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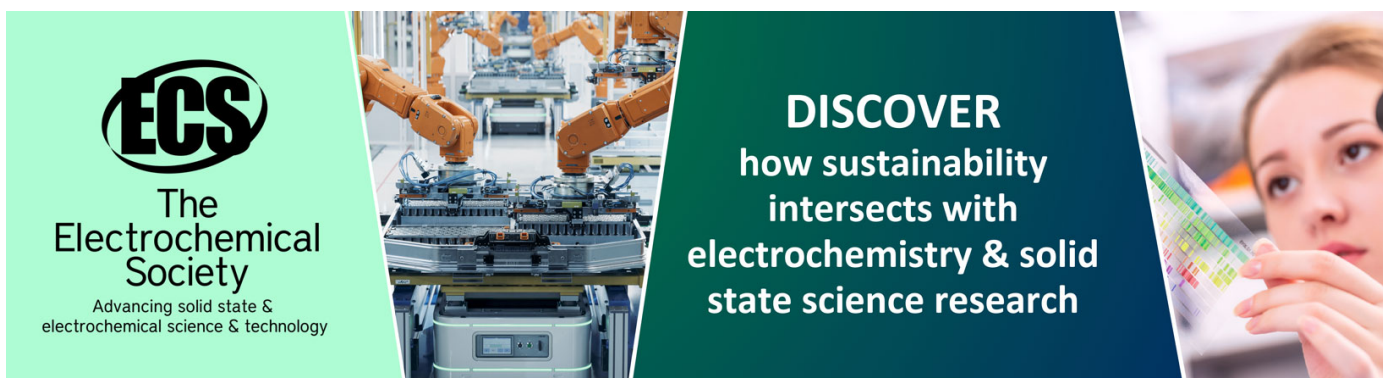
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Numerical simulation of viscosity/implicit large-eddy steady turbulence with the Reynolds number dependency

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Abstract. This study presents a numerical analysis that models small scale turbulence using numerical viscosity or implicit large-eddy simulation (LES). The motivation for focusing on these models is that the sub-grid scale components of LES are assumed to have a sufficiently high Reynolds number turbulence. The Reynolds number dependence of steady isotropic turbulence is used to validate the present analysis. Here, this dependency ranges from low to high Reynolds numbers. The results of this analysis are validated by comparing them with those of direct numerical simulation. The donor cell method and quick method are used as schemes of the numerical viscosity. Analysis based on the numerical viscosity can give accurate turbulent kinetic energy values at high Reynolds numbers and implicit LES at low Reynolds numbers. However, these models did not accurately predict static pressure fluctuations. These results were discussed by visualizing the large-scale turbulent structures.

1. Introduction

Numerical analysis is attracting attention as the third method, in addition to experimental (e.g., [1]) and theoretical techniques (e.g., [2]) as a means of research and development in the field of fluids engineering. The background of the study is the rapid development and spread of computers in recent years. Numerical analysis is widely adopted in engineering applications because it is often advantageous in terms of speed, safety, and low cost compared to experiments. In computational fluid dynamics, the governing equations of flow fields are solved numerically. The flow fields are reproduced in the numerical space to elucidate and predict various phenomena of the flow. Most of the flows around us are incompressible flows, which are in a turbulent state. Direct numerical simulation (DNS) and large-eddy simulation (LES) [3] are used for the numerical technique of the turbulence. DNS is a method of numerically solving the governing equations without any modelling. LES is a method of directly calculating only large-scale flow fields. LES has a smaller computational cost than DNS and is a more practical computational method for applying to engineering problems, but the accuracy of the resulting numerical solution depends on the model used.

Lundgren [4] has proposed a Linear forcing scheme, which is a force scheme constructed in physical space. This scheme is a method in which the external force vector in the governing equations is given to be proportional to the velocity vector, and the external force term is provided to maintain the turbulence steady. Later, Rosales and Meneveau [5] and Carroll and Blanquart [6] showed the applicability to steady homogeneous turbulence using this scheme. Also, the flow in the engineering is



often in a state of high Reynolds number, and high Reynolds number turbulence has attracted attention in previous studies. Turbulence with a high Reynolds number is locally isotropic and has universal properties for small-scale eddies. On the other hand, in recent years, in order to clarify the characteristics of turbulence, low Reynolds number turbulence in which local anisotropy does not hold has also been studied. There is a lot of research on models that can be applied to flows of laminar-turbulent transition and flows near the wall, for which sufficient accuracy cannot be obtained with the Smagorinsky model of LES. Vreman [7] proposed an SGS model that can be applied to these flows using only the first derivative of the local filter width and velocity components.

Most previous studies on the forcing method for generating steady turbulence focus on the isotropic field, and the number of the previous study for the effects of the anisotropy of external forces on steady turbulence is found to be insufficient. LES has sufficient accuracy in the high Reynolds number range. Still, the examination for the accuracy of LES in the lower Reynolds number range is inadequate. There may be a Reynolds number region where the LES accuracy decreased in the field. Therefore, examining models other than the SGS stress model may lead to application to a broader range of turbulence analysis.

In this study, isotropic external force term is given using the Linear forcing method, and the fundamental characteristics of the generated steady turbulence are numerically analyzed. Here, this study focuses on the Reynolds number dependence of the turbulent kinetic energy and static pressure fluctuation. Also, the small-scale numerical viscosity and implicit LES are applied as techniques of small-scale turbulence modeling. For the numerical viscosity, we attempt to set a fitted model constant value in the flow field in this study by examining the model constant dependence of the turbulent kinetic energy statistics. In the numerical results of DNS, implicit LES, and numerical viscosity, the Reynolds number dependence of turbulent kinetic energy and static pressure fluctuation is calculated. The results are compared to validate cases due to the numerical viscosity and implicit LES. This study is motivated by a problem with the SGS model. In the SGS model, the Reynolds number should be sufficiently high. However, this point may not be satisfied in engineering analysis. Therefore, if the Reynolds number is low, the accuracy of the SGS model may decrease. Thus, in this study, it is necessary to investigate whether a model other than the SGS model can sufficiently model a small-scale turbulence field.

2. Methods

The governing equations of the flow field are the continuity equation and the Navier-Stokes equation shown as follows:

$$\nabla \cdot \mathbf{u} = 0 \text{ and} \quad (1)$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \frac{1}{\text{Re}} \nabla^2 \mathbf{u} + \mathbf{F}. \quad (2)$$

Here, $\mathbf{u}$, p and Re are velocity vector, pressure, and Reynolds number, respectively, and $\mathbf{F}$ is an external force term according to the linear forcing method [4,5], and is given as follows.

$$\mathbf{F} = C\mathbf{u}^F. \quad (3)$$

The linear forcing method is a method of generating steady turbulence by giving a component of the external force term in proportion to the velocity component.

The components (u_x^F, u_y^F, u_z^F) of the analytical solution $\mathbf{u}^F$ given by Goto and Vassilicos [8] as follows were focused on in the setting of the external forcing term.

$$u_x^F = -\cos(x) \sin(y), u_y^F = \sin(x) \cos(y), \text{ and } u_z^F = 0. \quad (4)$$

Here, this velocity component is proportional to Taylor's analytical solution. Therefore, the external force terms are found to give the velocity components of Taylor's analytical solution to the external force terms by the linear forcing method. This external force terms have the following properties.

$$\nabla \cdot \mathbf{u}^F = 0 \text{ and } \nabla^2 \mathbf{u}^F = -\mathbf{u}^F. \quad (5)$$

The external force term that generates isotropic steady-state turbulence was set by combining the components of Taylor's analytical solution. Specifically, components of the force term are shown as follows:

$$\begin{aligned} u_x^F &= (2/\sqrt{3})(-\cos(x) \sin(y) + \cos(x) \sin(z)) \\ u_y^F &= (2/\sqrt{3})(-\cos(y) \sin(z) + \cos(y) \sin(x)) \\ u_z^F &= (2/\sqrt{3})(-\cos(z) \sin(x) + \cos(z) \sin(y)) \end{aligned} \quad (6)$$

Numerical viscosity is widely known to be effective for computational stability. This study focuses on the effect of numerical viscosity as the role of small-scale modelling in LES. When the convection terms are the divergence type, the first derivative of uf is approximated by the second-order central difference scheme:

$$[(uf)]_i = \frac{-[uf]_{i-1/2} + [uf]_{i+1/2}}{\Delta x}. \quad (7)$$

Here Δx is grid spacing for x direction. The following equation can be obtained by using numerical viscosity as an upwind difference scheme, which is donor cell scheme:

$$[uf]_i = u_i \frac{f_{i-\frac{1}{2}} + f_{i+\frac{1}{2}}}{2} - [u_i] \frac{-f_{i-\frac{1}{2}} + f_{i+\frac{1}{2}}}{2} \quad (8)$$

The second item on the right side in equation (8) always dissipate the turbulence energy, and the numerical viscosity term in the governing equations is added to the viscous terms. In this study, the model constant C_D is introduced to control the magnitude of the numerical viscosity terms.

$$[uf]_i = (\text{Skew.}) - C_D \times [u_i] \frac{-f_{i-\frac{1}{2}} + f_{i+\frac{1}{2}}}{2}. \quad (9)$$

Here (skew) denotes the skew-symmetric form of the discretized convective terms [9]. The optimum value of the constant C_D is arbitrarily determined in order to minimize the error of turbulent kinetic energy. In this study, Quick upwind scheme is also used to examine the influence of accuracy order of spatial resolution.

For the LES model, the Smagorinsky model (SM) and the implicit LES (ILES) is used. The Smagorinsky model will be the most basic SGS model and is an eddy-viscosity model whose characteristic length is the filter width Δ , and has been widely used. There is a parameter dependence that the optimum value of the model constant C_s , which may depend on the flow field. Since implicit LES does not require filtering or introducing the SGS stress model, it is easier to apply to complex flows than other LES methods, specifically to high Reynolds number flow around an aerofoil. The model constant of implicit LES is given in the form of the model constant $C_s = 0$ of the Smagorinsky model.

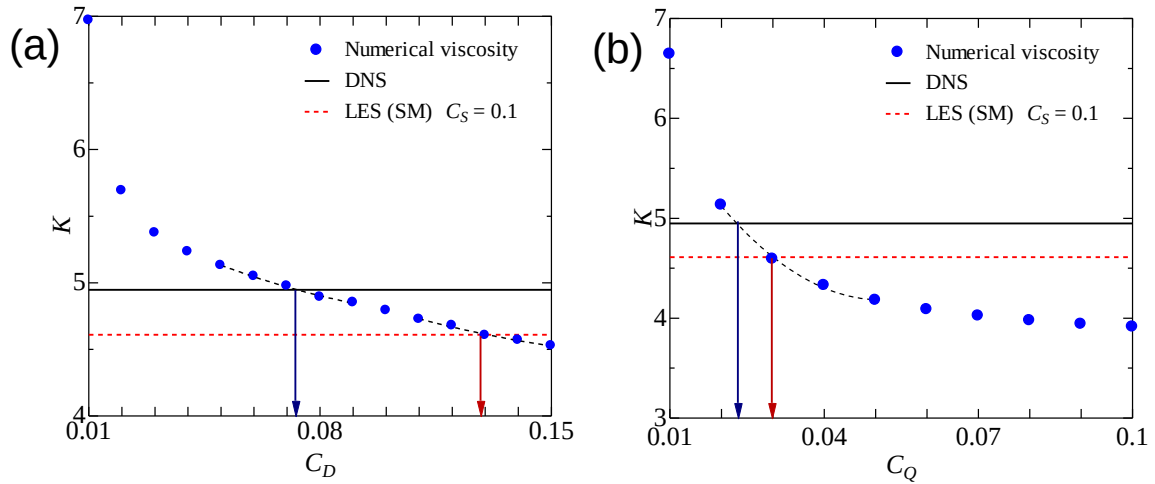


Figure 1. Model constant dependence of turbulent energy statistics. (a) donor cell method, (b) Quick method.

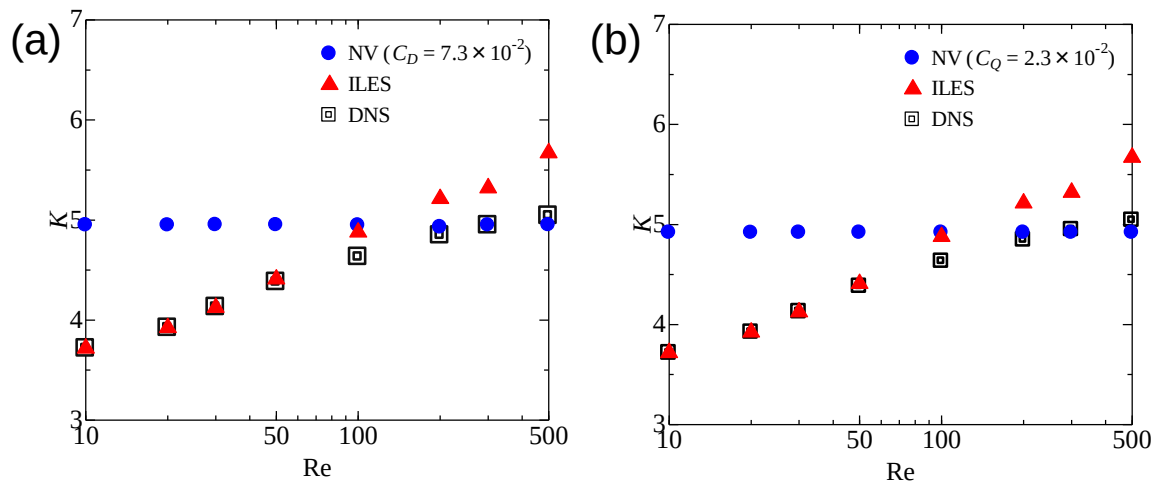


Figure 2. Reynolds number dependence of turbulent kinetic energy (a) donor cell method, (b) Quick method.

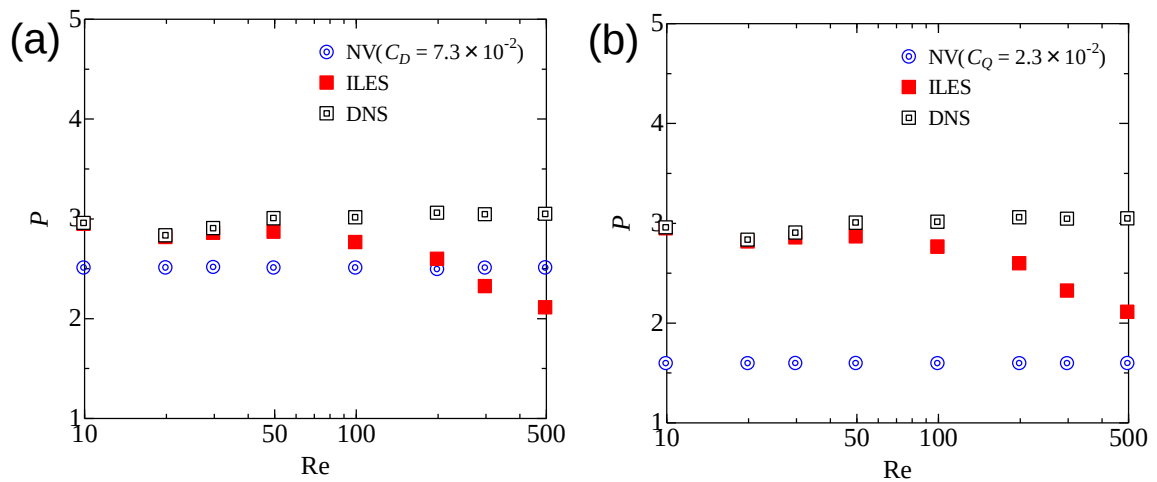


Figure 3. Reynolds number dependence of static pressure fluctuation rms. (a) donor cell method, (b) Quick method.

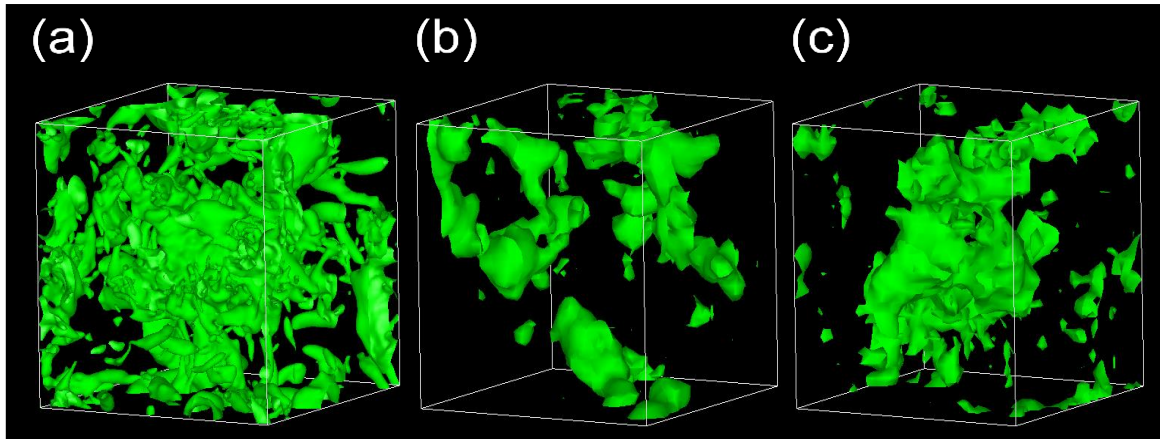


Figure 4. Visualization results using the isosurface of static pressure fluctuation (a) DNS, (b) donor cell method, (c) implicit LES.

In this study, as the numerical conditions, the computational grid system was set to the staggered grid, the computational domain was set to $L^3 = (2\pi)^2$, the time step width was set to $\Delta t = 0.005$, and the time was evolved to $t = 3000$. The implicit LES was used as the LES model. The constant C in the linear forcing method was set to $C = 1$. The initial velocity field is isotropic, and the components are the same as the form of the external force term components. The Reynolds number is $Re = 10, 20, 30, 50, 100, 200, 300, 500$, and the number of grid points is $N^3 = 128^3$ when $Re = 200 \sim 500$ in the numerical cases by DNS, and $N^3 = 64^3$ in the other cases. In the grid points analysis using LES and numerical viscosity, $N^3 = 32^2$ is set for all Reynolds numbers. The spatial difference method is the fourth-order central difference scheme [9-11], the time integration method is the low-storage fourth order Runge-Kutta method, the convection term is the skew-symmetric form in which the square is each component of velocity is sufficiently conserved, and the pressure equation is solved using the fast Fourier transform. The fractional step method was applied to solve the governing equation. The value of model constant of the LES is set to be 0.1. In our numerical analysis, all conservation laws are validated to be almost completely held by analysing inviscid homogeneous isotropic fluctuation field. It is probable that the present analysis has been validated by these results.

3. Results and Discussion

For the model constants in the donor cell method and the Quick method of numerical viscosity in the isotropic field, the model constants are optimized by comparing the turbulent kinetic energy in the flow field with $Re = 300$ with those obtained by DNS. Here, the model constants calculated for the donor cell method and the Quick method are shown to be C_D and C_Q , respectively. We also simulate the flow with the condition where the value of the model constant of numerical viscosity corresponding to the model constant value $C_S = 0.1$ of the Smagorinsky model of LES. Figure 1 show the dependence of the turbulent kinetic energy given by the donor cell method and the Quick method on the model constants C_D and C_Q , respectively. For the values calculated by each method of numerical viscosity, an approximate function was found using the least square fitting method near the intersection with each DNS result, the intersection with the DNS value can be obtained. Unlike the LES model, there is no standard model constant value for numerical viscosity. Therefore, the model constant value of the numerical viscosity corresponding to the model constant value $C_S = 0.1$ of the commonly used Smagorinsky model (SM) of LES is calculated and compared with the Smagorinsky model. From figure 1, it was confirmed that the value of the optimum model constant value is lower than the value of the model constant corresponding to $C_S = 0.1$ in each method of numerical viscosity.

We analysed the Reynolds number dependence of turbulent kinetic energy in an isotropic field obtained by DNS, numerical viscosity, and implicit LES. Figure 2 shows the Reynolds number

dependence of the turbulent kinetic energy by the donor cell method and the Quick method, respectively. As shown in the figure, results of the implicit LES agree with those of the DNS in the low Reynolds number range, whereas in the high Reynolds number range, results of the numerical viscosity are close to the DNS results. From these results, the numerical viscosity and implicit LES may not agree with the DNS results in the entire Reynolds number range of 10 to 500. The implicit LES is sufficiently accurate in the Reynolds number range of 10 to 50. On the other hand, results of the numerical viscosity are close to the DNS result in the range of 200 to 500.

Figure 3 shows the Reynolds number dependence of the static pressure fluctuation (e.g., [12-14]) obtained by the donor cell method and the Quick method, respectively. In Figures 2 and 3, the dependence of turbulent kinetic energy and static pressure fluctuation rms on the Reynolds number has been shown. As found in Bernoulli's theorem, static pressure has an energy dimension by dividing by density. Therefore, the static pressure fluctuation rms can be regarded as observing the energy of the turbulent flow from another point of view. As shown in the figure, results of the implicit LES agree with those of the DNS in the low Reynolds number range up to $Re = 50$. However, the value of the numerical viscosity is slightly smaller than that of the DNS. In the higher Reynolds number range, values of the implicit LES and numerical viscosity are different from those of the DNS. As shown in these results, the numerical viscosity and implicit LES do not match the DNS result in the entire Reynolds number range of 10 to 500. Here, values obtained using the numerical viscosity are roughly constant as the Reynolds number changes. This characteristic found in the results is qualitatively similar to that of the DNS results.

Then, visualization results are shown in Figure 4. The static pressure fluctuation is visualized using isosurface at $Re = 200$. Figure 4 shows the visualization results of DNS, the numerically viscous based on the donor cell method, and the implicit LES. The green surface in the figure denotes the negative value of instantaneous static pressure. As shown in the figure, finer structures are found in the results of DNS because the finer grid resolution is set in the simulation. Large scale structure obtained in the implicit LES is roughly similar to that of DNS. Therefore, large-scale structure in the implicit LES results may be more accurate than that in numerical viscosity. Since these visualization results are instantaneously obtained, the shape of the observed vortical structures is slightly changed at each instant.

4. Conclusions

In this study, we numerically studied turbulence modelling based on numerical viscosity and the implicit LES using the fundamental characteristics of the steady turbulence by applying isotropic external force term using the linear forcing method in the steady turbulence. This study focused on the Reynolds number dependence of the turbulent kinetic energy and static pressure fluctuation. The small-scale model numerical viscosity and implicit LES were also applied as the role of small-scale turbulence modelling. Here, the optimized value of the model constant for the numerical viscosity terms is calculated by using the DNS result.

The optimum values of the model constant are calculated by using the turbulent kinetic energy at the high Reynolds number condition. The analysis based on the numerical viscosity gives accurate results at the high Reynolds number region. On the other hand, the implicit LES is found to be accurate at the lower Reynolds number. Also, when the statistics of static pressure fluctuation were examined, results of the numerical viscosity did not agree with those of the DNS, although the value is slightly constant for the Reynolds number change as similar to that of the DNS. Then, large-scale turbulent structures are visualized using the isosurface of static pressure fluctuation.

In this study, we applied numerical viscosity and implicit LES to the isotropic steady turbulence as the small-scale modelling. As shown in the present results, there was no complete agreement with the DNS results in the entire Reynolds number range. Therefore, as future works, a factor causing this inagreement should be approached. Specifically, the difference in vortical structures should be more clearly by expanding the Reynolds number range to be visualized.

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