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Effect of the kinetic energy conservation error on unsteady/steady isotropic turbulence: a numerical simulation study

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Abstract. This study presents the effects of the conservation error of turbulent kinetic energy on the isotropic turbulence field. Here, the present research focuses on the effects on the unsteady turbulence field in addition to the steady field. This unsteady turbulence field is given by changing the magnitude of the external forcing term using a linear forcing scheme with a sinusoidal function. The conservation error is caused by using the Crank-Nicolson method, which is an implicit time integration method. This result is examined by comparing those with the results with negligible conservation error using the fourth-order Runge-Kutta scheme. For this unsteady turbulence, the period and amplitude of the external forcing term are changed as computational conditions. In contrast to steady turbulence, unsteady turbulence is shown to be affected by turbulent kinetic energy conservation error. These effects are significant when the period is short and the amplitude is large.

1. Introduction

Numerical analysis in addition to experimental and theoretical techniques (e.g., [1,2]) is used as the means of previous studies in fluids engineering. In the experiment, the parameters to be obtained are measured from the occurring phenomena, so highly reliable measurement results can be obtained. On the other hand, in numerical analysis, it is necessary to examine the validity of the simulation results if the constructed mathematical model can reproduce the actual flow accurately [3,4]. In numerical analysis, it is possible to study complicated phenomena that are difficult to measure in experiments. It can have the advantage of lower cost than experiments. Therefore, in addition to investigations, numerical analysis is also engineeringly applicable and necessary. The turbulence model, such as the sub-grid scale model of large-eddy simulation (LES), exists as a method for numerical analysis of turbulence. LES can also analyze the flow around the wing, which is locally unsteady.

A flow solver is often used for simulating the flows in engineering applications. An example of fluid solvers is OpenFOAM (e.g., [4]). OpenFOAM is a numerical research platform for fluid and continuum simulations. The time integration methods used for these fluid solvers include an implicit way, such as the Crank-Nicolson scheme and an explicit way. This time integration method has problems that the conservation of kinetic energy may not be sufficiently maintained. The kinetic energy conservation should be held in discretized governing equations (e.g., [5,6]). It is necessary to maintain the kinetic energy conservation law in the discretized space to perform numerical analysis with high accuracy.



Previous studies (e.g., [5]) have shown that the conservative property of a discretization scheme used for numerical analysis may not be sufficiently held when the time integration method is implicit. The Crank-Nicolson method is a typical example of the implicit time integration method and is often used in fluid solvers. As shown in the previous study, the effects of the kinetic energy conservation error on the velocity field of steady turbulence may not be significant. On the other hand, there is a lack of examination for the effects of the error on unsteady turbulence. The present research focuses on the effects of the conservation error on unsteady turbulence.

The study aims to investigate the effects of kinetic energy conservation error on unsteady isotropic turbulence. Here the unsteady turbulence was set by using the isotropic external forcing term with using a coefficient of which the magnitude is varied based on a sinusoidal wave. By changing the amplitude and period of the wave for the unsteady external force term, the effects on the turbulent field are investigated.

2. Methods

The governing equations to be analyzed are the continuity and the Navier-Stokes equations, where the governing equations are non-dimensional and include the external force term $\mathbf{F}$ by linear forcing [7-9], which is a method of generating steady turbulence by giving the component of the external force term in proportion to the velocity vector $\mathbf{u}^F$.

$$\mathbf{F} = C(t)\mathbf{u}^F. \quad (1)$$

Here $C(t)$ is a coefficient which varies temporally. The present turbulence as the analysis target is based on the two-dimensional Taylor-Green vortex (TGV). The two-dimensional TGV is an isotropic flow and the boundary conditions are periodic.

The velocity components (u_x^F, u_y^F, u_z^F) of the velocity vector $\mathbf{u}^F$ given as follows were used as the setting of the external force term that generates the isotropic steady-state turbulence.

$$\begin{aligned} u_x^F &= (2/\sqrt{3})(-\cos(x)\sin(y) + \cos(x)\sin(z)) \\ u_y^F &= (2/\sqrt{3})(-\cos(y)\sin(z) + \cos(y)\sin(x)) \\ u_z^F &= (2/\sqrt{3})(-\cos(z)\sin(x) + \cos(z)\sin(y)) \end{aligned} \quad (2)$$

These velocity components are constructed based on Taylor's analytical solution [10]. Then, in order to apply the part of the forcing term to generating unsteady turbulence, the following terms were applied to the forcing term.

$$C(t) = 1 + A \sin(2\pi t/L). \quad (3)$$

Here, in the above relation, the constant A is the amplitude of the unsteady external forcing term and L is the period of the term. Here, in order to vary the magnitude of the forcing term, we used a form of sinusoidal function.

A large-eddy simulation (LES) model is used in the present study to simulate the turbulence. The governing equations of LES are obtained by filtering the governing equations of direct simulation. An eddy viscosity model can be obtained by approximating the sub-grid scale (SGS) stress generated based on the eddy viscosity hypothesis. The Smagorinsky model is a classical model that assumes turbulence. In this study, the Smagorinsky model was applied as the present SGS model.

The computational conditions in this study are shown in this paragraph (7). In the computational domain, the periodic boundary condition was used as the boundary condition. This boundary condition is a necessary condition for using the fast Fourier transform, which is used for solving the pressure equation at each fractional step (e.g., [11,12]). The size of the computational domain was set to $L^3 = (2\pi)^3$, and the number of grid points was set to $N^3 = 32^3$. The spatial discretization scheme is the fourth-

order central difference scheme [6]. The time step is $\Delta t = 0.002$. The value of the model constant of the SGS model was set to $C_s = 0.0573$, where this value is given by optimizing in steady turbulence. The value of the computational Reynolds number was set to $Re = 300$. At this Re number condition, the value of the turbulent Re number is about 200, which is considered to be a sufficiently high Re number, as shown in previous studies. In the present analysis, steady turbulence as well as the unsteady turbulence is simulated. For the steady turbulence, a value of A in the external forcing term is set to zero. The Fourth-order Runge-Kutta scheme and the Crank-Nicolson scheme are used to proceed temporally the turbulence fields. The kinetic energy conservation error can be negligible for using the Runge-Kutta scheme, in contrast to using the Crank-Nicolson scheme. When the Crank-Nicolson scheme is applied, a significant conservation error can be produced.

3. Results and Discussion

In contrast to using the Runge-Kutta method, the Crank-Nicolson method produces a significant magnitude of kinetic energy conservation error. The present study starts with verifying the conservation error using periodic inviscid flow. Here the initial velocity field was set using a random number series, which satisfies the continuity equation. Figure 1 (a) shows the time evolution of turbulent kinetic energy for using the Runge-Kutta and Crank-Nicolson schemes. In using the Runge-Kutta scheme, the turbulent kinetic energy is held to be constant. On the other hand, in the CN method, the energy increases as time increases, where the turbulent kinetic energy is analytically constant because of the inviscid characteristic. Figure 1 (b) shows the rate of initial change of turbulent kinetic energy. As shown in the figure, the Crank-Nicolson scheme allows the significant magnitude of the conservation error for the kinetic energy. Using the periodic inviscous fluctuation field, the conservation error due to using the Crank-Nicolson method was compared with that of the second-order Adams-Bashforth method in the preliminary analysis. When the second-order Adams-Bashforth scheme is used, the rate of initial change of the kinetic energy is proportional to the square of the time step and is sufficiently smaller than that of the Crank-Nicolson method. This study focuses on the effects of kinetic energy conservation error on the turbulence field. Therefore, this study considered that the conservation error should be as small as possible under the reference condition. Therefore, the fourth-order Runge-Kutta method is used for this reference analysis. Since the time step is sufficiently small in all cases, and the effects of the difference in the accuracy order of the time integration scheme seems to be not significant (B).

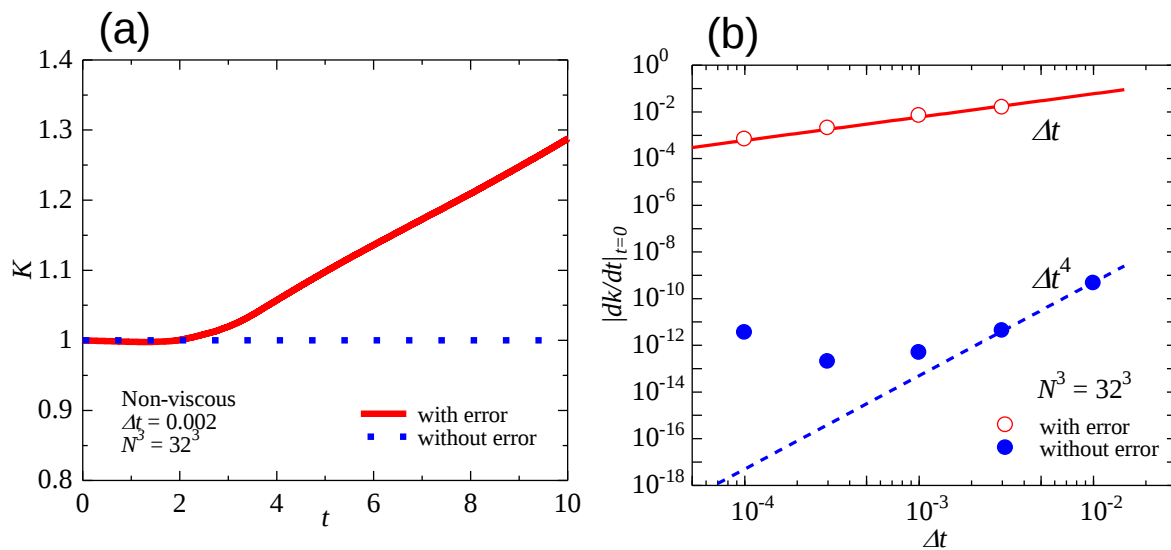


Figure 1. Results of the kinetic energy conservation using the periodic inviscid flow. (a) Temporal evolutions of the kinetic energy. (b) The rate of initial change of the kinetic energy.

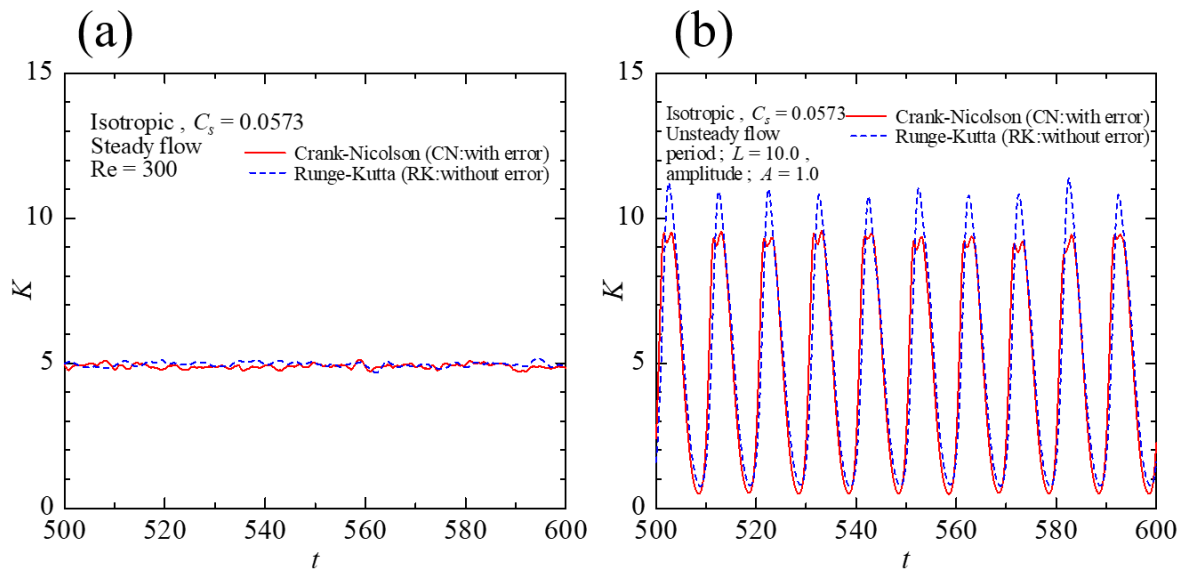


Figure 2. Time series of turbulent kinetic energy. (a) steady turbulence. (b) unsteady turbulence.

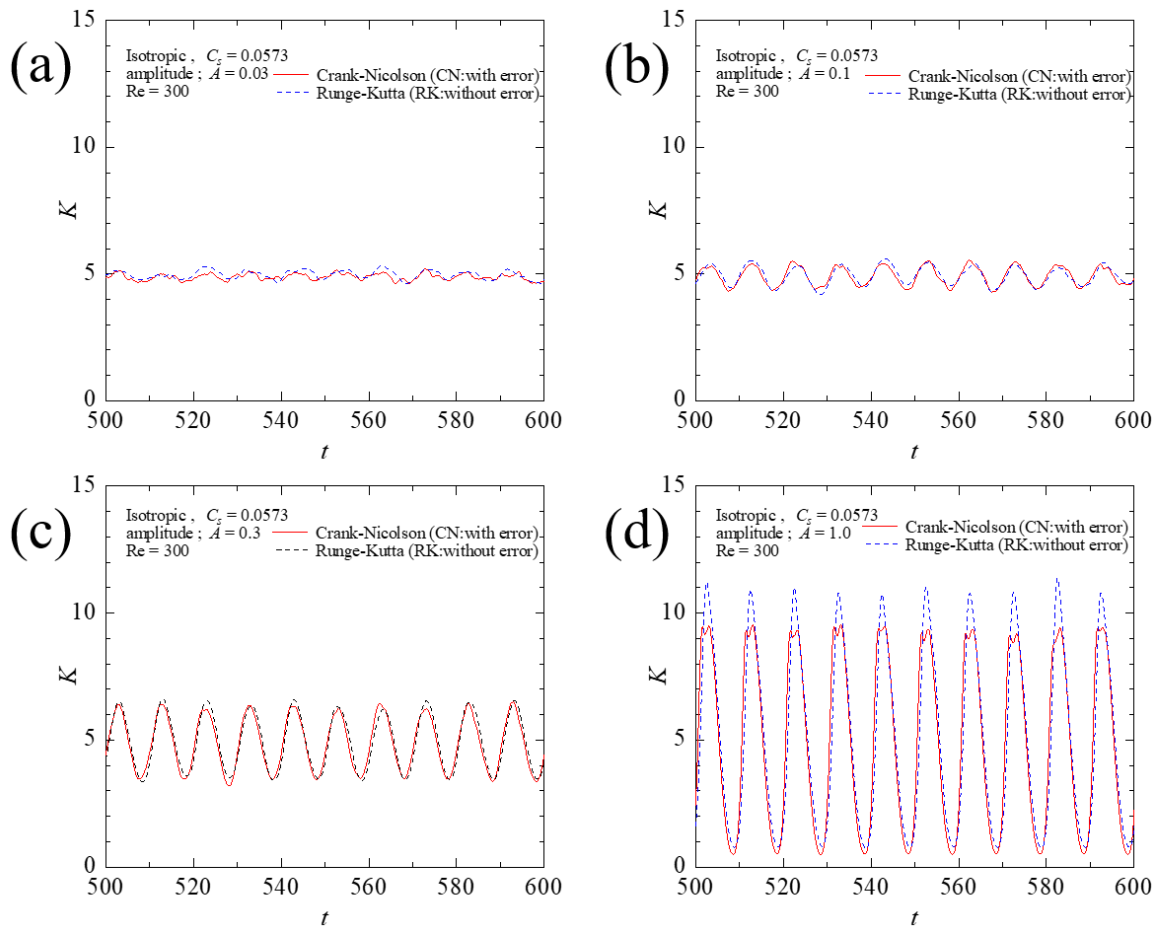


Figure 3. The dependence of the amplitude in the forcing term on the time series of turbulent kinetic energy in the unsteady turbulence. (a) $A = 0.03$. (b) $A = 0.1$. (c) $A = 0.3$. (d) $A = 1.0$. Here the period of the forcing term is fixed to $L = 10.0$.

The present study compares the time series of turbulent kinetic energy in the steady and unsteady flow fields, where the turbulent kinetic energy is calculated as spatial mean value at each instant. Figure 2 (a) shows the time series in the steady turbulence field. In the steady turbulence, the turbulent kinetic energy was found to be almost constant in both of the time integration schemes. Here a mean value of the turbulent kinetic energy is similar between results of Crank-Nicolson and Runge-Kutta schemes. From the results, the turbulent kinetic energy is hardly affected by the conservation error of kinetic energy in the steady turbulence. Figure 2 (b) shows the time series in the unsteady turbulent flow fields. In the unsteady flow, the maximum value obtained using the Crank-Nicolson scheme is around 9.5, while the maximum value of the Runge-Kutta scheme is about 11. As shown in the figure, it was found that the conservation error of kinetic energy the turbulent kinetic energy can affect the field of the unsteady turbulent flow.

The results of changing the amplitude in the unsteady flow are shown. The amplitude is changed to $A = 0.03, 0.1, 0.3, 1.0$ with constant period of $L = 10.0$. The results are shown in Fig. 3. The effects of the conservation error on the turbulent kinetic energy are not significant for the conditions of $A = 0.03, 0.1$, and 0.3 . Therefore, the effects of the energy conservation appear when the amplitude of the unsteady forcing term is large. The observed effects of conservation error are reduced as the amplitude decreases.

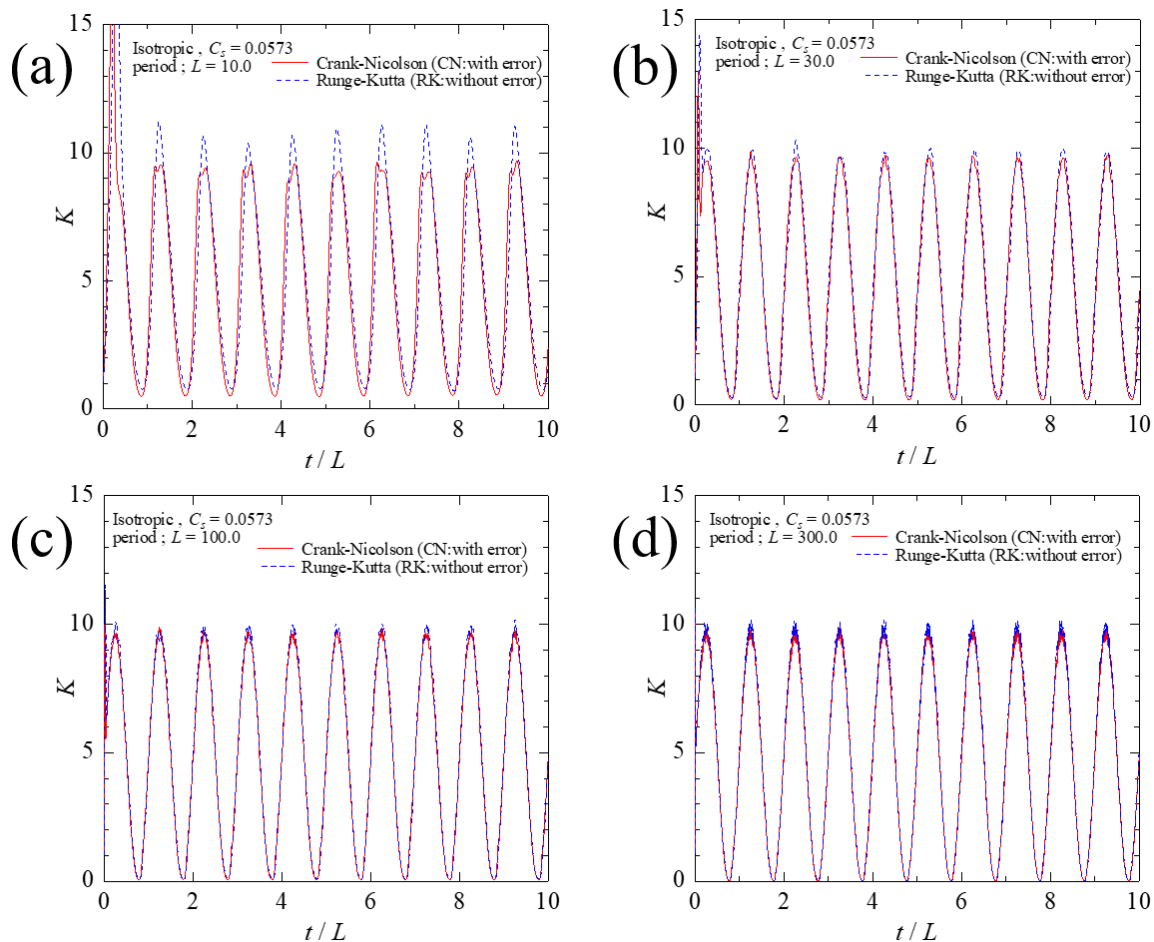


Figure 4. The dependence of the period in the forcing term on the time series of turbulent kinetic energy in the unsteady turbulence, where the temporal coordinate is normalised by each period. (a) $L = 10.0$. (b) $L = 30.0$. (c) $L = 100.0$. (d) $L = 300.0$. Here the amplitude of the forcing term is fixed to $A = 1.0$.

Figure 4 shows results with changing the period, which is more than or equal to $L = 10.0$, where the amplitude was set to $A = 1.0$. The value of period is changed to $L = 10.0, 30.0, 100.0$, and 300.0 . Here the temporal coordinate is normalized by each period in the figure. In the conditions of $L = 100.0$ and 300.0 , the effects of the conservation error of the time series are sufficiently small. The turbulent kinetic energy of steady-state turbulence does not seem to be significantly affected by conservation error. If the period is large enough, the turbulent field at each time may be considered as locally steady. On the other hand, the effects on the time series cannot be negligible in the conditions of $L = 10.0$ and 30.0 . From the results, the effects of the conservation error can be more significant when the temporal period is smaller.

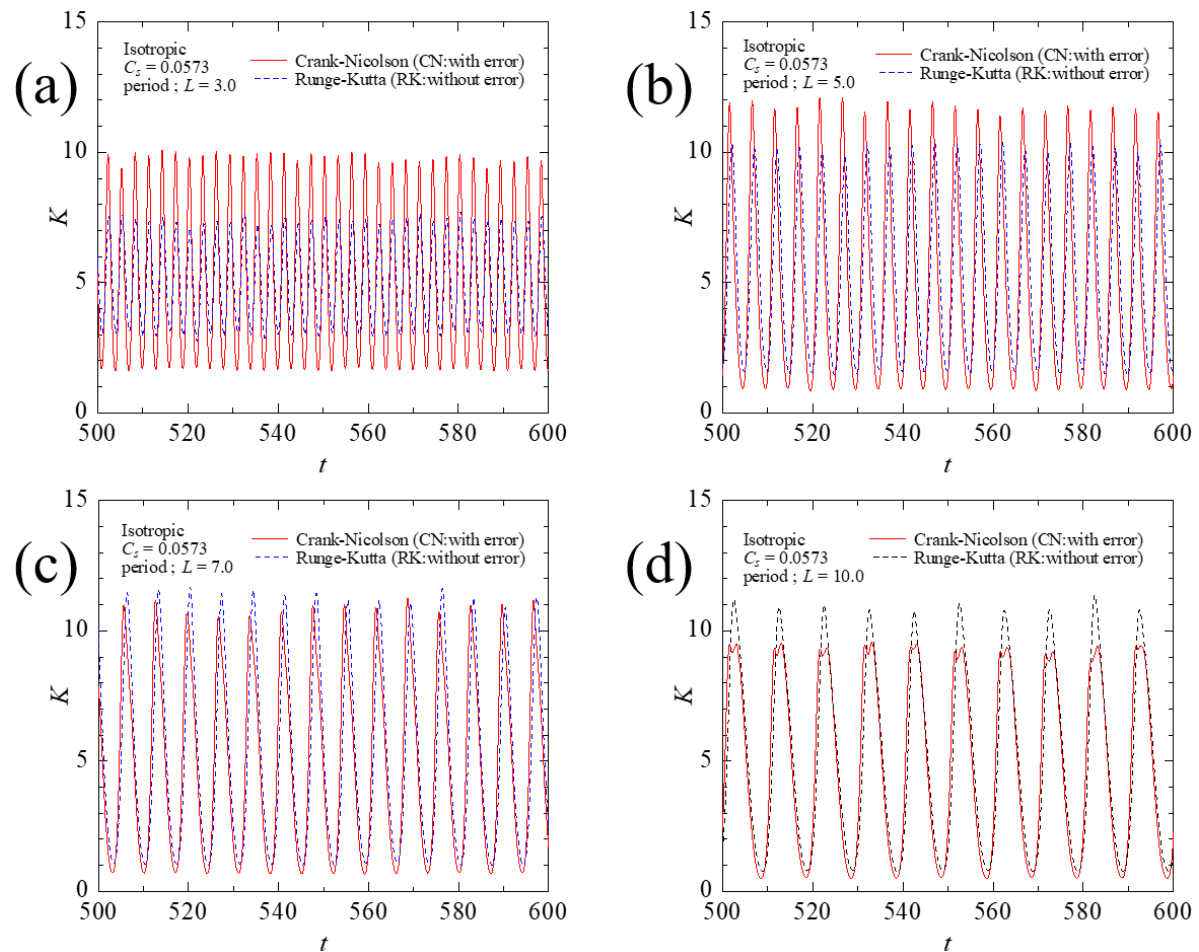


Figure 5. The dependence of the period in the forcing term on the time series of turbulent kinetic energy in the unsteady turbulence, where the temporal coordinate is normalised by each period. (a) $L = 3.0$. (b) $L = 5.0$. (c) $L = 7.0$. (d) $L = 10.0$. Here the amplitude of the forcing term is fixed to $A = 1.0$.

Figure 5 shows the effects on the turbulent kinetic energy for the conditions of a smaller period, where the period is set to $L = 3.0, 5.0, 7.0$, and 10.0 . Here the amplitude of the forcing term is also set to $A = 1.0$. As shown in the figure, when the period is decreased, there are significant effects on the turbulent kinetic energy, in contrast to results with the larger temporal period. When the period is small, the conservation error of kinetic energy increases the amplitude of the varying turbulent kinetic energy. On the other hand, local maximum values of the time series are slightly decreased due to the conservation error for the conditions of $L = 3.0$ and 5.0 . The reason has not been found exactly, but it may be that the kinetic energy conservation error affects the dissipation of turbulent kinetic energy. In

our test analysis, not published, we have obtained the result that the kinetic energy conservation error affects the properties of the small-scale turbulent field.

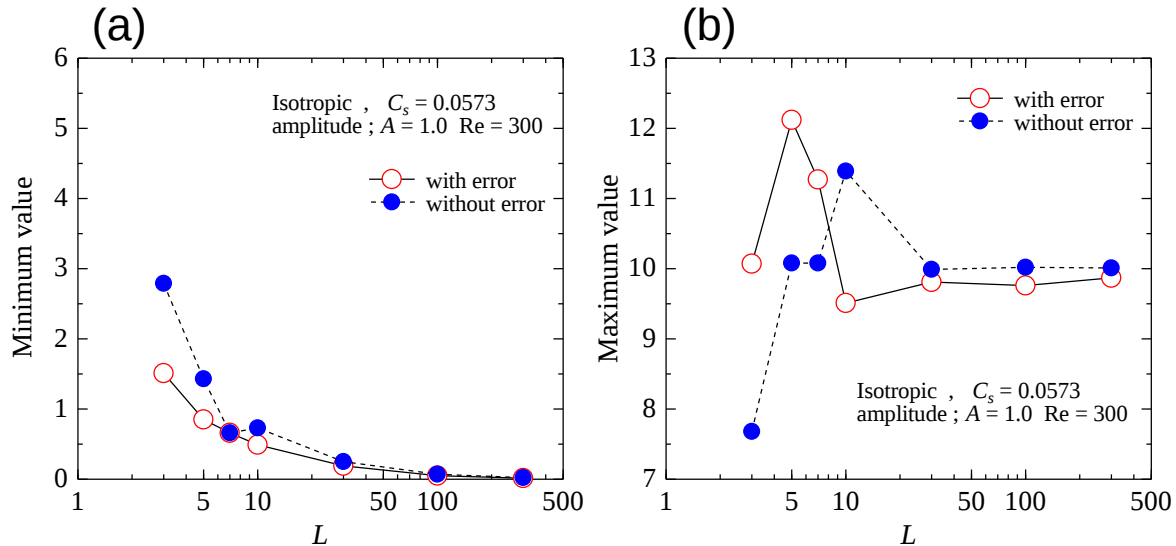


Figure 6. R The Effects of the conservation error on the local minimum and maximum values of the times series. (a) Minimum value. (b) Maximum value. Here the amplitude is fixed to $A = 1.0$.

The results on the dependence of the period are shown in Figure 6. Here minimum and maximum values of time series are shown in the figure. As shown in the figure, the minimum values of the time series are decreased as the period increases. As shown in Figure 6, the kinetic energy conservation error may shift the distribution laterally. Therefore, under each condition, the magnitude of the effect of conservation error may apparently change. The conservation error decreases, the minimum values of the time series are decreasing as the period increases (18). The maximum values have a local maximum around $L = 10$, as shown in the results for using the Runge-Kutta scheme. The conservation error can decrease the time gives the locally maximum value.

Although results with conservation error agree qualitatively with those without the error, results obtained using the Crank-Nicolson scheme do not quantitatively agree with those of the Runge-Kutta scheme.

4. Conclusions

This study aimed to investigate the effects of conservation error of kinetic energy on unsteady isotropic turbulence. The present isotropic flow field is set using the external forcing term based on linear forcing. This forcing term to produce the unsteady turbulence is formed by using a part of the sinusoidal function with the forcing term. The Smagorinsky model is used as the LES model. The Crank-Nicolson method, an implicit time integration method, is used to set analysis with significant conservation error. The computational domain is the periodic box. Values of period and amplitude in the forcing term are set to be varied in the present analysis.

By using the Crank-Nicolson method, the significant conservation error can be validated to be produced. On the other hand, the negligible conservation error is given by using the Runge-Kutta scheme. By comparing the steady and unsteady turbulent flows, the effects of the conservation error can be observed in the unsteady condition. The effects of the conservation error are not negligible when the amplitude is large and the period is small. The dependence of the observed effects on the period is shown for local minimum and maximum values of the time series.

The present study focuses on the anisotropy of the turbulence to approach the results of this research

as future works, where isotropic turbulence was used in the present study. Also, discussion using the transport equation of turbulent kinetic energy may produce further insights into the present observations.

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