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Attempt to quantify the impact of seasonal air density variation on operating tip-speed ratio of small wind turbines

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Abstract. This study presents the impact of seasonal variation in air density on the operating tip-speed ratio of small wind turbines. The air density, which varies depending on the temperature, atmospheric pressure, and relative humidity, has an annual amplitude of about 5% in Tokyo, Japan. This study quantified this impact using the rotational speed equation of motion in a small wind turbine informed by previous work. This governing equation has been simplified by expanding the aerodynamic torque coefficient profile for a wind turbine rotor to the tip-speed ratio. Furthermore, this governing equation is simplified by using non-dimensional forms of the air density, inflow wind velocity, and rotational speed with their characteristic values. In this study, the generator's load is set to be constant based on a previous analysis of a small wind turbine. By considering the equilibrium between the aerodynamic torque and the load torque of the governing equation at the optimum tip-speed ratio, the impact of the variation in the air density on the operating tip-speed ratio was expressed using a simple mathematical form. As shown in this derived form, the operating tip-speed ratio was found to be less sensitive to a variation in air density than that in inflow wind velocity.

1. Introduction

Wind energy is one of the promising renewable energy sources; hence it is essential to obtain knowledge critical to enhancing the operation and production of wind energy. There are two main types of wind turbines used for wind energy production: a large wind turbine and a small wind turbine [1]. The characteristics of wind energy using small wind turbines are different from those of large wind turbines. For example, pitch angle control, often used in large wind turbines, is not often used in small wind turbines [2-4]. In addition, there are various types of small wind turbines, such as a horizontal axis wind turbine and a vertical axis wind turbine [1]. Therefore, this study considers that obtaining a universal result is challenging in wind energy using small wind turbines. The density of the air flowing into the actuator disc of a small wind turbine may not be constant throughout the year



[5]. This seasonal air density variation depends on the temperature, pressure, and relative humidity of the air.

The impact of inflow wind velocity variation on wind power production using small wind turbines has often been studied. Karasudani et al. [6-8] used a mathematical form to study the rotational speed in response to a variation in inflow wind velocity. Suzuki and Hasegawa [9] derived the equation governing rotational speed in small wind turbines by applying a series expansion to a profile of the aerodynamic torque coefficient. Toshimitsu et al. [10] studied a small wind turbine with and without shroud using a wind tunnel experiment. In recent years, the impact of changes in air density on operating a wind turbine has been reported. Pourrajabian et al. [11] have shown the effects of change in air density caused by altitude-dependent changes in air pressure on the performance of a small wind turbine. Ulazia et al. [5] estimated the wind energy potential by focusing on seasonal changes in air density. Meanwhile, the rotational speed of a small wind turbine is reportedly controlled using a load of a generator rather than the pitch angle [2-4]. This load torque of the generator is set to equilibrium with the aerodynamic torque at the optimum tip-speed ratio. The value of air density may thus affect the equilibrium characteristic between the two terms in the governing equation. A previous study has attempted to reduce the effects of variation of air density on the performance of a small wind turbine by controlling the load torque [12].

Air density depends on the temperature, air pressure, and relative humidity, and these quantities vary throughout the year. Hence, the seasonal variation of these quantities subsequently influences air density. Moreover, the magnitude of seasonal variation is not always negligible. For example, the absolute value of seasonal variation in air density in Tokyo, Japan, is about 5%. On the other hand, the load characteristics of a generator may not vary throughout the year. Juxtaposing these facts imply that even if inflow wind velocity is constant throughout the year, the operating tip-speed ratio of a small wind turbine may not be kept constant. In addition, previous studies on the characteristics of small wind turbines have been conducted experimentally and numerically. Since there are a wide variety of small wind turbine types, the characteristics of small wind turbines are not universal. Therefore, this study considers that increasing the universality of results obtained by an investigation is essential to obtain knowledge about the characteristics of the small wind turbine.

The purpose of this study is to quantify the impact of seasonal variation in air density on the operating tip-speed ratio associated with a small wind turbine using a simplified equation of motion informed by previous studies. Here, this simplified equation is derived by expanding the aerodynamic torque coefficient in the vicinity of the optimum value of the tip-speed ratio. Using this equation to quantify this impact, we attempt to increase the universality of this study findings. Based on previous work, a generator load is kept constant for the rotational speed of a small wind turbine. As described later, by deriving a simple equation, the operating tip-speed ratio is clarified to be affected by both changes in air density and inflow wind velocity. Specifically, the operating tip-speed ratio is more sensitive to inflow wind speed than to air density.

2. Method

In this study, the equation of motion, expressed as follows, is used to describe the rotational speed of a small wind turbine [1,9].

$$I \frac{d\omega'}{dt^r} = T^r - T_L^r. \quad (1)$$

Here, I and t^r are the moment of inertia for the rotation axis of the wind turbine rotor and the dimensional time, respectively. Also, ω' is dimensional rotational speed of a wind turbine rotor. T^r and T_L^r are aerodynamic and load torques, respectively. The aerodynamic torque is determined by the aerodynamic characteristics of a wind turbine disc of a small wind turbine. The load torque is given by the load characteristics of the generator. This governing equation is also used in a previous study [9]. This study analyzes a case where the temporal variation of air density is sufficiently slow, and the

inflow wind velocity is steady. Furthermore, in the present case of a small wind turbine, the unsteady nature of the wake of a wind turbine can hardly affect the aerodynamic characteristics of the wind turbine rotor. Based on these, the aerodynamic torque coefficient included in the governing equation T^r can be given using the steady torque coefficient as introduced in the previous research [9].

$$T^r = \frac{1}{2} \rho' A R C_Q U'^2. \quad (2)$$

Here, ρ' , A , R , C_Q and U' are the air density, the area of actuator disc, rotor radius, the aerodynamic torque coefficient, and inflow wind velocity, respectively. This aerodynamic torque coefficient depends on the tip-speed ratio λ defined as follows.

$$\lambda = \frac{R\omega'}{U'}. \quad (3)$$

Here, the tip-speed ratio that often gives the maximum value of the aerodynamic torque coefficient is called the optimum tip-speed ratio λ_o . Inflow wind velocity and rotational speed which give the optimum tip-speed ratio are considered to be characteristic values, U_o' and ω_o' as follows.

$$\lambda_o = \frac{R\omega_o'}{U_o'}. \quad (4)$$

Also, there is a condition that $\partial C_Q / \partial \lambda \gg \partial C_Q / \partial \text{Re}$, used in the present analysis [9], where Re is the Reynolds number of the blade element.

This study approaches a small wind turbine operating with an optimum tip-speed ratio. Therefore, this study focuses on the fact that a variation in air density deviates the operating value of the tip-speed ratio from the optimum value λ_o . When this deviation of the operating tip-speed ratio is sufficiently slight, the relation between the aerodynamic torque coefficient value between the operating and optimum tip-speed ratios, λ and λ_o , can be approximated as follows.

$$C_Q(\lambda) = C_Q(\lambda_o) + \left(\frac{dC_Q}{d\lambda} \right) (\lambda - \lambda_o) + \dots. \quad (5)$$

Although a profile of the aerodynamic torque coefficient to the tip-speed ratio differs significantly among small wind turbines, the operating aerodynamic torque coefficient value can be described as a linear function using the deviation of the tip-speed ratio and the first derivative value of the aerodynamic torque coefficient as shown in the above equation. As shown in the previous study [9], since the higher-order terms following the right side of the above relation can be considered to have a negligible impact on the rotational speed, the higher-order terms are not used in this study. Furthermore, as described later, the operating tip-speed ratio deviation from the optimum value can be given in a form that does not include the aerodynamic torque coefficient. Therefore, in this study, a profile of aerodynamic torque coefficients is not required.

In this study, the load torque whose magnitude is kept constant is used as a generator load [11]. Using this constant load can simplify the governing equation and thus enable an analytical solution to the governing equation. When the small wind turbine is operated with the characteristic velocity and rotational speed, the magnitude of this constant load is equilibrated with that of the aerodynamic torque. Therefore, this constant load torque is given as follows.

$$T_L' = K \omega_o'^2, \text{ where } K = \frac{1}{2} \rho_o' A C_Q(\lambda_o) R^3 / \lambda_o^2. \quad (6)$$

Table 1. Seasonal values related to air density in Tokyo, Japan, where air density value of standard atmosphere is 1.225 [kg/m³].

Physical quantities	Summer	Winter
Air pressure [hPa]	1005.8	1012.8
Air temperature [°C]	22.4 - 29.9	1.2 - 9.8
Relative humidity [%]	76	51
Air density [kg/m ³]	1.143 - 1.177	1.245 - 1.285
Nondimensional air density [-]	0.933 - 0.961	1.016 - 1.049

Here, ρ_o' is the air density value given as a characteristic value in this study. In an actual small wind turbine, the value of this characteristic air density corresponds to that of the standard atmosphere.

Also, the following air density ratio is defined to investigate the impact of this variation in air density on the operating tip-speed ratio.

$$\rho = \rho' / \rho_o' \quad (7)$$

This non-dimensional ratio represents the magnitude of the air density value to the characteristic value. This study considers that making the physical quantity dimensionless. By using the non-dimensional physical quantities included in the governing equation, this study envisaged an analytical solution may be determined. This study defines the ratios of the inflow velocity and the rotational speed to the characteristic values, U and ω as follows.

$$U = U' / U_o' \text{ and } \omega = \omega' / \omega_o' \quad (8)$$

Based on the non-dimensional forms of inflow velocity and rotational speed, the operating tip-speed ratio can thus be expressed using the optimum value.

$$\lambda = \frac{\omega}{U} \lambda_o \quad (9)$$

3. Results and discussion

As stated in the Introduction, air density depends on the temperature, air pressure, and relative humidity. Table 1 present the temperature, pressure, and relative humidity mean values in the summer and winter periods measured in Tokyo by the Japan Meteorological Agency for over 30 years. Air density values were calculated from these mean values. As shown in the table, air density differs between summer and winter as a seasonal variation, and the amplitude magnitude of the change in air density is about 5% in Tokyo, Japan.

The approximate expression of the aerodynamic torque coefficient is used in this study based on the non-dimensional inflow wind velocity and rotational speed. By using this equation, the aerodynamic torque coefficient at the operating tip-speed ratio is expressed as follows.

$$C_Q(\lambda) = C_Q(\lambda_o) + a_1 \lambda_o \left(\frac{\omega}{U} - 1 \right) + \dots, \text{ where } a_1 = \left. \frac{dC_Q}{d\lambda} \right|_{\lambda_o} \quad (10)$$

Here, a_1 is a constant. Using the aerodynamic torque coefficient given by the above equation and the present form of constant load torque, the non-dimensional governing equation can be obtained as follows.

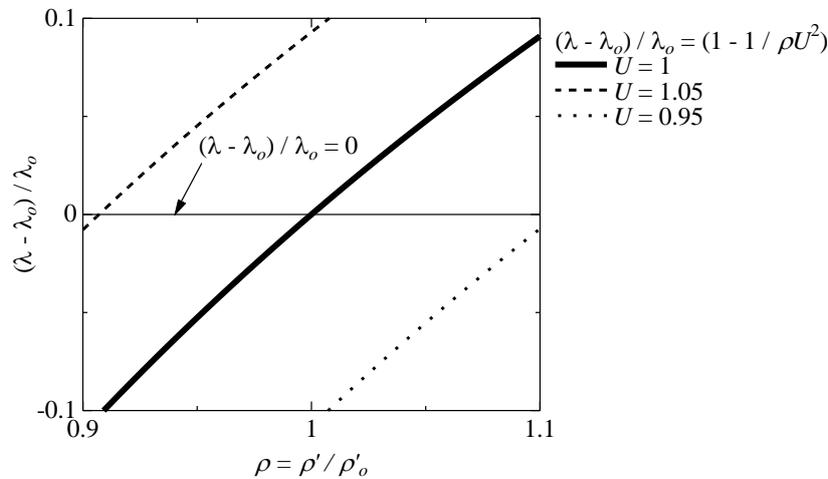


Figure 1. The quantified relative deviation of operating tip-speed ratio is derived as Eq. (13), where λ and λ_o are the operating and optimum tip-speed ratios, respectively. ρ and U are the non-dimensional density and inflow velocity, respectively.

$$\frac{d\omega(t)}{dt} = P + \omega(t)Q, \text{ where} \tag{11}$$

$$P = \frac{\lambda_o^2}{2T_o} a_1, Q = \frac{\lambda_o}{2T_o} \left[\left(1 - \frac{1}{\rho}\right) C_Q(\lambda_o) - a_1 \lambda_o \right] \text{ and } T_o = \frac{(1/2)I\omega_o'^2}{(1/2)\rho'ARU_o'^2}.$$

Here T_o included in the above equation represents a ratio of the rotational energy of a wind turbine rotor of small wind turbines to the volume integral of the kinetic energy of the inflow wind. Also, the present air density ratio ρ is included in Q in the above equation. This part of the form describes the influence of the air density ratio deviating from unity. The above equation is found to be a first-order linear differential equation. Therefore, this study considers that an analytical solution may be derived from this differential equation.

Since the rotation speed is steady in the small wind turbines approached in this study, the time change rate of the dimensionless rotation speed on the left side can be regarded as zero in the dimensionless governing equation. In addition, if the tip-speed ratio is maintained at the optimum value, the following simple relation holds.

$$1 = -\frac{a_1 \lambda_o}{C_Q(\lambda_o)}. \tag{12}$$

By using these relations with Eq. (11), the following relation can be obtained.

$$\lambda = \lambda_o + \lambda_o \left(1 - \frac{1}{\rho U^2}\right). \tag{13}$$

This relation quantifies the relative effects of the inflow wind speed and air density deviations from their respective characteristic values on the operating tip-speed ratio. In Figure 1, the second term in the above equation obtained in this study is shown as a function of the air density ratio. Consequently, it can be seen from the figure that the magnitude of the operating tip-speed ratio to the optimum value increases as the air density ratio increases. In Tokyo, Japan, the magnitude of the amplitude of the

variation of air density during the year is about 5%, similar in magnitude to the relative deviation of the tip-speed ratio due to the air density variation.

As shown in the figure, the operating tip-speed ratio is affected by the variation of air density and the inflow wind velocity. This study further investigated whether the operating tip-speed ratio is more sensitive to variations in air density or inflow wind velocity. The following relation can be obtained by expanding the above equation around the characteristic values of air density and inflow wind velocity.

$$\lambda = \lambda_o + \lambda_o [2(U - 1) + (\rho - 1)] + \dots \quad (14)$$

The value of the coefficient related to the deviation of the inflow wind velocity is larger than that of the air density. This result implies that a variation of inflow wind velocity may significantly affect the operating tip-speed ratio than air density in small wind turbines.

In an actual small wind turbine, the tip-speed ratio may not be the optimum value due to the ageing deterioration in the load torque of a generator. In this case, since Eq.(12) does not hold, this study considers that the following coefficient should be used to express the effect of air density variation.

$$\alpha = -\frac{a_1 \lambda_o}{C_Q(\lambda)} \quad (15)$$

Using this relation, the relative operating tip-speed ratio affected by variations in air density and inflow wind velocity is given as follows.

$$\lambda = \lambda_o + \lambda_o \left(1 - \frac{1}{\rho U^2}\right) + \lambda_o \left(1 - \frac{1}{\rho U^2}\right) (\alpha - 1) + \dots \quad (16)$$

Comparing the above equation with Eq. (13), this study can consider that the effects of the non-optimal tip-speed ratio value are described by the third term of the above right-hand side. On the other hand, if a coefficient deviates from unity does not affect the form itself of the equation that expresses the effects of the variations. This point suggests that the non-optimal tip-speed ratio hardly affect the effects of variations in air density and inflow wind speed qualitatively.

The effects of the air density variation on the operating tip-speed ratio in this study are further evaluated comparatively with previous studies. Pourrajabian et al. [11] reported that the change in air density caused by changes in air pressure depends on the altitude of the operating characteristics of small wind turbines. Their work involved the mean deviation of air density rather than the seasonal variation in air density. I.e., this study quantitatively investigated the effect of seasonal air density variation based on the impact of changes during the year rather than its mean deviation; hence, differentiating it from the previous study.

Evaluating this study from an aerodynamic torque coefficient perspective, the present relation describing the effect of variations in the air density and the inflow wind velocity on the operating tip-speed ratio does not include an aerodynamic torque coefficient itself, as shown in Eq.(13). Aerodynamic torque coefficient profiles vary among small wind turbines. This point implies that the present effect of these variations does not depend on the value of the aerodynamic torque coefficient in the small wind turbines. Therefore, this study considers that the present relation can have a high universality among small wind turbines. Also, a constant generator load was used in this study guided by a previous study [11]. Therefore, the present load of a generator is independent of rotational speed. Moreover, there are small wind turbines with a generator load that changes as a function of rotational speed. Since the load characteristics of these small wind turbines are different from those of this study, this study considers that the present result may not apply to those small wind turbines.

4. Conclusion

The purpose of this study was to investigate the effect of seasonal air density changes on the driving tip-speed ratio of small wind turbines quantitatively. Air density varies during the year, and the amplitude of this variation is about 5% in Tokyo, Japan. The governing equation of the rotational speed of the small wind turbines shown in the previous research was employed to investigate the aim of this study. The aerodynamic torque coefficient included in this equation is determined by approximating the deviation from the optimum peripheral speed ratio value. Also, the load torque of the generator is constant to the rotational speed, which has equilibrium with the aerodynamic torque coefficient at the optimum tip-speed ratio. Also, the governing equation was simplified by using non-dimensional forms of air density, inflow wind velocity, and rotation speed.

First, the air density values in summer and winter were calculated using temperature, atmospheric pressure, and relative humidity in Tokyo, Japan. Then, a relation was derived that represents the effect of air density variation on the operating tip-speed ratio. This relation showed that the operating tip-speed ratio was affected by both the variations of air density and inflow wind velocity. Also, the amplitude of the operating tip-speed ratio due to the air density variation during one year in Tokyo is known to be about 5%. By considering this relation, in small wind turbines, the operating tip-speed ratio is more sensitive to inflow wind velocity than the seasonal air density variation. Further, the present results were discussed, factoring in the optimum tip-speed ratio and air density change on the operating characteristics of small wind turbines and the aerodynamics and load torque coefficients.

The results of this study are obtained under the condition that the generator's load is constant to the rotation speed. Although, there are some actual small wind turbines whose generator load depends on the rotation speed. Consequently, results obtained in this study may not apply to such wind turbines. As a future study, the effect of seasonal changes in air density on the operating peripheral speed ratio of small wind turbines should be investigated using a generator load that depends on the rotational speed. In addition, the seasonal variation in air density may also affect the output power of small wind turbines. Elucidating this could also constitute future works in this research space.

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