right]$$

## 3. Results and discussion

The resulting system of nonlinear differential equations (13) allows the study of the excitation processes of autoparametric oscillations of different frequencies in steady and transition states for symmetric three-phase and multi-phase electropherromagnetic circuits. Its generalized second-order differential equation is expressed as follows.

$$\begin{aligned}&\frac{d^2\Phi_\nu}{dt^2} + \frac{1}{\mathrm{R}C}\frac{d\Phi_\nu}{dt} + \frac{R_0\alpha}{\mathrm{R}C}\sum_{n=1}^{3}\Phi_\mathrm{n} + \frac{R_0\boldsymbol{\beta}}{\mathrm{R}C}\sum_{n=1}^{3}\Phi_n^3 + \frac{R_0}{\mathrm{R}}\sum_{n=1}^{3}\frac{d^2\Phi_n}{dt^2} + \frac{\alpha}{\mathrm{C}}\Phi_\nu + \frac{\boldsymbol{\beta}}{C}{\Phi_\nu}^3 = \\ &= \frac{U_{m\nu}}{\mathrm{R}C}\cos\left[\omega t + \varphi - 2\pi\frac{(\nu-1)}{3}\right]\end{aligned} \tag{15}$$

For three-phase subharmonic oscillations in three-phase ferroresonance circuits connected in parallel with zero wire we define it $\omega t = 2\tau$ or $t = \frac{2\tau}{\omega}$. In that case $t^2 = \frac{4\tau^2}{\omega^2}$ be equal $\frac{d^2\Phi}{dt^2} = \frac{\omega^2}{4}\frac{d^2\Phi}{d\tau^2}$ we multiply all the terms of the system of equations (14) by $\frac{4}{\omega^2}$ and form the following system of equations.

$$\begin{aligned}&\frac{d^2\Phi_\nu}{dt^2} + \frac{4}{\omega^2}\frac{1}{\mathrm{R}C}\frac{d\Phi_\nu}{dt} + \frac{4}{\omega^2}\frac{R_0\alpha}{\mathrm{R}C}\sum_{n=1}^{3}\Phi_\mathrm{n} + \frac{4}{\omega^2}\frac{R_0\boldsymbol{\beta}}{\mathrm{R}C}\sum_{n=1}^{3}\Phi_n^3 + \frac{4}{\omega^2}\frac{R_0}{\mathrm{R}}\sum_{n=1}^{3}\frac{d^2\Phi_n}{dt^2} + \frac{4}{\omega^2}\frac{\alpha}{\mathrm{C}}\Phi_\nu + \\ &+ \frac{4}{\omega^2}\frac{\boldsymbol{\beta}}{C}{\Phi_\nu}^3 = \frac{4}{\omega^2}\frac{U_m}{\mathrm{R}C}cos\ (2\tau + \ \varphi - 2\pi\frac{(\nu-1)}{3})\end{aligned} \tag{16}$$

According to the initial condition,when $\tau = 0$ we create the following system of equations.

$$\frac{4}{\omega^2}\frac{\alpha}{\mathrm{C}}\Phi_\nu + \frac{4}{\omega^2}\frac{\boldsymbol{\beta}}{C}{\Phi_\nu}^3 = \frac{4}{\omega^2}\frac{U_m}{\mathrm{R}C}\cos\ (2\tau + \varphi - 2\pi\frac{(\nu-1)}{3}) \tag{17}$$

By multiplying both sides of this system of equations (16) by $\frac{\omega^2}{4}\frac{C}{\beta}$ we obtain the following formula

$$\Phi_\nu^3 + \Phi_\nu \frac{\alpha}{\beta} - \frac{U_{m2}}{\text{RC}\beta} cos\left(2\tau + \varphi - 2\pi\frac{(\nu - 1)}{3}\right) = 0 \tag{18}$$

This expression (17) solves the process of formation of the second-order subharmonic oscillation as a solution of equation (14) on the basis of the program of the steady-state equation in the package "Matlab".Using Cardano's formula for solving tertiary equations, we find the solution of equation (17) [5].

$$\Phi_\nu = \sqrt[3]{\frac{\mathrm{U_m C}}{2\mathrm{RC}\beta}\cos\left(2\tau + \varphi - 2\pi\frac{(\nu-1)}{3}\right) + \sqrt{\frac{\mathrm{U_m}^2\mathrm{C_0}^2}{4\mathrm{R}^2\mathrm{C}^2\beta^2}\cos^2\left(2\tau + \varphi - 2\pi\frac{(\nu-1)}{3}\right)\frac{\alpha^3}{27\beta^3}}} - \\ - \frac{\alpha}{3\beta\cdot\sqrt[3]{\frac{\mathrm{U_m C}}{2\mathrm{RC}\beta}\cos(2\tau + \varphi - 2\pi\frac{(\nu-1)}{3}) - \sqrt{\frac{\mathrm{U_m}^2\mathrm{C_0}^2}{4\mathrm{R}^2\mathrm{C}^2\beta^2}\cos^2(2\tau + \varphi - 2\pi\frac{(\nu-1)}{3}) + \frac{\alpha^3}{27\beta^3}}}} \tag{19}$$

Connection parameters $U_m = 170$ В; $C = 60$ kmF; R = 25 Ом; $\mathrm{R_0} = 10$ Om; $\alpha = 1.0$; $\beta = 0.4$; ω=314.15926 $c^{-1}$ in connections $\left(\frac{\omega}{2}\right)$ – a steady state of subharmonic ferroresonance at the frequency occurs (figure 2).

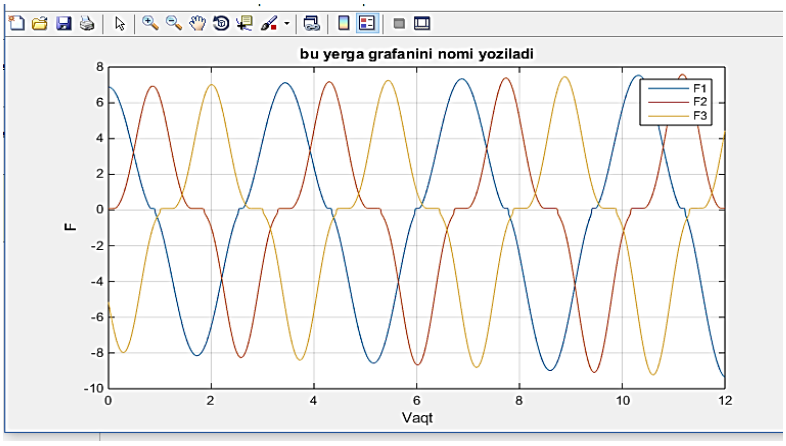


**Figure 2.** In ferroresonance circuits connected in parallel with a neutral wire $(\omega/2)$ –is a steady state of subharmonic ferroresonance at a frequency.

## 4. Conclusion

Hence, the solution of equation (16) is found using Cardano's formula for solving tertiary equations.These solution elements prevent the occurrence of faults caused by ferroresonance in power transmission networks and systems by solving the subharmonic ferroresonances of the frequency$\left(\frac{\omega}{2}\right)$ –frequency generated by three-phase circuits connected in parallel.

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