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Resonances of electromagnetic-induced transparency and electromagnetic-induced absorption at the transition with level momenta J=1/2 in unidirectional wave spectroscopy

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Abstract. The physical processes that form the saturated absorption resonance spectra on the atomic transition with level momenta J=1/2 in the field of unidirectional waves of arbitrary intensities are investigated both analytically and numerically. It is shown that the narrow structures of the nonlinear resonance spectra (resonances of electromagnetic-induced transparency and absorption) and the processes forming them are determined by the direction of the light wave polarizations, degree of openness of the atomic transition, and the saturating wave intensity. The conditions under which the nonlinear resonance is exclusively coherent, due to the magnetic coherence of transition levels, are revealed.

1. Introduction

Studies of the nonlinear spectroscopic effects under the resonant interaction of several light fields with degenerate atomic transitions have been conducted for a long time. This interest is due to the variety of physical processes occurring in these systems, including the interference of atomic states, which manifests itself as narrow structures in the studied spectra. Note that the occurrence of atomic state coherence in two-photon processes has been known since pre-laser times. With the advent of lasers, the studies of such coherent phenomena have significantly expanded (see, for example, [1]). Subsequently, the resonances caused by the atomic state coherences in the presence of laser radiation were called as the electromagnetic-induced transparency (EIT) or electromagnetic-induced absorption (EIA) resonances. Many of the previously discovered phenomena in those years were "rediscovered" and renamed in the contributions on EIT and EIA. This, as well as the misconceptions in the interpretation of a number of results of those times, were drawn by us in the Introduction to the work [2].

An important example of coherent phenomena at transitions from the ground state of alkali metal atoms is the EIT resonances, which are based on the phenomenon of coherent captivity of level populations (CPT) [3, 4], as well as resonances of the opposite sign are the EIA resonances found in [5]. The occurrence of these EIA resonances was explained in [6] on the basis of the spontaneous transfer process of the level magnetic coherence (MC) of the excited atom state to the ground one, which manifestation regularities in saturated absorption spectroscopy were first considered in [7]. Subsequently, the observation of EIA resonances was reported in [8, 9]. However, the anomalies of the EIA resonances on degenerate atom transitions recorded in experiments were not always explained

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due to the level MC transfer mechanism [6], (in particular, [8, 9]). Therefore, to explain these effects, other processes, such as optical pumping and CPT [8], and collisions [10], were also considered and not always justified, but gave externally similar to the experiment resonance structures. Nevertheless, in the resent paper [11], in the development of the concept [6], the authors stated that the main reason for the formation of EIA resonances at closed transitions with any level moment values is precisely the spontaneous transfer of the level MC of the upper to the lower state.

However, our further studies have shown that when forming structures of nonlinear resonances at degenerate transitions, including EIT and EIA resonances, the other processes are more important than the process considered in [4, 6, 11]. In this case, the nature of processes depends on the values of the total level moment, the degree of openness of the atomic transition, the mutual directions of polarizations and the intensities of optical fields. Thus, it was shown in [12] that in a simple system of two levels, the narrow resonance structure in the field of two unidirectional waves manifested itself as an EIP resonance at the open transition, and as an EIA resonance at the closed transition. The reason for the appearance of these structures is the coherent beats of the level populations in the field of two frequencies [13, 14]. In the case of transitions with level momenta J=1, as established in [15], the narrow structures of the saturation absorption resonance are formed in the Λ -schemes of transition as EIT resonances, and their shapes are determined by the coherent level population beats in parallel polarizations of fields, as in [12], and by the nonlinear interference effect (NIEF) in orthogonal polarizations of fields [13]. The level MC effect forms the EIT and EIA resonances in the magnetic scanning spectra. In this case, the main contribution is made by the MC of lower state levels, and the contribution of the spontaneous transfer of coherence from the upper state to the lower one is small and manifests itself only in the form of an additive. The results of work [15] are also valid for the $J \rightarrow J$ and $J \rightarrow J - 1$ type of transitions, since the nonlinear resonance spectra at these transitions are formed also in the open Λ -schemes.

Another situation occurs on the $J\rightarrow J+1$ transition type where the nonlinear resonance spectrum is formed mainly in the V-transition schemes created by sublevels with the maximum magnetic number M [16, 17]. The closed two-level transitions are realized just in V-schemes, leading to the narrow spectrum structures in the form of EIA resonances [12]. The transfer effect of the level MC at these transitions also does not affect qualitatively on appearance of the narrow resonance structures. At the same time, it was found that the effect of probe field intensity can change the type of narrow structures (from EIT to EIA and *vice versa*) [16].

A review of the performed studies on the nonlinear and coherent effects in two-photon processes at various types of atomic transitions shows that the transition between levels with the total momenta J = 1/2 fell out from consideration when studying the effects. The present paper fills this gap. The simple level structure of this transition makes it possible to carry out an analytical analysis and establish quantitative relations between the processes that form the spectrum structures of the saturated absorption resonance in the probe field method and to determine the contributions of such processes as the saturation effect of level populations, coherent beatings of the level populations, induction of magnetic coherence of the levels by optical fields and its transfer between the levels of different states. The obtained relations are important for identifying the contributions of these processes in the more complex atomic systems, including clarifying the formation mechanism of the EIA resonance in [5].

2. The probe field absorption spectrum in a system of two levels with momenta J=1/2

Let us consider the problem of the probe field absorption spectrum in a gas of two-level atoms with level momenta J = 1/2 in the presence of a strong wave field. The scheme of transition levels is shown in figure 1. The strong wave is assumed to be plane, monochromatic, linearly polarized (frequency ω , wave vector **k**, electric field strength **E**) and resonant with the *m-n* atomic transition (transition frequency ω_{mn}). The probe wave is also monochromatic (frequency ω_{μ} , wave vector **k**_{μ}, electric field strength **E**) with the linear polarization directed parallel or orthogonal to the strong-field polarization. The gas is assumed to be sufficiently rarefied so that collisions can be neglected.

We will consider the problem in a coordinate system with the quantization axis directed along the

strength vector **E** of the strong wave (**E** along the Z-axis). In this coordinate system, a strong field induces transitions between magnetic sublevels with a change in the magnetic quantum number $\Delta M = 0$, and the probe field causes transitions with a change of $\Delta M = 0$ in case of parallel field polarizations, or with $\Delta M = \pm 1$ in case of orthogonal field polarizations (see figure 1).



Figure 1. Schematic diagram of kinetic processes at the transition with level momenta J=1/2. Solid and dashed arrows correspond to the transitions under the action of strong and probe field, respectively; dotted lines represent spontaneous transitions (velocities are A₁, A ₂and A_c); arc arrows represent the level magnetic coherence.

When solving the problem, we will use the kinetic equations for the density matrix of the atomic system [13]. In the case of a four-level quantum system (figure 1) interacting with a bichromatic (strong and probe) field, the kinetics of diagonal elements ρ_{mi} , ρ_{nk} and off-diagonal elements ρ_{ik} of the density matrix in the relaxation constant model are described by the system of equations:

$$\left(\frac{d}{dt} + \Gamma_m\right)\rho_{mi} = Q_{mi} - 2\operatorname{Re}(i\sum_k \rho_{ik}V_{ki}),\tag{1}$$

$$\left(\frac{d}{dt} + \Gamma_n\right)\rho_{nk} = Q_{nk} + \sum_i A_{ik}\rho_{mi} + 2\operatorname{Re}(i\sum_i \rho_{ki}V_{ik}), \qquad (2)$$

$$\left(\frac{d}{dt} + \Gamma_{ik}\right)\rho_{ik} = -i\sum_{j}\left(V_{ij}\rho_{jk} - \rho_{ij}V_{jk}\right) + \delta\rho_{ik}^{s},\tag{3}$$

Here, indices *i* and *k* denote the magnetic sublevels of the upper and lower states, Γ_m , Γ_n are the level relaxation constants; Γ_{ik} are the relaxation constants of polarizations on allowed ($\Gamma_{ik} = \Gamma$) and forbidden ($\Gamma_{ik} = \Gamma_m$, Γ_n) transitions between magnetic sublevels of *m* and *n* states; Q_{mi} , Q_{nk} are the excitation rates of these sublevels, assumed to be given; A_{ik} are the rates of spontaneous decay for the sublevels of the *m* state for each of the channels; $V = G \exp(-i\Omega t) + G^{\mu} \exp(-i\Omega_{\mu} t)$ is the atom interaction operator with the strong and probe fields, where $G = d_{mn} E/2\hbar$, $G^{\mu} = d_{mn} E_{\mu}/2\hbar$, and d_{mn} is the reduced matrix element of the transition dipole moment. The frequencies in the interaction

operator are written for fixed atoms: $\Omega = \omega - \omega_{mn}$, $\Omega_{\mu} = \omega_{\mu} - \omega_{mn}$. Taking into account the atomic motion it is confined to replacing in equations $\Omega \rightarrow \Omega - \mathbf{kv}$, $\Omega_{\mu} \rightarrow \Omega_{\mu} - \mathbf{k}_{\mu}\mathbf{v}$, where **v** is the atom

velocity vector. In the case of the ground state, Γ_n is replaced by the average transit width determined by the transfer size of light beams and the most probable particle velocity v_T .

The equation (3) for the non-diagonal elements of the density matrix includes a term $\delta \rho_{ik}^s$ that determines the spontaneous magnetic coherence transfer of the *m*-state sublevels to the *n*-state with decay rate A_c . This term is present in the equation for orthogonal polarizations and is absent for parallel polarizations of optical fields.

According to the standard procedure of the probe field method [13], the solutions of equations (1) - (3) with an accuracy up to the linear terms in G^{μ} , are found in the form:

$$\rho_{mi} = \rho_{mi}^{0} + r_{mi} \exp(-i\varepsilon t) + r_{mi}^{*} \exp(i\varepsilon t), \qquad (4a)$$

$$\rho_{nk} = \rho_{nk}^0 + r_{nk} \exp(-i\varepsilon t) + r_{nk}^* \exp(i\varepsilon t)), \qquad (4b)$$

$$\rho_{ik} = \rho_{ik}^{0} \exp(-i\Omega t) + r_{ik} \exp[-i(\Omega + \varepsilon)t] + r_{ik}^{1} \exp[-i(\Omega - \varepsilon)t], \qquad (4c)$$

or
$$\rho_{ik} = r_{jik} \exp(-i\varepsilon t] + r_{jik}^1 \exp(i\varepsilon t]$$
 (here $j = m, n$), (4d)

 $\varepsilon = \Omega_{\mu} - \Omega$.

Solutions (4a and 4b) describe the sublevel populations of the *m*-and *n*-states. Solution (4c) describes the polarization at the allowed transitions between the sublevels of *m*- and *n*-states, and solution (4d) describes the polarization at the forbidden transitions between sublevels of one state. Note that the polarization at the transitions between sublevels of the same state determines the so-called magnetic coherence (MC) of the levels.

In the considered system of levels (figure 1), we have the following relations on the spontaneous decay rates of magnetic sublevels A₁ and A₂ and the magnetic coherence A_c: A₁= A_{mn}/3, A₂=2A_{mn}/3, A₁+A₂ = A_{mn}, A_c = $-A_{mn}/3$, where A_{mn} is the first Einstein transition coefficient [18]. The matrix elements of the interaction operator G between the magnetic sublevels $mM_m \rightarrow nM_n$ are $G_{mM_m nM_n} = G_{mM_m nM_n}$ which we denote as: $G_{mM_m nM_n} = G_{mM_m nM_n} = G_{mM_m nM_n}$

 $G_{m\pm 1/2n\pm 1/2}$, which we denote as: $G_{m+1/2n+1/2} = G_+, G_{m-1/2n-1/2} = G_-, G_{m+1/2n-1/2} = G_+, G_{m-1/2n+1/2} = G_+$. The following relations are valid for these elements: $G_+ = -G_-, G_{+-} = -G_{-+}$.

We also concretize the density matrix elements of solution (4) and denote the population coefficients of the sublevels of *m*- and *n*- states as: $\rho_{mi}^0 = \rho_{m\pm}^0$, $r_{mi} = r_{m\pm}$, $\rho_{ni}^0 = \rho_{n\pm}^0$, $r_{ni} = r_{n\pm}$; the polarization coefficients at the allowed transitions between the sublevels of *m*- and *n*-states as: $\rho_{ik}^0 = \rho_{mn\pm}^0$, $r_{ik} = r_{\pm}$, $r_{ik}^1 = r_{\pm}^1$ (parallel polarizations of fields), or $r_{ik} = r_{\pm\mp}$, $r_{ik}^1 = r_{\pm\mp}^1$ (orthogonal polarizations of fields); and the polarization coefficients at forbidden transitions between sublevels of the same state as: $r_{jik} = r_{j\pm\mp}$, $r_{iik}^1 \rightarrow r_{i\pm\mp}^1$ (j = m, n).

According to the solution procedure of equations (1-3) by the probe field method for $G >> G^{\mu}$, the system of equations of zero-approximation at the transition between the magnetic sublevels with M = 1/2 has the form:

$$\Gamma_{m} \rho_{m+}^{0} + 2\operatorname{Re}(iG_{+}^{*} \rho_{mn+}^{0}) = Q_{m+},$$

$$\Gamma_{n} \rho_{n+}^{0} - A_{1} \rho_{m+}^{0} - A_{2} \rho_{m-}^{0} - 2\operatorname{Re}(iG_{+}^{*} \rho_{mn+}^{0}) = Q_{n+},$$

$$(\Gamma - i\Omega) \rho_{mn+}^{0} + iG_{+}(\rho_{m+}^{0} - \rho_{n+}^{0}) = 0.$$
(5)

The system of equations for similar quantities at transition between the sublevels with M = -1/2 has also the same form.

Due to the symmetry relative to the signs of the magnetic state numbers and the relations $\rho_{m+}^0 = \rho_{m-}^0$, $A_1+A_2 = A_{mn}$, the solutions of the equations (5) are the same as for the two-level system [18]:

$$\rho_{m+}^{0} = N_{m} + \left[\frac{N_{nm}}{\Gamma_{m}T_{mn}}\frac{\kappa\Gamma^{2}}{\Gamma_{s}^{2} + \Omega^{2}}\right], \quad \rho_{n+}^{0} - \rho_{m+}^{0} = N_{nm}\left[1 - \frac{\kappa\Gamma^{2}}{\Gamma_{s}^{2} + \Omega^{2}}\right], \tag{6}$$

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$$\rho_{mn\pm}^{0} = i G_{\pm} (\rho_{n+}^{0} - \rho_{m+}^{0}) / (\Gamma - i\Omega), \quad \rho_{m+}^{0} = \rho_{m-}^{0}, \quad \rho_{n+}^{0} = \rho_{n-}^{0}, \quad N_{nm} \equiv N_n - N_m.$$

where: $\Gamma_s = \Gamma \sqrt{1 + \kappa}$, $\kappa = 2|G|^2 \gamma_{mn} / \Gamma \Gamma_n \Gamma_m$, $\gamma_{mn} = \Gamma_m + \Gamma_n - A_{mn}$, (7)
and N_n , N_m are the populations of the sublevels of *n*- and *m*-states in the absence of a strong field.

The systems of equations of the next approximation in terms of G^{μ} , determining the probe field absorption spectrum, depend on the mutual direction of strong and probe field polarizations. In the case of parallel polarizations, the equations for the coefficients linear in G^{μ} from (4) for populations r_{m+} , r_{n+} and polarizations r_+ , r_+^1 at the $mM = 1/2 \rightarrow nM = 1/2$ transition have the following form:

$$(\Gamma_{m} - i\epsilon)r_{m+} + iG_{+}^{*}r_{+} - iG_{+}r_{+}^{1*} = iG_{+}^{\mu}\rho_{nm+}^{0},$$

$$(\Gamma_{n} - i\epsilon)r_{n+} - A_{1}r_{m+} - A_{2}r_{m-} - iG_{+}^{*}r_{+} + iG_{+}r_{+}^{1*} = -iG_{+}^{\mu}\rho_{nm+}^{0},$$

$$(p - i\epsilon)r_{+} + iG_{+}(r_{m+} - r_{n+}) = -iG_{+}^{\mu}(\rho_{m+}^{0} - \rho_{n+}^{0}),$$

$$(p^{*} - i\epsilon)r_{+}^{1*} + iG_{+}^{*}(r_{m+} - r_{n+}) = 0, \text{ where } p = \Gamma - i\Omega.$$

$$(8)$$

For the $mM = -1/2 \rightarrow nM = -1/2$ transition, we have a similar (8) system of equations, but with the replacement of the signs $+ \leftrightarrow -$ in the indices.

The system of equations (8) is uniquely solvable relative to the unknown density matrix coefficients [18]. The solution for the r_+ coefficient is as follows:

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$$r_{+} = (\mathbf{p} - i\varepsilon)^{-1} [i G_{+}^{\mu} (\rho_{n+}^{0} - \rho_{m+}^{0}) + iG_{+}(r_{n+} - r_{m+})] = i G_{+}^{\mu} \frac{\mathbf{N}_{nm}}{p - i\varepsilon} (1 - \frac{\mathbf{M}^{-1}}{\Gamma_{s}^{2} + \Omega^{2}}) (1 - \mathbf{J}_{\parallel}(\varepsilon)), \quad (9)$$

where:
$$\mathbf{J}_{\parallel}(\varepsilon) = (\left|G_{+}\right|^{2} / \Delta_{\varepsilon})(2\Gamma - i\varepsilon)(\Gamma_{m} + \Gamma_{n} - \mathbf{A}_{mn} - 2i\varepsilon)[\Gamma - i(\varepsilon - \Omega)] / (\Gamma + i\Omega),$$
$$\Delta_{\varepsilon} = [\Gamma - i(\varepsilon + \Omega)][\Gamma - i(\varepsilon - \Omega)](\Gamma_{m} - i\varepsilon)(\Gamma_{n} - i\varepsilon) + 2(\Gamma - i\varepsilon)(\Gamma_{m} + \Gamma_{n} - \mathbf{A}_{mn} - 2i\varepsilon) \left|G_{+}\right|^{2}. \quad (10)$$

In the case of orthogonal field polarizations, the equations that determine the values linear in G^{μ} are formed from (4) by the density matrix coefficients r_{+-} , r_{+-}^1 , r_{-+} , r_{-+}^1 at allowed transitions and by the coefficients r_{j+-} , r_{j+-}^1 , r_{j-+} , r_{j-+}^1 , r_{j-+}^2 , r_{j

$$(\Gamma_{n} - i\epsilon)r_{m+-} + iG_{-}r_{+-} - iG_{+}r_{+-} = iG_{+-}\rho_{nm-},$$

$$(\Gamma_{n} - i\epsilon)r_{n+-} - A_{c}r_{m+-} - iG_{+}r_{n+-} + iG_{-}r_{+-}^{1} = -iG_{+-}^{\mu}\rho_{nm+}^{0},$$

$$(p - i\epsilon)r_{+-} - iG_{+}r_{n+-} + iG_{-}r_{m+-} = -iG_{+-}^{\mu}(\rho_{m+}^{0} - \rho_{n-}^{0}),$$

$$(p^{*} - i\epsilon)r_{+-}^{1} + iG_{-}r_{n+-} - iG_{+}r_{m+-} = 0.$$
(11)

The term $A_c r_{m+-}$ in the second equation of (11) describes the level MC transfer effect of the upper state to the lower one. The equations for the coefficients at the $m M = -1/2 \rightarrow n M = 1/2$ transition are obtained from equations (11) by replacing the signs $+ \leftrightarrow -$ in the indices. Both systems of equations are solvable uniquely with respect to the coefficients.

The solution of the equation system (11) gives the following value for coefficient r_{+-} :

$$\mathbf{r}_{+-} = i G_{+-}^{\mu} \frac{\mathbf{N}_{nm}}{p - i\varepsilon} \left(1 - \frac{\kappa \Gamma^2}{\Gamma_s^2 + \Omega^2} \right) (1 - \mathbf{J}_{\perp}(\varepsilon)), \tag{12}$$

where:
$$J_{\perp} = (|G_{+}|^{2}/\Delta_{\perp})(2\Gamma - i\epsilon)(\Gamma_{m} + \Gamma_{n} + A_{c} - 2i\epsilon)[\Gamma - i(\epsilon - \Omega)]/(\Gamma + i\Omega),$$

$$\Delta_{\perp} = [\Gamma - i(\epsilon + \Omega)][\Gamma - i(\epsilon - \Omega)](\Gamma_{m} - i\epsilon)(\Gamma_{n} - i\epsilon) + 2(\Gamma - i\epsilon)(\Gamma_{m} + \Gamma_{n} + A_{c} - 2i\epsilon)|G_{+}|^{2}.$$
(13)

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The probe field absorption spectrum is determined by its work as: $P_{\mu} = -4\hbar \omega_{\mu} \text{Re} \langle i \sum r_{+} G_{+}^{\mu*} \rangle$

(for parallel polarizations), or $P_{\mu} = -4\hbar \omega_{\mu} \text{Re} \langle i \sum r_{+-} G_{+-}^{\mu*} \rangle$ (for orthogonal polarizations of fields)

[13]. Using solutions from (9, 12), we obtain the following expressions for the probe field work:

$$P_{\mu\parallel} = 4\hbar \omega_{\mu} \left| G_{+}^{\mu} \right|^{2} \operatorname{Re}\left[\frac{N_{nm}}{\Gamma - i\Omega_{\mu}} \left(1 - \frac{\kappa \Gamma^{2}}{\Gamma_{s}^{2} + \Omega^{2}} \right) (1 - J_{\parallel}(\varepsilon)) \right], \text{ (parallel polarizations);}$$
(14)

$$P_{\mu\perp} = 4\hbar\omega_{\mu} \left| G_{+-}^{\mu} \right|^{2} \operatorname{Re}\left[\frac{N_{nm}}{\Gamma - i\Omega_{\mu}} \left(1 - \frac{\kappa\Gamma^{2}}{\Gamma_{s}^{2} + \Omega^{2}} \right) (1 - J_{\perp}(\varepsilon)) \right], \text{ (orthogonal polarizations).}$$
(15)

Expressions (14, 15) describe the absorption spectra of the probe field in a system with the level momenta J = 1/2, due to the incoherent saturation effect of the level populations and coherent processes, that lead to resonances of electromagnetic-induced transparency (EIT) or electromagnetic-induced absorption (EIA). The differences between expressions (14) and (15) appear only in the coherent terms $J_{\parallel}(\varepsilon)$ and $J_{\perp}(\varepsilon)$. These differences are significant at transitions with the relaxation constants ratio of $\Gamma_m \gg \Gamma_n$ in the case of unmoved atoms, and when taking into account the atomic motion, in the case of unidirectional light waves, due to difference of terms $J_{\parallel}(\varepsilon)$ and $J_{\perp}(\varepsilon)$. In the case of counter-directed waves, the contribution of coherent processes to the probe field absorption shape is known to be insignificant and appears only at a large saturating field ($\kappa \gg 1$) [13, 19].

2.1. The case of unmoved atoms

In the case of unmoved atoms, for level relaxation constant ratios $\Gamma_m \gg \Gamma_n$ and parallel polarizations of fields the probe field work (absorption spectrum) in a weak saturation field ($\kappa \ll 1$) is determined from (14) near the line center ($\epsilon \ll \Gamma$, Γ_m ; $\Omega = 0$), as in the two-level system [12], by the expression:

$$\mathbf{P}_{\mu\parallel} = 4\hbar \omega_{\mu} \left| G_{+}^{\mu} \right|^{2} \operatorname{Re} \left\{ \frac{\partial \mathbf{N}_{nm}}{\Gamma - i\varepsilon} \left[1 - \frac{2|\mathbf{G}_{+}|^{2}}{\Gamma \Gamma_{m}} \left(\frac{\Gamma_{m} + A_{mn}}{\Gamma_{m}} + \frac{\Gamma_{m} - A_{mn} - \Gamma_{n}}{\Gamma_{n} - i\varepsilon} \right) \right] \right\}, \quad (16)$$

where: $\partial \mathbf{N}_{nm} = \mathbf{N}_{nm} \left(1 - \frac{\kappa \Gamma^{2}}{\Gamma^{2} + \Omega^{2}} \right) = \frac{\mathbf{N}_{nm}}{1 + k_{0}}, \ \kappa_{0} = \kappa \Gamma^{2} / (\Gamma^{2} + \Omega^{2}).$

In this case, the absorption spectrum has a Lorentzian shape with a half-width of Γ and a structure with a half-width of the lower level Γ_n near the field frequency detunings $\varepsilon = 0$. The structure amplitude is determined on the value $S_{\parallel} = \Gamma_m - \Gamma_n - A_{mn}$ that, depending on the relations between the values of relaxation constants and the first Einstein coefficient A_{mn} , can change its sign. When the ratio of constants is $\Gamma_m - A_{mn} > \Gamma_n$, the S_{\parallel} value is positive (> 0), and the structure manifests itself as a narrow dip (EIT resonance). When the ratio $\Gamma_m - A_{mn} < \Gamma_n$, the S_{\parallel} value is negative (< 0), and the structure manifests itself as a peak (EIA resonance). When $A_{mn} = \Gamma_m - \Gamma_n$ there is no narrow structure in the probe field spectrum, and the nonlinear resonance shape is determined by the population term in (16).

In the case of orthogonal field polarizations, the probe field work under the same conditions will be determined from (15) by the following expression:

$$\mathbf{P}_{\mu\perp} = 4\hbar\omega_{\mu} \left| G_{+-}^{\mu} \right|^{2} \operatorname{Re}\left\{ \frac{\partial N_{nm}}{\Gamma - i\varepsilon} \left[1 - \frac{2|G_{+}|^{2}}{\Gamma\Gamma_{m}} \left(\frac{\Gamma_{m} - A_{c}}{\Gamma_{m}} + \frac{\Gamma_{m} + A_{c} - \Gamma_{n}}{\Gamma_{n} - i\varepsilon} \right) \right] \right\}.$$
 (17)

It follows from (17) that a narrow Lorentz type structure with a half-width of Γ_n and an amplitude determined by the multiplier $S_{\perp} = \Gamma_m - \Gamma_n + A_c$ is also formed in the absorption line shape. Since at considered transition $A_c = -A_{mn}/3$ (see above), then at the constant ratios $\Gamma_m \gg \Gamma_n$, the value $S_{\perp} \approx \Gamma_m - A_{mn}/3 > 0$ is always positive, and the structure manifests itself at any transitions as a narrow dip (EIT resonance).

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The ratio of the narrow structure amplitudes is determined from relations (16) and (17) by the value $\delta S = (\Gamma_m - \Gamma_n - A_{mn})/(\Gamma_m - \Gamma_n - A_c)$. For the constant ratios $\Gamma_m >> \Gamma_n$ and $\Gamma_m - A_{mn} > \Gamma_n$ (open transitions), this value is equal to $\delta S \approx (\Gamma_{mn} - A_{mn})/(\Gamma_m + A_{mn}/3) < 1$, i.e., the difference in the amplitudes of the coherent dips (EIT resonances) is small, in particular, for the branching parameter $a_0 = 0.5 \ \delta S \approx 0.4$. In the case of $\Gamma_m = A_{mn}$ (closed transitions), the amplitude ratio is as follows: $\delta S \approx 3\Gamma_n/4\Gamma_m >>1$, i.e. the amplitude of the coherent dip (EIT resonance) in orthogonal polarizations significantly exceeds the coherent peak amplitude (EIA resonances) in parallel polarizations of fields.

In the case of orthogonal field polarizations, the contribution of the level MC transfer process from the upper to the lower state in the EIT resonance is determined from (17) as $A_c / (\Gamma_m - \Gamma_n) \approx -A_{mn} / 3\Gamma_m$. This means that the MC transfer leads to an increase in the absorption line center (since $A_c < 0$). At the same time, the maximum change in the absorption shape is realized on the closed transitions and is equal to 30% of the EIT resonance amplitude due to the MC of levels. The change in the absorption shape on the open transitions is significantly less (< 10%).

2.2. The case of moving atoms

In the case of unidirectional waves, when averaging expressions (14, 15) over velocities with the large Doppler broadening ($kv_T \gg \Gamma_s$) and in the approximation of the first nonlinear disturbances over the saturating field ($\kappa \ll 1$), the probe wave work in the region of field detuning frequencies ($\Omega_{\mu} \ll kv_T$, $\Omega \ll kv_T$) is described by the following expressions:

$$\langle \mathbf{P}_{\mu\parallel,\perp} \rangle = 4\hbar \omega_{\mu} \left| G^{\mu}_{+,+-} \right|^2 \langle \mathbf{N}_{nm} \rangle \frac{\sqrt{\pi}}{k \mathbf{v}_{T}} \exp[-(\Omega_{\mu}/k \mathbf{v}_{T})^2] \operatorname{Re}(\mathbf{F}_{\parallel,\perp}(\varepsilon)), \tag{18}$$

where:
$$F_{\parallel,\perp}(\varepsilon) = \frac{\kappa \Gamma}{2\Gamma - i\varepsilon} - \frac{2|G|^2 \eta_{\parallel,\perp}(\varepsilon)}{2\Gamma - i\varepsilon} ,$$
 (19)

$$\eta_{\parallel} = \frac{\Gamma_m + \Gamma_n - A_{mn} - 2i\varepsilon}{(\Gamma_m - i\varepsilon)(\Gamma_n - i\varepsilon)} = \frac{\Gamma_m - \Gamma_n + A_{mn}}{\Gamma_m - \Gamma_n} \cdot \frac{1}{\Gamma_m - i\varepsilon} + \frac{\Gamma_m - \Gamma_n - A_{mn}}{\Gamma_m - \Gamma_n} \cdot \frac{1}{\Gamma_n - i\varepsilon}, \quad (20)$$

$$\eta_{\perp} = \frac{\Gamma_m + \Gamma_n + A_c - 2i\varepsilon}{(\Gamma_m - i\varepsilon)(\Gamma_n - i\varepsilon)} = \frac{\Gamma_m - \Gamma_n - A_c}{\Gamma_m - \Gamma_n} \cdot \frac{1}{\Gamma_m - i\varepsilon} + \frac{\Gamma_m - \Gamma_n + A_c}{\Gamma_m - \Gamma_n} \cdot \frac{1}{\Gamma_n - i\varepsilon} \cdot (21)$$

It follows from expressions (18 - 21) that, in a weak saturating field, a resonance is formed on the Doppler absorption line of the probe wave as a hole with a half-width of 2Γ centered on the frequency difference $\varepsilon = 0$. In this case, the structures of an interference nature are formed near the line center, the shapes of which are determined by the multipliers $\eta_{\parallel,\perp}(\varepsilon)$, depending on the transition relaxation constants and the directions of field polarizations. The shapes of the resonance structures differ significantly at the transitions with a significant difference in the level relaxation constants (at $\Gamma_m \gg \Gamma_n$). In this case, the multipliers $\eta_{\parallel,\perp}(\varepsilon)$ are representable near the line center (for $\varepsilon/\Gamma_m \ll 1$) as:

$$\eta_{\parallel}(\varepsilon) \approx \frac{1}{\Gamma_{m}} \left(\frac{\Gamma_{m} + A_{mn}}{\Gamma_{m}} + \frac{\Gamma_{m} - \Gamma_{n} - A_{mn}}{\Gamma_{n} - i\varepsilon} \right),$$

$$\eta_{\perp}(\varepsilon) \approx \frac{1}{\Gamma_{m}} \left(\frac{\Gamma_{m} - A_{c}}{\Gamma_{m}} + \frac{\Gamma_{m} - \Gamma_{n} + A_{c}}{\Gamma_{n} - i\varepsilon} \right).$$
(22)
(23)

It follows from relations (22, 23) that, in the line center of a wide dip (19) the narrow Lorentz type structures with a half-width of Γ_n are formed. The amplitudes of the structures and their signs are determined, as in the case of unmoved atoms, by the level relaxation constant values, the first Einstein coefficient A_{mn} and the relaxation constant A_c of the level MC from the upper state.

In the case of parallel field polarizations and the ratios of relaxation constants $A_{mn} < \Gamma_m - \Gamma_n$ (open transitions), the structure amplitude in (22) is positive, and the structure manifests itself in the shape of a resonance (18) as a narrow dip (EIT resonance) against the background of a wide hole. When the

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constant ratios $\Gamma_m \ge A_{mn} > \Gamma_m - \Gamma_n$ (closed transitions), the amplitude of the structure in (22) is negative, and the structure manifests itself as a narrow absorption peak (EIA resonance). At the constant ratio $A_{mn} = \Gamma_m - \Gamma_n$ there is no narrow structure shaped as a resonance (18).

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In case of orthogonal field polarizations and $\Gamma_m >> \Gamma_n$ the value of multiplier in (23) $\Gamma_m - \Gamma_n + A_c = \Gamma_m - \Gamma_n - A_{mn}/3$ is always positive, and the structure at any transitions manifests itself shaped as a narrow dip (EIT resonance).

Let us compare the contributions of populational and coherent terms to the resonance (18) near the line center. For parallel polarizations of fields, the values of these contributions are as follows: $(1+2\Gamma_n/(\Gamma_m+\Gamma_n-A_{mn}))$: $(\Gamma_m-\Gamma_n-A_{mn})/(\Gamma_m+\Gamma_n-A_{mn})$. Hence, at the constant values $\Gamma_m \gg \Gamma_n$, the ratio of the contribution of amplitudes at the open transitions with the branching parameter $a_0 = 0.5$ is equal to one, and on the closed transition ($a_0 = 1$), the ratio of amplitudes of the wide dip and the narrow peak is correlated as 3:1. These relations are observed also in the two-level system [12].

In the case of orthogonal field polarizations, the ratio of the populational and coherent amplitudes of dip is defined as: $(1+2\Gamma_n/(\Gamma_m+\Gamma_n-A_{mn}))$: $(\Gamma_m-\Gamma_n-A_{mn}/3)/(\Gamma_m+\Gamma_n-A_{mn})$. At the open transition with $a_0 = 0.5$, this ratio correspond approximately to 1:2, and at the closed transition $(a_0 = 1)$ the contribution of the level MC to the resonance amplitude will significantly exceed the contribution of the incoherent term (at the ratio of $\Gamma_m/3\Gamma_n$ times). In this case, the contribution of the level MC transfer process from the upper state to the lower one to the resonance amplitude (18) is determined by the value $A_c/(\Gamma_m-\Gamma_n)$. At a value $A_c= -A_{mn}/3$, the contribution of the level MC transfer leads to an increase in the absorption line center (due to $A_c < 0$), and its maximum value is realized at the closed transition and is equal to ~ 30% of the resonance amplitude due to the level MC.

Thus, the EIT resonance at a closed transition between levels with momenta J = 1/2 with ortho field polarizations in a weak saturating field is exclusively coherent (interference) in its nature and is caused by the MC of the transition levels. The resonance will also have a coherent character in a strong saturating field, and it will be demonstrated below.

2.3. Numerical simulation of the nonlinear resonance shape in a saturating field of arbitrary intensity in unidirectional waves

To identify the features of the saturated absorption resonance of the probe wave and the processes forming its shape, we performed numerical simulations of the probe wave absorption spectrum based on the exact solution according to formulas (14, 15) when the saturating intensity and the atomic transition parameters changed in a wide range of values. When modeling the shapes of resonances, the contributions of the incoherent saturation effect of the level populations and coherent processes, such as beating of the level populations (for para polarizations), or effects of the level MC, induced by the optical fields, and its transfer from the upper to the lower state (for ortho polarizations of fields), as well as the effect of level splittings by the saturating field, were determined.

The shapes of absorption lines (calculated for per atom) were determined by the relation: $\alpha_{\mu} / \alpha_{0} = \frac{\Gamma}{4\hbar\omega_{\mu}} \left\langle P_{\mu\parallel,\perp} / \left| G_{+,+-}^{\mu} \right|^{2} \right\rangle$, where <...> denotes averaging by the Maxwell distribution of the

particle velocities, $\alpha_0 = \sigma_0 N_{mn}$ and $\sigma_0 = 4\pi \omega_{mn} d^2/(c \hbar \Gamma)$ is the resonant absorption cross section.

The calculations were carried out at the following parameters of atomic transitions: $\Gamma_m = 5.5 \cdot 10^7 \text{ c}^{-1}$, $\Gamma_n = (10^{-2} \div 1) \Gamma_m$, $\Gamma = (\Gamma_m + \Gamma_n)/2$; the ratio of the initial level populations was $N_m/N_n \sim 10^{-2}$, and the Doppler line width was assumed to be $kv_T = 5 \cdot 10^9 \text{ c}^{-1}$. When integrating, the particle velocity range was $\pm 3kv_T$ with a step of $\Delta kv_T = (10^{-3} \div 10^{-4})kv_T$, the strong field saturation parameter κ varied within the range of $\kappa = 0.01 \div 50$, and the transition branching parameter value $a_0 = A_{mn}/\Gamma_m$ varied in the range of $a_0 = 0 \div 1$.

The calculations revealed significant differences in the spectra of resonances and processes forming them for the open $(a_0 < 1)$ and closed $(a_0 = 1)$ atomic transitions. The characteristic behavior of the saturated absorption resonance shapes and the contributions of the incoherent saturation effect of the

level populations are shown for two types of transitions: for open ($a_0 = 0.5$, figure 2) and closed ($a_0 = 1$, figure 3) transitions with parallel and orthogonal polarizations of fields, half-width ratio $\Gamma_n/\Gamma_m = 2 \cdot 10^{-2}$ and values of the saturation parameter in the range of $\kappa = 0.01 \div 10$.

In the case of open transitions (figure 2), both for parallel and orthogonal polarizations of fields a resonance is formed on the Doppler absorption contour of the probe wave shaped as a wide hole and a narrow structure (dip) near the line center. From the above dependences of the processes forming the resonance spectrum, it is clear that the wide hole is due to the incoherent saturation effect of the level populations by a strong field action and due to the coherent processes that occur only near the line center (at $\varepsilon \sim 0$) shaped as a narrow dip structure (EIT resonance) and a low-amplitude lining. The calculations showed that, at low saturation parameters $\kappa \leq 0.1$ (curves 1), the narrow resonance structure (EIT resonance) is due to the coherent beats of the level populations (at parallel polarizations of fields) and to the MC between the magnetic sublevels of the same state induced by the fields of unidirectional waves (at orthogonal polarizations of fields). In this case, the width of narrow structures (EIT resonances) are approximately the same and are determined by the width of the long-lived lower level, and the amplitude of the structure at orthogonal polarizations is still greater than the amplitude of the structure at parallel polarizations of fields. The maximum ratio of these amplitudes at the saturation parameters $\kappa \leq 0.1$ was ~ 2 .



Figure 2. Shapes of the population part (dotted line) and total resonance on the open transition ($a_0 = 0.5$) at $\Omega = 0$, $\Gamma_n / \Gamma_m = 0.02$, $\kappa = 0.1$ (1), 0.5 (2), 1.0 (3), 5.0 (4), 10 (5); solid lines are parallel polarizations, dashed lines are orthogonal polarizations of fields.

An increase in the strong wave intensity (in the range of saturation parameters $\kappa = 0.01 \div 10$) lead to an increase in the width and amplitude of the population part of a resonance, as in the case of a twolevel system [13, 19], and to an increase in the widths and amplitudes of narrow coherent structures (EIT resonances), as well as to a decrease in the ratio between the amplitudes of narrow structures and

their contrast with respect to the incoherent dip. At the same time, in the case of saturation parameters $\kappa > 1$, a small amplitude lining appears in the spectrum of narrow structures (see curves 3-5). It is established from the processing of the spectra of figure 2 that the half-widths of coherent dips (EIT resonances) obey to the linear law as $\Gamma = \Gamma_n(1+\alpha\kappa_s)$, where $\alpha \approx 1$ (for parallel polarizations, as in [20], or $\alpha \approx 2$ (for orthogonal polarizations of fields). The estimation of the broadening coefficient from the relations (12, 13) for the transition parameters used in the calculations gives a value $\alpha \approx 5/3$.

In the case of closed transitions, the probe wave absorption resonance shapes (figures 3, 4) turn out to be qualitatively different, depending on the directions of light wave polarizations, and are mainly due to the contributions of coherent processes. If the incoherent process, as in open transitions, forms a wide hole with the half-width of Γ in the absorption spectrum (figure 3, dotted curves), then the coherent processes form a complex contour (figure 4) containing narrow structures near the line center (at $\varepsilon = 0$), which appear as an additional dip (EIT resonance), in case of orthogonal polarizations (dashed curves), and as a peak of small amplitude (EIA resonance) in case of parallel polarizations of fields (continuous curves). At the same time, at low saturation parameters ($\kappa < 0.1$, curves 1, 2), the contribution of the saturation effect to the absorption resonance is small, and a resonance spectrum is formed exclusively by the coherent processes. For parallel polarizations of fields, these are the beats of the level populations forming a narrow peak structure (EIA resonance), and, these are the MC of the levels forming a narrow dip structure (EIT resonance for orthogonal polarizations). Moreover, the EIT resonance amplitude significantly exceeds the EIA resonance amplitude.



Figure 3. Shapes of population part (dotted line) and total resonance on the closed transition ($a_0 = 1$) at $\Omega = 0$, $\Gamma_n / \Gamma_m = 0.02$, $\kappa = 0.01$ (1), 0.1 (2), 0.5 (3), 1.0 (4), 5 (5); solid lines are parallel polarizations, dashed lines are orthogonal polarizations of fields.

An increase in the strong wave saturation parameter values of $\kappa = 0.01 \div 10$ lead, as in the case of open transitions, to an increase in the amplitude and width (according to the law close to the square root of κ) of the incoherent part of the resonance, but it manifested itself differently in the spectra of coherent processes (see figures 3, 4).

In the case of orthogonal field polarizations and saturation parameters $\kappa = 0.01 \div 0.5$ (figure 4, curves 1-3), the amplitude of the EIT resonance and its width change (according to the linear law in κ), and, at saturation parameters $\kappa \sim 1$ (curve 4), the additional structures appear in the line wings. The frequency interval between the maxima of these structures is $\Delta \omega \sim 10^{-2} \Delta \omega_D \approx \Gamma_m \approx 2 \Gamma$, and that corresponds roughly to the transition width used in the calculations. With a further increase in the saturation parameter, the resonance shape is represented as three spectral components (curves 4, 5). With an increase of κ , the amplitude of the central component decreases and the amplitudes of the extreme components increase in their magnitude. The frequency distance between their maxima obeys the root dependence on parameter κ (linear dependence on the interaction parameter *G*). This fact indicates that the extreme spectrum components are caused by the field splitting of transition levels.



Figure 4. Shapes of coherent part of the resonance on the closed transition ($a_0=1$) at $\Omega = 0$, $\Gamma_n/\Gamma_m = 0.02$, $\kappa = 0.01$ (1), 0.1 (2), 0.5 (3), 1.0 (4), 5.0 (5); solid lines are parallel polarizations, dash-dotted line are orthogonal polarizations of fields.

In the case of parallel field polarizations with an increase of parameter κ , the amplitude of the EIA resonance and its contrast, as in the system of two levels [20], first increase also, reach their maximum at values of $\kappa \sim 0.5 \div 1$ and then decrease. In this case, there is a slight narrowing of the EIA resonance spectrum. At saturation parameters $\kappa > 1$, an increase in the amplitudes and widths of the minima in the line wings is observed, as well as their shift relative to the line center. The shift magnitude obeys, as in the case of orthogonal polarizations of fields, the root dependence on parameter κ values.

Note that, for saturation parameters $\kappa > 1$, the shapes of resonant wings (for detuning frequencies $\epsilon > 2\Gamma$) are almost the same for both orthogonal and parallel field polarizations. In this case, the frequency shifts of the wing maxima obey to the root dependence on parameter κ . These facts indicate that at saturation parameters $\kappa > 1$, the spectrum in the resonance wings is caused by the field splitting of transition levels (a coherent process), and, near the line center, it is caused by the level population

beats (with parallel polarizations), or by the induced MC of levels (with orthogonal polarizations of fields).

A change in the lower level width (at $\Gamma_n \rightarrow \Gamma_m$, $\Gamma_{mn} \rightarrow \Gamma_m$) leads at any transitions to an increase in the widths of the main resonance hole and its narrow coherent structures with a decrease in their amplitudes at any polarizations of fields.

Calculations of the field splitting dependences in the MC transfer spectra according to formula (15) showed that the splitting value in the range of saturation parameters $\kappa = 0.01 \div 1$ exceeds the root dependence, but it is less than the linear one. In this case, the minimum splitting value in the MC transfer spectrum is determined by the relaxation constant Γ_n and is due to the field splitting of the lower transition levels. Thus, the level MC transfer spectrum is more sensitive to the saturating field intensity, since the field splitting effect begins to manifest itself already at the splitting of the lower long-lived levels here, while the splitting in the spectrum of the total absorption resonance (figure 4) manifests itself at the splitting values larger than the uniform transition width, that is determined, in the absence of collisions, by the relaxation constants of both state levels [13].

3. Conclusion

The presented analytical and numerical studies of the saturated absorption spectra in the probe field method of unidirectional light waves on transition with level momenta J = 1/2 demonstrate their dependence on the level relaxation constant values, the degree of transition openness, the intensities and the mutual orientation of strong and probe wave polarizations. In this case, the mutual orientation of wave polarizations is decisive in the formation of the saturated absorption spectra, since the specific features of the spectra are determined by the contributions of coherent processes, the character of which at a considered transition depends on the directions of the light wave polarizations.

In the case of parallel wave polarizations, these are coherent beats of the level populations in two level transition schemes, and their specific relaxation determines the type of narrow resonance structures: a peak (EIA resonance) at a closed transition and a dip (EIT resonance) at an open transition.

In the case of orthogonal wave polarizations, this is the magnetic coherence (MC) of levels induced by the light fields and its transfer from the upper to the lower state, and that creates a narrow structure shaped as a dip (EIT resonance) at any transitions. Moreover, the main contribution to the resonance amplitude is made by the lower state levels. The contribution of the magnetic coherence transfer from the upper state to the lower one is small and manifests itself only in the form of an additive near the line center. At the same time, the shape of the MC transfer has an alternating interference character inherent to coherent processes. The MC transfer spectrum is more sensitive to the saturating field intensity, since, here, the level splitting effect begins to manifest itself already when the lower longlived state levels are split, while, in the full resonance spectrum, this effect manifests itself in the wings of resonance when all transition levels are split larger than the uniform transition width.

Note that the above studies of the saturated absorption resonance at transitions with level momenta J = 1/2 are important for studying the nonlinear resonance formation processes in the more complex atomic systems and, in particular, for clarifying the EIA resonance formation mechanism in [5].

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