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Statistical analyses of the energy demand and thermal comfort for multiple uncertain input parameters, performed using transformed variable and perturbation method

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Abstract. In the calculations of buildings' thermal comfort, the input parameters are usually considered as strictly determined values. Numerous of them may be characterized by certain probability density functions. In the energy related problems, the uncertainty analyses are usually performed using the Monte Carlo method. However, this method requires multiple calculations and, therefore, may be very time-consuming. In the proposed work, two approaches are applied for the probabilistic studies: the stochastic perturbation method and the transformed random variables method. The stochastic analysis is based on the response functions and their derivatives with respect to all random input parameters. The relation between the thermal comfort and the input (random) variables have been calculated using the *Energy Plus* software. Afterwards, the response functions were estimated using the polynomial regression. The expected value and central moments of the response functions were calculated by means of the perturbation method and the transformed random variable theorem. The latter method allowed to obtain, using the same response functions, the implicit form of probability distributions function of the output parameter.

1. Introduction

Widely used simulation programs such as *Energy Plus* enable to predict the buildings' energy demand, heating, HVAC systems' loads, thermal comfort, etc. Such calculations are performed for some deterministic input parameters and boundary conditions, which are usually not entirely certain and are characterized by some randomness. Therefore, the building's actual performance may differ from the predicted one. If the randomness of the input parameters is considered, instead of obtaining one value of energy demand or thermal comfort, we may obtain their probability distribution functions (PDF) instead.

In the calculations of the thermal comfort response to the input parameters' uncertainty, the stochastic sampling methods are usually used. Chen et al. [1] applied Latin Hypercube Sampling for the uncertainty analyses in the naturally ventilated office building. The energy demand uncertainty has been also widely studied in the past [2, 3, 4]. In order to perform such analyses, the software code does not have to be modified strongly – the calculations are repeated for a given number of times for different sets of input parameters and a histogram of the outcome function may be elaborated. However, for complicated models, the calculation time cost necessary to obtain robust results may be too high. In this work,

alternative approaches: perturbation method and transformed variable method, which also do not require strong modification of the software, are compared.

2. Mathematical model

2.1. Perturbation method

In the perturbation method [5], function f is approximated using the Taylor series of a given order as:

$$f(\mathbf{b}) \cong f^0(\overline{\mathbf{b}}) + \varepsilon \Delta \mathbf{b} \cdot \nabla f(\overline{\mathbf{b}}) + \frac{\varepsilon^2}{2!} \Delta \mathbf{b}^T \cdot H(\overline{\mathbf{b}}) \cdot \Delta \mathbf{b} + .. \tag{1}$$

where ∇ stands for gradient vector, ε for a small perturbation and H for the *Hessian* matrix. The approximations for expected value and variance may be defined, assuming two uncertain parameters and limiting the solution for clarity to the second order, as:

$$E\left[f(b_1, b_2)\right] \cong f(\overline{b}_1, \overline{b}_2) + \frac{f'^{b_1 b_1}(\overline{b}_1, \overline{b}_2)}{2}\mu_2(b_1) + \frac{f'^{b_2 b_2}(\overline{b}_1, \overline{b}_2)}{2}\mu_2(b_2) \tag{2}$$

$$Var\left[f(b_1, b_2)\right] = \varepsilon^2\left[f^{b_1}(\overline{b}_1, \overline{b}_2)\right]\mu_2(b_1) + \varepsilon^2\left[f^{b_2}(\overline{b}_1, \overline{b}_2)\right]\mu_2(b_2) \tag{3}$$

where $\mu_2(b_i) = \int_{-\infty}^{\infty}\left[b_i - E(b_i)\right]^2 p(b_i)db_i$ is the second central moment of the input variable. The derivatives can be obtained by means of the direct differential method or the response function. Since the calculations were performed with the *Energy Plus* software, which is characterized by a high level of complexity of the equations, the latter method has been chosen.

2.2. Function of random variables

Let us assume that b_1 and b_2 are two input uncertain parameters and $u(b_1, b_2)$, $v(b_1, b_2)$ are two output variables of C^1 class for (b_1, b_2). Further, let us assume that the set $b_1 = b_1(u, v)$; $b_2 = b_2(u, v)$ possess the unique solution with continuous partial derivatives and the Jacobian:

$$\frac{D(b_1, b_2)}{D(u, v)} = \begin{vmatrix} \dfrac{\partial b_1}{\partial u} & \dfrac{\partial b_1}{\partial v} \\ \dfrac{\partial b_2}{\partial u} & \dfrac{\partial b_2}{\partial v} \end{vmatrix} \neq 0 \quad \text{for}(u, v) \in \Delta \tag{4}$$

where Δ is the image of D.

The probability density of bivariable (U, V), where $U = u(B_1, B_2)$ and $V = v(B_1, B_2)$, assuming that the PDF of bivariate (B_1, B_2) is continuous almost everywhere in D, is defined as:

$$k(u, v) = f\left[b_1(u, v), b_2(u, v)\right]\left|\frac{D(b_1, b_2)}{D(u, v)}\right| \quad \text{for }(u, v) \in \Delta \tag{5a}$$

$$k(u, v) = 0 \quad \text{for }(u, v) \notin \Delta \tag{5b}$$

3. Numerical simulations

A representative working open space, of dimensions 10.8x5.4x3.0m, with glazing of 80% of the exterior facade, has been chosen as the case study. Typical Meteorological Year for Warsaw has been used in the calculations which have been performed using the *Energy Plus* software.

Two parameters of windows, namely thermal transmittance (U_w) and solar heat gain coefficient (*SHGC*), have been considered as uncertain. Both parameters have been assumed as normally distributed with distribution N(μ, σ): N(1, 0.03) [W/(m^{2}K)] for U_w and N(0.34, 0.023) [-] for *SHGC*.

Thermal comfort has been predicted using the *Energy Plus* software for the entire year, with an hourly time step. Afterwards, the percentage time of thermal comfort (*TC*) has been determined as a

relation of a number of the working hours, in which the predicted mean value (PMV) index of thermal comfort is within the so-called neutral range, to the total number of the working hours in the year. The threshold values for the PMV index have been determined as [-0.5, +0.5] as for the 2nd category of thermal comfort. For both input uncertain parameters considered, 11 uniformly distributed points from the interval of $[\mu - 5\sigma, \mu + 5\sigma]$ have been chosen. Afterwards, the calculations in the *Energy Plus* software have been performed for 121 sets of values and dependence of thermal comfort on the input parameters has been determined by polynomial regression analysis with 4th order approximation.

The expected TC, obtained by means of the transformed variables method and the perturbation method, was equal to 86.50% and 86.42%, accordingly. The standard deviation was equal to 0.66% and 0.68%, respectively. As can be noticed, good accordance of the results has been noted. Using the transformed random variables method, additionally, the PDF can be obtained. Results of the PDF and the cumulative distribution of the thermal comfort is presented in Figure 1. It can be noticed that the percentage of comfort hours is influenced by the material uncertainties.
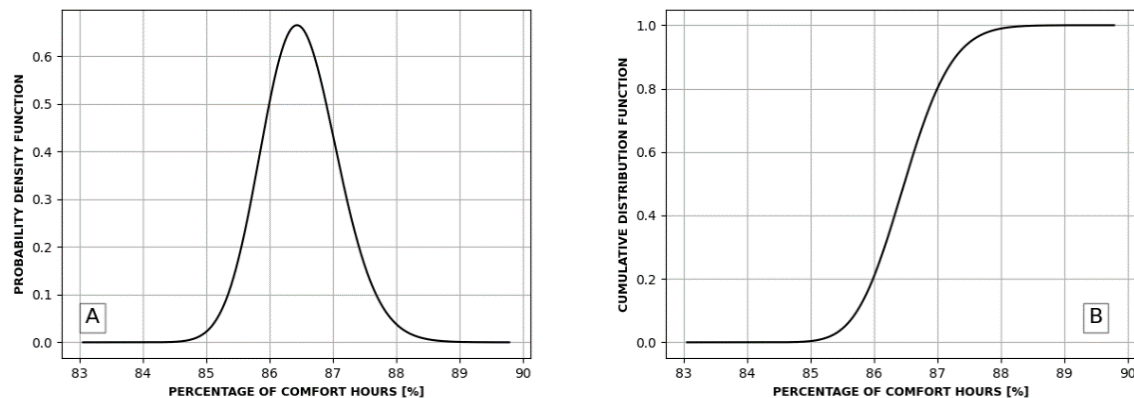

Figure 1. The probability density (A) and the cumulative distribution (B) of thermal comfort obtained by means of the transformed random variables method.

4. Conclusions

Two different methods of stochastic analyses have been compared. The perturbation method allows to obtain expected values and central moments, while the method deploying transferred random variable theorem allows to obtain also exact probability density functions of the results. Both methods require calculation time much shorter than in case of the Monte Carlo method.

Uncertainty of the windows parameters impact the thermal comfort conditions in the examined building. The assumed distributions of the material parameters affect the percentage of comfort hours up to approximately 5% comparing with its mean value. Based on the results, it seems reasonable to analyze the propagation of the uncertainties in such calculations and put effort to limit the uncertainties at the project stage.

Based on the probability density functions, any statistical parameter might be calculated. The presented methods allowed to obtain quickly the distribution of the response functions in the thermal comfort analyses and may be successfully applied in the software, allowing to account for uncertainties in the input parameters.

References

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