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Resolution in in–line Digital Holography

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Abstract. Digital in-line holography is a 3D imaging technique which has been widely developed during the last two decades. This technique achieves the 3D reconstruction of volume objects from a 2D image-hologram. It is a metrological tool and therefore the improvement of resolution is one of the current challenges. However the resolution depends on several experimental parameters and the experimenters have to choose the parameters which will lead to the best resolution. This paper presents the study of resolution in in–line digital holography from the asymptotical bounds of the covariance of estimators used in hologram reconstruction.

1. Introduction

Digital in-line holography is a metrological tool widely used in experimental mechanics, biology or fluid dynamics and therefore the improvement of resolution is one of the key issues of this field ([1], [2], [3], [4]). The resolution depends on several experimental parameters such as the sensor definition, recording distance, object size, location of the object in the field of view ... and the experimenters have to choose the parameters which will lead to the best resolution. This paper describes an approach to the theoretical study of the resolution issue in in–line digital holography.

The concept of resolution is usually associated with the ability to distinguish two overlapping image-patterns of the same kind of objects. The resolution limit of lenses, microscopes, and telescopes are mostly based on the Rayleigh criterion (two-point resolution), which relates to the resolving power of the human eye to distinguish the images of two closely located point sources in observations of the sum of the images. Modern definitions of resolution, on the other hand, are based on statistical approaches. In this field two definitions of resolution have been distinguished [5]: (1) differential resolution (or two point resolution) is defined as the system's capability to determine the separation of two sources (Rayleigh's resolution), (2) the single-source resolution is defined as the system's capacity to determine the position of a point-source object that is observed in a background of noise. The latter is the most relevant in in-line digital holography, as the technique is mostly used to study volumes of isolated objects.

We use a statistical point of view to study single source resolution, and derive here the resolution on the optical axis.

2. Cramér-Rao lower bound on the covariance of maximum likelihood estimators

We will consider resolution from the view point of statistical parameter-estimation theory ([5], [6], [7]). The (optical) system response to a point source can be described by a parametrical model $\mathbf{g}_\theta(x_k, y_k)$, where θ is a vector of parameters defining both the optical system and source location, and (x_k, y_k) are the pixel coordinates on the sensor. Due to noise, the measured image is a perturbed version of the model $\mathbf{g}_\theta(x_k, y_k)$. The estimation of the source location from a measured image can be performed based on a maximum likelihood (ML) criterion. Under a hypothesis of additive white Gaussian noise, this amounts to minimizing the squared residuals between measurements and model \mathbf{g}_θ . ML estimators are unbiased and asymptotically efficient in the sense that their covariance reaches Cramér-Rao lower bound (CRLB). The covariance of any unbiased estimator is lower bounded by CRLB. The CRLB therefore provide an interesting insight of the achievable accuracy. Note that a ML approach has recently been introduced ([8], [9]) in particle digital holography.

3. Single-source resolution on the optical axis

We describe here the methodology of CRLB computation in the simple case of a point source located on the optical axis. The extension to a single source located outside of the optical axis is similar, but omitted here due to the paper length limit.

The diffraction pattern of a point source located at $\theta(x, y, z)$ can be approximated by:

$$\mathbf{g}_{x,y,z}(x_k, y_k) = \alpha \cdot \sin\left(\pi \frac{(x-x_k)^2 + (y-y_k)^2}{\lambda z}\right) \quad (1)$$

Under the assumption of a white Gaussian noise, the analytical form of Fisher information matrix element $[\mathbf{I}(\theta)]_{i,j}$ is given [10] by:

$$[\mathbf{I}(\theta)]_{i,j} = \frac{1}{\sigma^2} \left(\frac{\partial \mathbf{g}}{\partial \theta_i} \right)^t \left(\frac{\partial \mathbf{g}}{\partial \theta_j} \right) \quad (2)$$

where \mathbf{g} is a vector of components $\mathbf{g}_\theta(x_k, y_k)_{k=1..N}$. Ignoring the sampling on the sensor equation (2) can be rewritten as follows:

$$[\mathbf{I}(\theta)]_{i,j} = \frac{1}{\sigma^2} \frac{1}{L^2} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \left(\frac{\partial \mathbf{g}_{x,y,z}(x_k, y_k)}{\partial \theta_i} \frac{\partial \mathbf{g}_{x,y,z}(x_k, y_k)}{\partial \theta_j} \right) \Bigg|_{\substack{x=0 \\ y=0}} dx_k dy_k \quad (3)$$

where L is the sensor width (considered square here). Using the fact that the point source is located on the optical axis $x = y = 0$ and the hypothesis $L^2 \ll \lambda z$, most often valid in in-line digital holography, Fisher matrix can be written:

$$\mathbf{I}(x, y, z) = \frac{\alpha^2}{\sigma^2} \begin{pmatrix} \frac{\pi^2 L^2}{6\lambda^2 z^2} & 0 & 0 \\ 0 & \frac{\pi^2 L^2}{6\lambda^2 z^2} & 0 \\ 0 & 0 & \frac{7\pi^2 L^4}{360\lambda^2 z^4} \end{pmatrix} \quad (4)$$

Fisher information matrix is diagonal and so is its inverse: the covariance matrix. Estimation errors on the parameters x , y and z are not correlated. The lateral and axial accuracy are given by the standard deviations:

$$\sigma_x = \sigma_y = c \frac{\sigma}{\alpha} \frac{\lambda}{L/z}, \quad \sigma_z = 6\sqrt{5/21}c \frac{\sigma}{\alpha} \frac{\lambda}{(L/z)^2} \quad (5)$$

with $c = \sqrt{6}/\pi$.

This result is in agreement with classical formulas [11]: $\sigma_x \propto \frac{\lambda}{\Omega}$ and $\sigma_z \propto \frac{\lambda}{\Omega^2}$ and furthermore

indicates that the resolution improves proportionally with the signal to noise ratio: $\frac{\alpha}{\sigma}$.

4. Conclusion :

This study describes a general methodology which can be used to derive the resolution or the estimation accuracy in digital holography. It is based on the computation of CRLB. This gives the classical relations for on-axis resolution. The resolution can also be computed outside of the optical axis. Let us note that the CRLB can be reached in practice using ML-based algorithms such as in ([8], [9]), provided that the global minimization problem can be solved. Sampling and quantization effects will also be modeled in a further work.

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