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# Mathematical Morphology and Modeling of Random Media

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**Abstract.** This paper is a short introduction to the modeling of complex microstructures by models of random sets, and to their use for predicting the effective properties of materials by means of numerical simulations.

## 1. Introduction

Since the seminal work of G. Matheron on porous media [11], and thanks to its implementation by means of image analysis [15], many applications based on mathematical morphology were developed. In this lecture, we introduced some models of random sets, and their use in the solution of homogenization problems.

## 2. Models of random sets

An efficient way to model random media is based on the theory of random sets [11, 13], giving a strong theoretical framework for modeling heterogeneous microstructures by means of a probabilistic approach. A random closed set $A$ is characterized by means of its Choquet capacity $T(K)$, defined on the compact sets $K$, as introduced by G. Matheron [11, 13]:

$$T(K) = P\{K \cap A \neq \varnothing\} = 1 - P\{K \subset A^c\}$$

The probability depending on $A$ and on $K$ can be theroretically calculated on some models, and can be estimated on real or on simulated media by image analysis [3, 4, 11, 15, 16]. This gives the link between theory and applications, and the way to estimate the parameters of a model from images.

Models of random sets provide representations of microstructures and ways for simulations, in order to predict morphological as well as physical properties of complex media. This purpose is illustrated by some basic models (using a combination of a Poisson point process and of random grains) and their applications to real data: the Boolean model [11, 13, 15, 16, 4], the dead leaves model [3, 4], and multiscale versions obtained from a Cox point process [6, 7, 8]. They can show very different percolation thresholds [6, 7], which control their transport or physical properties. Other models, derived from solutions of reaction-diffusion equations, provide striking natural textures [2].

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## 3. Homogenization

The theory of random sets also provides tools to solve homogenization problems, in order to predict the macroscopic properties of random media [11, 1, 10, 5]. Variational techniques give bounds of effective properties of random media, depending on a limited amount of information on the microstructure. The availability of 3D images, e.g. obtained by microtomography or generated by simulations of realistic models of microstructures, combined to solvers of PDE, gives access to the direct estimation of the effective properties, instead of bounds. The elastic moduli and the thermal conductivity can be obtained by finite elements [9]; using iterations of FFT [14], it is possible to solve in 3D the equations of electrostatics or of elasticity on images without meshing the microstructure; this was applied to the estimation of the dielectric permittivity of composites [8] and of the elastic bulk modulus of a porous Boolean models containing spherical pores or rigid inclusions, and therefore showing an infinite contrast [17]. For all these numerical estimations on finite domains must be solved the question of the RVE (Representative Volume Element), which we address by a statistical approach [9, 17]. Based on the use of the integral range introduced in Geostatistics [12], it provides intervals of confidence of the estimates of properties depending on the microstructure, the properties and the contrast of components. Numerical homogenization of random sets opens the way to the optimization of microstructures with respect to given physical properties.

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