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The double elastic foundation beam model for analysis of load transfer systems of racks and nut columns in shiplifts

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The double elastic foundation beam model for analysis of load transfer systems of racks and nut columns in shiplifts

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Abstract. The load transfer systems of racks and nut columns are important components of the rack and pinion vertical shiplifts which bear the ship chambers and transfer the ship chamber loads in special cases to the tower columns. Aiming at exploring a design based calculation method for the nut column, racks and the associated second stage embedded parts, this paper proposes the double elastic foundation beam model for both the rack load transfer systems and nut column load transfer systems by which the formulas for describing the axial distribution of internal forces and deflections of the rack, column nut and the associated second stage embedded parts are derived. The calculation method is demonstrated by the description of the application in the Three-Gorges shiplift.

1. Introduction

In engineering design of a hydro-power project with the requirement of keeping water transportation, a navigation building is need to be considered as a part of the project[1]. As one of two forms of navigation facilities, ship lifts have obtained rapid development in past thirty years due to their advantage in excellent technical and economic performance for navigation in high dams over ship locks, and the achievements in construction of shiplifts in hydro-power projects have been recognized in the world[2]. The construction of the largest shiplifts in China, the Three-Gorges shiplift with tonnage of 3000t and hoisting height of 113m[3]，and the Xiangjiaba shiplift with tonnage of 1000t and hoisting height of 114.2m[4], finished respectively in 2016 and 2018. Both shiplifts are the type of the full balance rack and pinion vertical shiplifts.

The ship chambers of full balance rack and pinion vertical ship lifts are hoisted by the pinion–rack drive mechanisms during the lifting process, and secured by the screw rod and nut column safety mechanisms in case the ship chambers are unbalanced in the vertical direction as the accidents such as leakage of ship chambers occur, which is the essential feature of this type of shiplifts. So the racks and nut columns are important bearing components of the ship chambers. Besides the function of drive and vertical support of ship chambers during lifting, the racks act also as the guide rails of the lateral guide mechanisms of ship chambers and bearing the lateral loads of ship chambers, among which the seismic couple force between the ship chambers and tower columns is the largest one and is far more than the other lateral loads.

The research of the racks in the rack and pinion vertical ship lifts focused mostly on the mechanical strength of the teeth of the racks under the vertical loads[5,6]. Wang Zhi-hao, Shi Duan-wei et al built a finite element model[7], which included a pinion, a rack, a section of adjusting beam that is the second stage embedded part, and concrete in the rack load-transfer system(RLTS) of the Three Gorges shiplift, for studying the static stress of main parts under the contact loads between the pinion and rack and temperature variation, and the fatigue strength of the pinion and the rack under the contact force. As for the nut columns, a physical model was built for test research of the stress and deformation of the parts of the nut column load transfer system(NCLTS) of the Three Gorges shiplift[8]. A model with finite element method of NCLTS of the Three Gorges shiplift was built by Li Zhi et al[9], which studies the static stress and deformation of the main parts in the system under the mechanical loads resulting from the accidents mentioned above and the temperature variation load. The site testing results for measurements of stress and deformation of NCLTS which aiming at testifying the safety of the shiplift under the accident of ship chamber empty conducted during shiplift's commissioning is also introduced in this paper.

Considering the structural complexity of the RLTSs and the NCLTSs, computation of the stress and deformation of the structure using the finite element method is necessary because it supplies the powerful means to testify accurately the strength and rigidity of the designed structure. On the other hand, the computation using finite element method is very time-consuming and is not convenient to conduct parameter study. So this method is not suitable to the primary design stage in which repeated tentative calculations under different groups of parameters are needed to determine primarily the proper design parameters (dimensions of cross sections for example) of the racks, nut columns and adjusting beams. In order to pursue a simpler design-oriented calculation method for primary design stage, this paper proposes a so-called double-elastic-foundation-beam model (DEFMB) for analysis of the mechanical characteristics of both the RLTSs and the NCLTSs, and derives the formulas for calculation of the internal forces and deflections of the racks and the adjusting beams of the RLTSs under the maximum horizontal load, and of the internal forces and deflection of the nut column and adjusting beams of the NCLTSs s under the maximum vertical load. These design-oriented methods supply the reference for primary design of the RLTSs and NCLTSs of the rack and pinion vertical shiplifts.

2. Modelling of the DEFMB for both RLTSs and NCLTSs

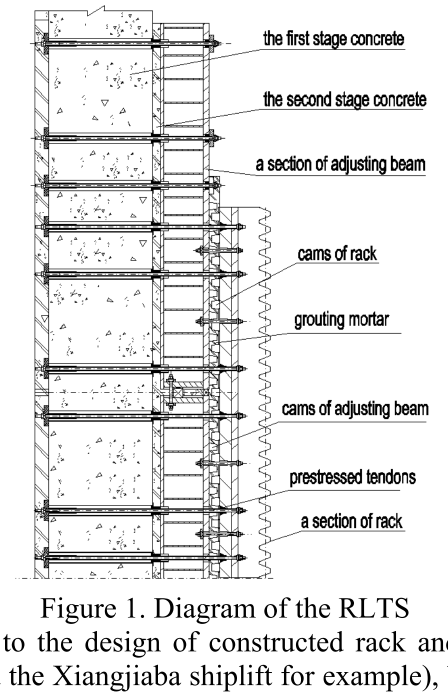


Figure 1. Diagram of the RLTS

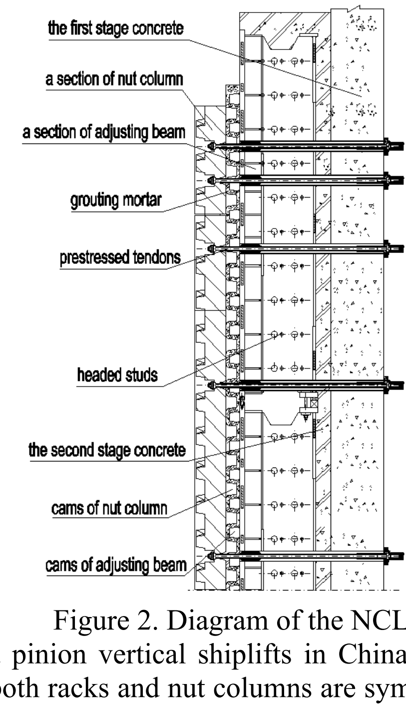


Figure 2. Diagram of the NCLTS

According to the design of constructed rack and pinion vertical shiplifts in China(the Three-Gorge shiplift and the Xiangjiaba shiplift for example), both racks and nut columns are symmetrically placed on the longitudinal walls in four grooves in the column towers, and cover the range of the hoist height. Each of the four lines of racks and four nut columns consists of dozens of sections of racks or nut columns.

As shown in Figure 1, a RLTS consists of dozens of rack units with lateral guiding rails, adjust steel beams, grouting mortar, the second stage concrete, the first stage concrete and prestressed tendons, and so on. Racks and adjusting beams in a line are interlaced to obtain the superior performance in load transfer. The cams are evenly set on the opposite vertical surfaces of racks and adjusting beams, and the interval between adjacent cams are filled by the grouting mortar, so that the loads can be transferred continuously from racks to the adjusting beams, and further to the second stage concrete and the first stage concrete. Pretension is applied in the prestressed tendons, which yields the initial compressive stress inside the grouting mortar, the second stage concrete and the first stage concrete and the interfaces between the beams and their support foundations, and ensures that racks and adjusting beams are always supported compressively by the grouting mortar and second stage concrete.

A NCLTS consists of nut columns, adjusting steel beams, grouting mortar, the second stage concrete, the first stage concrete and the prestressed tendons, as shown in Figure 2. The structural features of the NCLTSs are similar to that of the RLTSs. The vertical loads on the flank surfaces of the threads of the nut columns from the safety mechanisms of ship chambers are transferred to the tower columns through the grouting mortar, the adjusting beams, the second stage concrete and the first stage concrete. For a section of the nut column, the bending moments caused by the vertical loads are balanced by the horizontal support force from grouting mortar unevenly distributed along the whole section. By producing the initial compressive stress inside the whole load bearing system, the forces on the interfaces between the nut columns and grouting mortar and the interfaces between adjusting beams and the second stage concrete for example, are always pressure rather than tension. So no matter which directions of external loads are, the constraints of racks, nut columns and adjusting beams by grouting mortar or the second stage concrete can be treated as bilateral ones. Based on this, the racks, nut columns and adjusting beams may be treated as beams on elastic foundations.

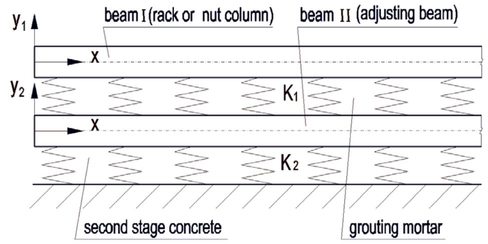
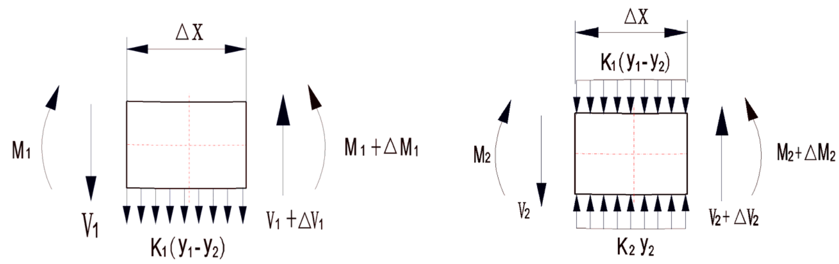


Figure 3. Diagram of a DEFMB

A DEFMB is shown in the Figure 3, in which “beam I” represents the racks or nut columns supported on the grouting mortar, “beam II” represents the adjusting beams in RLTSs or in NCLTSs which are embedded in the mortar and the second stage concrete. In this mechanical model, we ignore the interlacement of beam I and beam II for the convenience of calculation, the way in which the physical model for the NCLTS of the Three Gorges shiplift was fabricated[8]. The maximum lateral horizontal seismic design is treated as design load for RLTSs. The design load may occur at any position of the racks. When the design load is applies at the ends of a section of rack, the rack is mostly unfavourable as far as its static strength concerned. So in this research we consider the design load is applied at one end. For nut column load-bearing systems, the design bending moment is considered to be applied at one end of the nut column for the same reason. Based on this consideration, we model the rack, nut and the attached adjusting beams as the semi-infinite beams on Winkler foundations that behave like an array of independent springs as shown in Figure 3 [10]. The rationality for semi-infinite beam assumption will be verified by the calculation results described later.

The Figure 4 is the diagram to illustrate the forces in micro segments of the “beam I” and “beam II”, in which it is supposed that upward deflections are positive and the distributed reactions of the elastic foundations on the beams caused by upward deflections of the beams are positive. The signs of the bending moments and shear forces are regulated in accord with Material Mechanics.



(a) Illustration of the forces in a micro-section of beam I (b) Illustration of the forces in a micro-section of beam II

Figure 4 Illustrations of forces in micro-section of beams in DEFMB

According to the Figure 3(a) and Figure 3(b), we list the equilibrium equations for shear forces and bending moments of the micro segment of beam I and beam II as followings:

$$\Delta V_1 - K_1(y_1 - y_2)\Delta x = 0 \tag{1}$$

$$\Delta M_1 + V_1\Delta x = 0 \tag{2}$$

$$\Delta V_2 + K_1(y_1 - y_2) - K_2y_2\Delta x = 0 \tag{3}$$

$$\Delta M_2 + V_2\Delta x = 0 \tag{4}$$

Where y_1 and y_2 denote respectively the deflections of the beam I, V_1 and M_1 are respectively the shear force and the bending moment of the beam I, V_2 and M_2 are respectively the shear force and the bending moment of the beam II, K_1 and K_2 are respectively the spring constants for the mortar and the second stage concrete. For an elastic foundation beam with width b, the spring constant K can be determined as followings[11]:

$$K = bk \tag{5}$$

Where b is the width of beams, k is the coefficient of subgrade reaction and can been calculated by following formulas[12]:

$$k = \frac{E_0}{H(1 - \nu_0{}^2)} \tag{6}$$

$$E_0 = \frac{E_s}{1 - \nu_s{}^2} \tag{7}$$

$$\nu_0 = \nu_s(1 - \nu_s) \tag{8}$$

Where H is the thickness of the foundations, and E_s and ν_s are respectively the elastic modulus and the Poisson's rations of the foundation materials.

In the rack and pinion vertical shiplifts built in China, the material of the foundations of beam I, which are nut columns and racks, is PAGEL V1/50, a German product of mortar material. According to the product performance data and the results of the physical test on performances of PAGEL V1/50[13], the Yang's elastic modulus and the Poisson's ratio of PAGEL V1/50 are respectively $E_{s1}=3.8\times10^4$N/mm^2 and $\nu_{s1}=0.2$. The material of the foundations of beam II, which are adjusting beams for nut columns and racks, is ordinary concrete, of which the Yang's elastic modulus and the Poisson's ratio are respectively $E_{s2}=2.0\times10^4$N/mm^2 and $\nu_{s2}=0.1667$. The spring constants K_1 and K_2 for RLTSs and NCLTSs can be determined by substituting the parameters of the physical performance and the geometrical dimensions into formulas (5)~(8).

In formulas (1) and (2), let $\triangle x \to 0$ we get equilibrium differential equation for beam I:

$$\frac{dV_1}{dx} - K_1(y_1 - y_2) = 0 \tag{9}$$

$$\frac{dM_1}{dx} + V_1 = 0 \tag{10}$$

Taking the derivative of formula (10) to x, and substituting the result into the formula (9), one gets

$$\frac{d^2M_1}{dx^2}+K_1(y_1-y_2)=0 \tag{11}$$

According to the Material Mechanics, for beam I with small deflection, there exists following formula:

$$\frac{d^2y_1}{dx^2}=\frac{M_1}{EI_1} \tag{12}$$

Where E is the elastic modulus of steel and I_1 is the area inertial moment of the beam I. Substituting equation (12) into equation (11), one obtains

$$EI_1\frac{d^4y_1}{dx^4}+K_1\left(y_1-y_2\right)=0 \tag{13}$$

This is the deflection control equation for beam I.

For beam II according to the equation (3) and equation (4) we have the following equations:

$$\frac{dV_2}{dx}+K_2(y_1-y_2)-K_2y_2=0 \tag{14}$$

$$\frac{dM_2}{dx}+V_2=0 \tag{15}$$

Substituting (15) into (14), we obtain

$$\frac{d^2M_2}{dx^2}+(K_1+K_2)y_2-K_1y_1=0 \tag{16}$$

Similar to (12), there exists following identity for beam II

$$\frac{d^2y_2}{dx^2}=\frac{M_2}{EI_2} \tag{17}$$

Where I_2 is the area inertial moment of the beam II. We obtain the following equation by substituting (17) into (16)

$$\mathrm{EI}_2\frac{d^4y_2}{dx^4}+\left(K_1+K_2\right)y_2-K_1y_1=0 \tag{18}$$

Equation (13) can be changed into following form

$$y_2=\frac{\mathrm{EI}_1}{K_1}\frac{d^4y_1}{dx^4}+y_1 \tag{19}$$

Substituting equation (19) into equation (18), one obtains

$$\mathrm{EI}_2\frac{d^4}{dx^4}\left(\frac{\mathrm{EI}_1}{K_1}\frac{d^4y_1}{dx^4}+y_1\right)+\left(K_1+K_2\right)\left(\frac{\mathrm{EI}_1}{K_1}\frac{d^4y_1}{dx^4}+y_1\right)-K_1y_1=0 \tag{20}$$

By reorganization of (20), the following equation is derived:

$$\frac{\mathrm{E}^2\mathrm{I}_1\mathrm{I}_2}{K_1}\frac{d^8y_1}{dx^8}+\left(\mathrm{EI}_2+\frac{K_1+K_2}{K_1}\mathrm{EI}_1\right)\frac{d^4y_1}{dx^4}+K_2y_1=0 \tag{21}$$

Define

$$a=\frac{\mathrm{EI}_2+\frac{K_1+K_2}{K_1}\mathrm{EI}_1}{\frac{\mathrm{E}^2\mathrm{I}_1\mathrm{I}_2}{K_1}}=\frac{1}{\mathrm{E}}\left(\frac{K_1}{\mathrm{I}_1}+\frac{K_1+K_2}{\mathrm{I}_2}\right) \tag{22}$$

$$b=\frac{K_1K_2}{\mathrm{E}^2\mathrm{I}_1\mathrm{I}_2} \tag{23}$$

Equation（21）can be written in a simpler form

$$\frac{d^8y_1}{dx^8}+a\frac{d^4y_1}{dx^4}+by_1=0 \tag{24}$$

Because beam I and beam II are supposed to be semi-infinite beams, the deflections of the beams are zero when x→∞. The resolution of the equation (24) can be expressed as the following function of the deflection of beam I about argument x:

$$y_1 = e^{-\beta_1 x}\left(A\sin\beta_1 x + B\cos\beta_1 x\right) + e^{-\beta_2 x}\left(C\sin\beta_2 x + D\cos\beta_2 x\right) \tag{25}$$

Where

$$\beta_1 = \left(\frac{1}{2}\left(a + \sqrt{a^2 - 4b}\right)\right)^{\frac{1}{4}} \tag{26}$$

$$\beta_2 = \left(\frac{1}{2}\left(a - \sqrt{a^2 - 4b}\right)\right)^{\frac{1}{4}} \tag{27}$$

A、B、C、D are constants to be determined by the boundary conditions.

Substituting the function (25) into the formula (19), we obtain function of the deflection of beam II about argument x:

$$\begin{aligned} y_2 &= A\left(1-\frac{4\beta_1{}^4\mathrm{EI}_1}{k_1}\right)e^{-\beta_1 x}\sin\beta_1 x + B\left(1-\frac{4\beta_1{}^4\mathrm{EI}_1}{k_1}\right)e^{-\beta_1 x}\cos\beta_1 x \\ &+ C\left(1-\frac{4\beta_2{}^4\mathrm{EI}_1}{k_1}\right)e^{-\beta_2 x}\sin\beta_2 x + D\left(1-\frac{4\beta_{21}{}^4\mathrm{EI}_1}{k_1}\right)e^{-\beta_2 x}\cos\beta_2 x \end{aligned} \tag{28}$$

Define

$$\xi_1 = 1-\frac{4\beta_1{}^4 EI_1}{k_1} = 1-\frac{(a+\sqrt{a^2-4b})EI_1}{2k_1} \tag{29}$$

$$\xi_2 = 1-\frac{4\beta_2{}^4 EI_1}{k_1} = 1-\frac{(a-\sqrt{a^2-4b})EI_1}{2k_1} \tag{30}$$

The formula (28) can be simplified as following:

$$y_2 = \xi_1 Ae^{-\beta_1 x}\sin\beta_1 x + \xi_1 Be^{-\beta_1 x}\cos\beta_1 x + \xi_2 Ce^{-\beta_2 x}\sin\beta_2 x + \xi_2 De^{-\beta_2 x}\cos\beta_2 x \tag{31}$$

The internal bending moments of beam I and beam II are respectively derived:

$$\begin{aligned} M_1(x) &= EI_1\frac{d^2y_1}{dx^2} \\ &= 2EI_1\beta_1{}^2e^{-\beta_1 x}\left(B\sin\beta_1 x - A\cos\beta_1 x\right) + 2EI_1\beta_2{}^2e^{-\beta_2 x}\left(D\sin\beta_2 x - C\cos\beta_2 x\right) \end{aligned} \tag{32}$$

$$\begin{aligned} M_2 &= EI_2\frac{d^2y_2}{dx^2} \\ &= 2EI_2\xi_1\beta_1{}^2e^{-\beta_1 x}\left(B\sin\beta_1 x - A\cos\beta_1 x\right) + 2EI_2\xi_2\beta_2{}^2e^{-\beta_2 x}\left(D\sin\beta_2 x - C\cos\beta_2 x\right) \end{aligned} \tag{33}$$

The internal shear forces of beam I and beam II are respectively derived

$$\begin{aligned} V_1(x) &= -\frac{dM_1}{dx} = -EI_1\frac{d^3y_1}{dx^3} \\ &= 2EI_1\beta_1{}^3(B-A)e^{-\beta_1 x}\sin\beta_1 x - 2EI_1\beta_1{}^3(A+B)e^{-\beta_1 x}\cos\beta_1 x \\ &+ 2EI_1\beta_2{}^3(D-C)e^{-\beta_2 x}\sin\beta_2 x - 2EI_1\beta_2{}^3(C+D)e^{-\beta_2 x}\cos\beta_2 x \end{aligned} \tag{34}$$

$$\begin{aligned} V_2(x) &= -\frac{dM_2}{dx} = -EI_2\frac{d^3y_2}{dx^3} \\ &= 2EI_2\xi_1\beta_1{}^3(B-A)e^{-\beta_1 x}\sin\beta_1 x - 2EI_2\xi_1\beta_1{}^3(A+B)e^{-\beta_1 x}\cos\beta_1 x \\ &+ 2EI_2\xi_2\beta_2{}^3(D-C)e^{-\beta_2 x}\sin\beta_2 x - 2EI_2\xi_2\beta_2{}^3(C+D)e^{-\beta_2 x}\cos\beta_2 x \end{aligned} \tag{35}$$

3. Analysis of internal forces and deflections of NCLTS

As specified in the Chinese code for shiplift design[14], the most unfavourable case for design of the NCLTS is the one so-called “ship chamber emptying by water leakage”, or “ship chamber emptying” in simple. In this case, the unbalance load of ship chambers caused by losing weight of water in the chamber is applied on the NCLTSs through the contact of screw thread pairs. In the DEFMB for the NCLTSs shown in Figure 5, the concentrated moment M_{01} results from the transfer of the concentrated force applied at the left end of the nut column to the left end of the adjusting beam by the cams, and M_{02} results from the transfer of the force applied at the cams to the neutral axis of the adjusting beam. In the figure y_{n1} and y_{n2} are respectively the deflections of the nut column and the adjusting beam, and K_{n1} and K_{n2} are respectively the spring constants of the elastic foundations of the nut column and the adjusting beam.

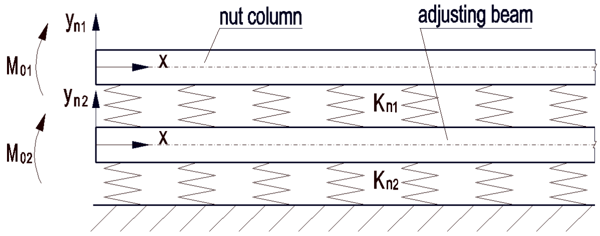


Figure 5 Diagram of DEFMB for NCLTS

The boundary conditions for NCLTS are given as followings:

For the nut column

$$M_{n1}(0) = EI_{n1} \left. \frac{d^2 y_{n1}(x)}{dx^2} \right|_{x=0} = M_{01} \tag{36}$$

$$V_{n1}(0) = -EI_{n1} \left. \frac{d^3 y_{n1}(x)}{dx^3} \right|_{x=0} = 0 \tag{37}$$

For the adjusting beam

$$M_{n2}(0) = \mathrm{EI}_{n2} \left. \frac{d^2 y_2(x)}{dx^2} \right|_{x=0} = M_{02} \tag{38}$$

$$V_{n2}(0) = -EI_{n2} \left. \frac{d^3 y_2}{dx^3} \right|_{x=0} = 0 \tag{39}$$

Where I_{n1} and I_{n2} denote respectively the area inertial moments of the cross sections of the nut column and the adjusting beam, $M_{n1}(x)$ and $M_{n2}(x)$ denote respectively the distribution functions of bending moments of the nut column and the adjusting beam, $V_{n1}(x)$ and $V_{n2}(x)$ denote respectively the distribution functions of shear forces of the nut column and the adjusting beam, M_{01} and M_{02} denote respectively the concentrated bending moments applied at the end of the nut column and adjusting beam can be calculated according to following formulas:

$$M_{01} = P_m.l_s \tag{40}$$

$$M_{02} = P_m.l_b \tag{41}$$

Where P_m denotes the maximum vertical load for single mechanism caused by ship chamber emptying, and l_s denotes the distance from the pitch line of the teeth of the nut column to the loading line of cams of the nut column and the adjusting beam, l_b is the distance from the loading line of cams of the nut column and the adjusting beam to the neutral axis of the adjusting beam, as shown in the Figure 6.

According to (32) and (34), the boundary condition for nut column (36) and (37) can be written in the following form:

$$-EI_{n1}\left(A\beta_{n1}^{2}+C\beta_{n2}^{2}\right)=\frac{M_{01}}{2} \qquad (42)$$

$$2\beta_{n1}^{3}\left(A+B\right)+2\beta_{n2}^{3}\left(C+D\right)=0 \qquad (43)$$

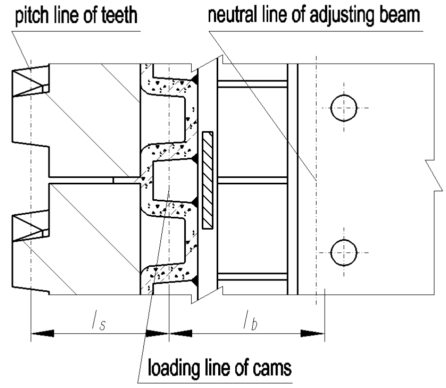


Figure 6 Illustration of the distances between loading lines of the forces

Where β_{n1} and β_{n2} are respectively the specified values of β_1 and β_2 for the NCLTS, obtained by applying equations (26), (27), (22) and (23) with I_1 and I_2 replaced respectively by I_{n1} and I_{n2}. Similarly the boundary conditions for adjusting beam of NCLTS can be derived as followings:

$$-EI_{n2}^{2}\left(\beta_{n1}^{2}\xi_{n1}A+\beta_{n2}^{2}\xi_{n2}C\right)=\frac{M_{02}}{2} \qquad (44)$$

$$2\beta_{n1}^{3}\xi_{n1}\left(A+B\right)+2\beta_{n2}^{3}\xi_{n2}\left(C+D\right)=0 \qquad (45)$$

Where ξ_{n1} and ξ_{n2} are respectively the specified values ofξ_1 and ξ_2 for NCLTS, obtained by applying equations (29) and (30) with I_1 and I_2 replaced respectively by I_{n1} and I_{n2}.

The equation (42) can be rewritten in the following form:

$$\beta_{n2}^{2}C=-\frac{M_{01}}{2EI_{n1}}-\beta_{n1}^{2}A \qquad (46)$$

Substituting (46）into (44) yields

$$\beta_{n1}^{2}\xi_{n1}A+\xi_{n2}\left(-\frac{M_{01}}{2EI_{n1}}-\beta_{n1}^{2}A\right)=-\frac{M_{02}}{2EI_{n2}} \qquad (47)$$

Then we obtain the expression for the constant A for NCLTS

$$A=\frac{\xi_{n2}\eta_{n}M_{01}-M_{02}}{2EI_{n1}\beta_{n1}^{2}(\xi_{n1}-\xi_{n2})\eta_{n}} \qquad (48)$$

By the same way, we obtain the expression for the constant C

$$C=\frac{\xi_{n1}\eta_{n}M_{01}-M_{02}}{2EI_{n1}\beta_{n2}^{2}(\xi_{n2}-\xi_{n1})\eta_{n}} \qquad (49)$$

Where

$$\eta_{n}=\frac{I_{n2}}{I_{n1}} \qquad (50)$$

Based on (43), (45), (48) and (49), the constants B and D for NCLTS can be derived:

$$B=-A=-\frac{\xi_{n2}\eta_{n}M_{01}-M_{02}}{2EI_{n1}\beta_{n1}^{2}(\xi_{n1}-\xi_{n2})\eta_{n}} \qquad (51)$$

$$D=-C=-\frac{\xi_{n1}\eta_{n}M_{01}-M_{02}}{2EI_{n1}\beta_{n2}^{2}(\xi_{n2}-\xi_{n1})\eta_{n}} \qquad (52)$$

Substituting expressions for A, B, C and D into (22) and (28)~32 yields the following formulas for description of the distributions of deflections, internal bending moments and internal shear forces of the nut columns and the adjusting beams in the NCLTS:

$$y_{n1}(x) = \frac{\xi_2\eta_n M_{01} - M_{02}}{2EI_{n1}\beta_1^2(\xi_{n1} - \xi_{n2})\eta_n} e^{-\beta_{n1}x}\left(\sin\beta_{n1}x - \cos\beta_{n1}x\right) + \frac{\xi_{n1}\eta_n M_{01} - M_{02}}{2EI_{n1}\beta_{n2}^2(\xi_{n2} - \xi_{n1})\eta_n} e^{-\beta_2 x}\left(\sin\beta_{n2}x - \cos\beta_{n2}x\right) \quad (53)$$

$$y_{n2}(x) = \frac{\xi_{n1}(\xi_{n2}\eta_n M_{01} - M_{02})}{2EI_{n1}\beta_{n1}^2(\xi_{n1} - \xi_{n2})\eta_n} e^{-\beta_{n1}x}\left(\sin\beta_{n1}x - \cos\beta_{n1}x\right) + \frac{\xi_{n2}(\xi_{n1}\eta_n M_{01} - M_{02})}{2EI_{n1}\beta_{n2}^2(\xi_{n2} - \xi_{n1})\eta_n} e^{-\beta_{n2}x}\left(\sin\beta_{n2}x - \cos\beta_{n2}x\right) \quad (54)$$

$$M_{n1}(x) = \frac{\xi_2\eta_n M_{01} - M_{02}}{(\xi_{n2} - \xi_{n1})\eta_n} e^{-\beta_{n1}x}\left(\sin\beta_{n1}x + \cos\beta_{n1}x\right) - \frac{\xi_{n1}\eta_n M_{01} - M_{02}}{(\xi_{n2} - \xi_{n1})\eta_n} e^{-\beta_{n2}x}\left(\sin\beta_{n2}x + \cos\beta_{n2}x\right) \quad (55)$$

$$M_2 = \frac{\xi_{n1}(\xi_{n2}\eta_n M_{01} - M_{02})}{\xi_{n2} - \xi_{n1}} e^{-\beta_{n1}x}\left(\sin\beta_{n1}x + \cos\beta_{n1}x\right) - \frac{\xi_{n2}(\xi_{n1}\eta_n M_{01} - M_{02})}{\xi_{n2} - \xi_{n1}} e^{-\beta_{n2}x}\left(\sin\beta_{n2}x + \cos\beta_{n2}x\right) \quad (56)$$

$$V_{n1}(x) = \frac{2\beta_{n1}(\xi_{n2}\eta_n M_{01} - M_{02})}{(\xi_{n2} - \xi_{n1})\eta_n} e^{-\beta_{n1}x}\sin\beta_{n1}x + \frac{2\beta_{n2}(\xi_{n1}\eta_n M_{01} - M_{02})}{(\xi_2 - \xi_1)\eta_n} e^{-\beta_{n2}x}\sin\beta_{n2}x \quad (57)$$

$$V_{n2}(x) = \frac{2\xi_{n1}\beta_{n1}(\xi_{n2}\eta_n M_{01} - M_{02})}{\xi_{n2} - \xi_{n1}} e^{-\beta_{n1}x}\sin\beta_{n1}x + \frac{2\beta_{n2}(\xi_{n1}\eta_n M_{01} - M_{02})}{\xi_{n2} - \xi_{n1}} e^{-\beta_{n2}x}\sin\beta_{n2}x \quad (58)$$

A calculation of the deflections, internal bending moments and internal shear forces of the nut column and the adjusting beam of the NCLTS of the Three Gorges shiplift has been performed as an illustrative example. The parameters needed for calculation are shown in Table 1. Both the lengths of the nut column and adjusting beam are 4950mm.

Table 1. The DEFMB parameters of NCLTS of the Three Gorge shiplift

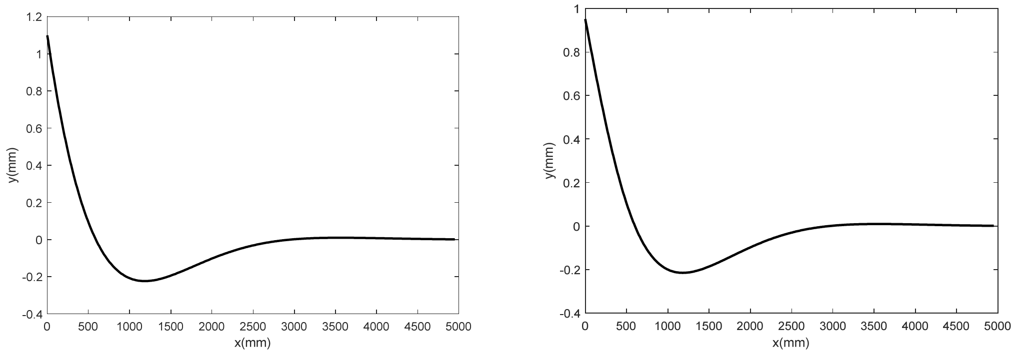
Components	b (m)	H (m)	K_n (N/mm^2)	I_n (mm^4)	M_0 (Nm)
Nut column	1.60	0.18	4.69×10^5	5.79×10^9	5.623×10^6
Adjusting beam	1.02	1.13	3.81×10^4	4.15×10^{10}	5.571×10^6

The results are shown in Figure 7~Figure 9. Figure 7 illustrates the distribution functions of the deflections of the nut column and adjusting beam along the axis direction. The deflection distribution curves supply the useful information for determination of the pretensions and arrangement of the prestressed tendons of the NCLTS. The maximum deflections of the nut column and adjusting beam are respectively 1.098mm and 0.951mm, both occurring at the left ends. At the right ends(x=4950mm), the deflections of the nut column and the adjusting beam are respectively 1.02×10^{-3}mm and 9.82×10^{-4}mm, which are far less than the maximum deflections. This fact verifies the rationality of the semi-infinite beam assumption in DEFMB for NCLTS.

Figure 8 shows the distributions of the internal bending moment $M_{n1}(x)$ of the nut column and the internal bending moment $M_{n2}(x)$ of the adjusting beam. The maximum internal bending moment of the nut column occurs at the left end, which equals to the concentrated moment M_{01}. The maximum internal bending moment of the adjusting beam is 2.21×10^7Nm, which occurs at x=471.4mm.

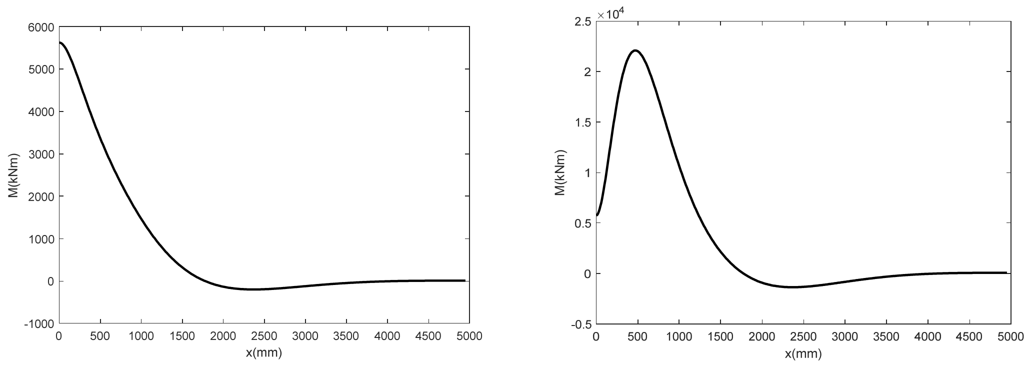
Figure 9 shows the distribution functions of the internal shear force $V_{n1}(x)$ of the nut column and the internal shear force $V_{n2}(x)$ of the adjusting beam. The maximum shear force of the nut column is

5.506×10^6N occurring at x=278.2mm, and the maximum shear force of the adjusting beam is -5.68×10^6N occurring at x=159.0mm.



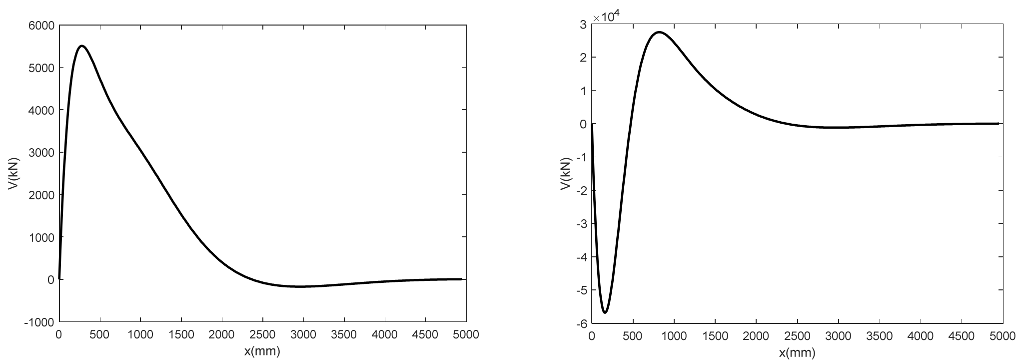
(a) Deflection of the nut column (b) Deflection of the adjusting beam

Figure 7 Deflection functions of the NCLTS of the Three Gorges shiplift



(b) Moment of the nut column (b) Moment of the adjusting beam

Figure 8 The ending moment functions of the NCLTS of the Three Gorges shiplift



(a) Shear force of the nut column (b) Shear force of the adjusting beam

Figure 9 The shear force functions of the NCLTS of the Three Gorges shiplift

4. Analysis of internal forces and deflections of RLTS

As mentioned above, the racks as the guiding rails supply the lateral guiding for ship chamber and bear the seismic forces between the ship chambers and tower columns. The seismic forces may be applied at the any location of any section of a rack in the whole elevation range within which the ship chamber is hoisted. In this paper, we treat the seismic force as a concentrated force that applied at the one end of the rack, considering most unfavourable loading condition on the safe side. By the same way in building the mechanical model for NCLTS, a DEFMB is built for RLTS as shown in Figure 10. In the figure y_{r1} and y_{r2} are respectively the deflections of the rack and the adjusting beam, and K_{r1} and K_{r2} are respectively the spring constants of the elastic foundations of the rack and the adjusting beam.

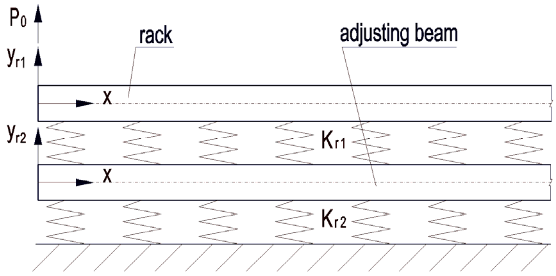


Figure 10 Diagram of DEFMB for RLTS

The boundary conditions for RLTS are given as followings:

For the rack

$$M_{r1}(0) = EI_{r1} \left. \frac{d^2 y_{r1}}{dx^2} \right|_{x=0} = 0 \qquad (59)$$

$$V_{r1}(0) = -EI_{r1} \left. \frac{d^3 y_{r1}}{dx^3} \right|_{x=0} = -P_0 \qquad (60)$$

For the adjusting beam

$$M_2(0) = EI_{r2} \left. \frac{d^2 y_{r2}}{dx^2} \right|_{x=0} = 0 \qquad (61)$$

$$V_{r2}(0) = -EI_{r2} \left. \frac{d^3 y_{r2}}{dx^3} \right|_{x=0} = 0 \qquad (62)$$

Where I_{r1} and I_{r2} denote respectively the area inertial moments of the cross sections of the rack and the adjusting beam, $M_{r1}(x)$ and $M_{r2}(x)$ denote respectively the distribution functions of the bending moments of the rack and the adjusting beam, $V_{r1}(x)$ and $V_{r2}(x)$ denote respectively the distribution functions of shear forces of the rack and the adjusting beam, P_0 denotes the seismic force on one set of the lateral guiding mechanism. Substituting the formulas (32)~(35) into the formulas (59)~(62) yields

$$\beta_{r1}^{\ 2} A + \beta_{r2}^{\ 2} C = 0 \qquad (63)$$

$$\beta_{r1}^{\ 3}(A + B) + \beta_{r2}^{\ 3}(C + D) = P_0 \qquad (64)$$

$$\xi_{r1}\beta_{r1}^{\ 2} A + \xi_{r2}\beta_{r2}^{\ 2} C = 0 \qquad (65)$$

$$\xi_{r1}\beta_{r1}^{\ 3}(A + B) + \xi_{r2}\beta_{r2}^{\ 3}(C + D) = 0 \qquad (66)$$

Where β_{n1} and β_{n2} are respectively the specified values of β_1 and β_2 for the RLTS obtained by applying equations (26), (27), (22) and (23) with I_1 and I_2 replaced respectively by I_{r1} and I_{r2}, and ξ_{r1} and ξ_{r2} are respectively the specified values of ξ_1 and ξ_2 for the RLTS obtained by applying equations (29) and (30) with I_1 and I_2 replaced respectively by I_{r1} and I_{r2}. Resolving the linear algebraic equations (63)~(66), we obtain the values of A and C and the expressions for B and D:

$$A = C = 0$$

$$B = \frac{\xi_2 P_0}{2EI_1\beta_1^{\ 3}(\xi_2 - \xi_1)} \qquad (67)$$

$$D = \frac{\xi_1 P_0}{2EI_1\beta_2^{\ 3}(\xi_1 - \xi_2)} \qquad (68)$$

Substituting the values of A and C and the expressions for B and D into formulas (25) and (31)~(35), we obtain the formulas for description of distributions of the deflection and the internal forces:

$$y_{r1}(x) = \frac{\xi_{r2}P_0e^{-\beta_{r1}x}\cos\beta_{r1}x}{2EI_1\beta_{r1}^3(\xi_{r2}-\xi_{r1})} + \frac{\xi_{r1}P_0e^{-\beta_{r2}x}\cos\beta_{r2}x}{2EI_{r1}\beta_{r2}^3(\xi_{r1}-\xi_{r2})} \quad (69)$$

$$y_{r2}(x) = \frac{\xi_{r1}\xi_{r2}P_0e^{-\beta_{r1}x}\cos\beta_{r1}x}{2EI_{r1}\beta_{r1}^3(\xi_{r2}-\xi_{r1})} + \frac{\xi_{r1}\xi_{r2}P_0e^{-\beta_{r2}x}\cos\beta_{r2}x}{2EI_{r1}\beta_2^3(\xi_{r1}-\xi_{r2})} \quad (70)$$

$$M_{r1}(x) = \frac{\xi_{r2}P_0}{\beta_{r1}(\xi_{r2}-\xi_1)}e^{-\beta_{r1}x}\sin\beta_1 x + \frac{\xi_{r1}P_0}{\beta_{r2}(\xi_{r1}-\xi_{r2})}e^{-\beta_{r2}x}\sin\beta_{r2}x \quad (71)$$

$$M_{r2}(x) = \frac{\xi_{r1}\xi_2\eta_rP_0}{\beta_{r1}(\xi_{r2}-\xi_{r1})}e^{-\beta_{r1}x}\sin\beta_{r1}x + \frac{\xi_{r2}\xi_{r1}\eta_rP_0}{\beta_{r2}(\xi_{r1}-\xi_{r2})}e^{-\beta_{r2}x}\sin\beta_{r2}x \quad (72)$$

$$V_{r1}(x) = \frac{\xi_{r2}P_0}{\xi_{r2}-\xi_{r1}}e^{-\beta_{r1}x}(\sin\beta_{r1}x-\cos\beta_{r1}x) + \frac{\xi_{r1}P_0}{\xi_{r1}-\xi_{r2}}e^{-\beta_{r2}x}(\sin\beta_{r2}x-\cos\beta_{r2}x) \quad (73)$$

$$V_{r2}(x) = \frac{\xi_{r1}\xi_{r2}\eta_rP_0}{\xi_{r2}-\xi_{r1}}e^{-\beta_{r1}x}(\sin\beta_{r1}x-\cos\beta_{r1}x) + \frac{\xi_{r2}\xi_{r1}\eta_rP_0}{\xi_{r1}-\xi_{r2}}e^{-\beta_{r2}x}(\sin\beta_{r2}x-\cos\beta_{r2}x) \quad (74)$$

Where

$$\eta_r = \frac{I_{r2}}{I_{r1}}$$

A calculation of the deflections, internal bending moments and internal shear forces of the rack and the adjusting beam of the RLTS of the Three Gorges shiplift has been performed as an illustrative example. The parameters needed for calculation are shown in Table 2. Both the lengths of the rack and adjusting beam are 4750mm.

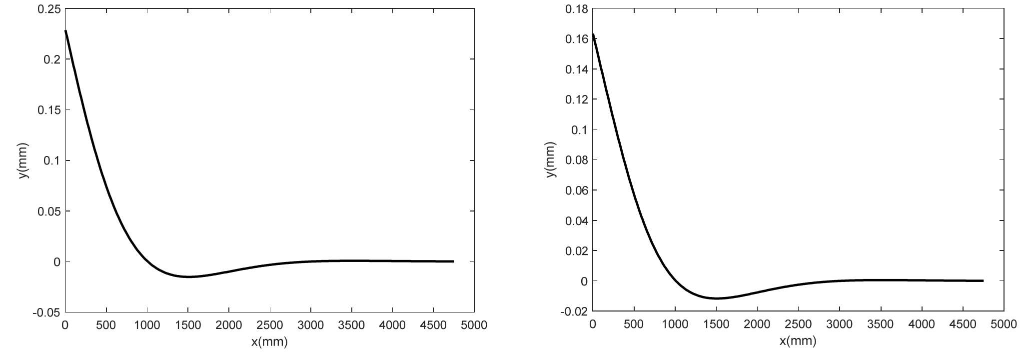
.Table 2. The DEFMB parameters of RLTS of the Three Gorges shiplift

Components	b (m)	H (m)	K_r (N/mm^2)	I_r (mm^4)	P_0 (N)
Rack	0.880	0.140	2.58×10^5	1.223×10^{10}	4.82×10^6
Adjusting beam	0.820	0.640	2.66×10^4	8.277×10^9	0

The results are shown in Figure 11~Figure 13. Figure 11 illustrates the distribution functions of the deflections of the rack and adjusting beam along the axis direction. The deflection distribution curves supply the useful information for determination of the pretensions and arrangement of the prestressed tendons of the RLTS. The maximum deflections of the rack and adjusting beam are respectively 0.229mm and 0.163mm, both occurring at the left ends. At the right ends(x=4750mm), the deflections of the rack and the adjusting beam are respectively 5.73×10^{-5}mm and 4.38×10^{-5}mm, which are far less than the maximum deflections, which shows the rationality of the semi-infinite beam assumption in DEFMB for RLTS.

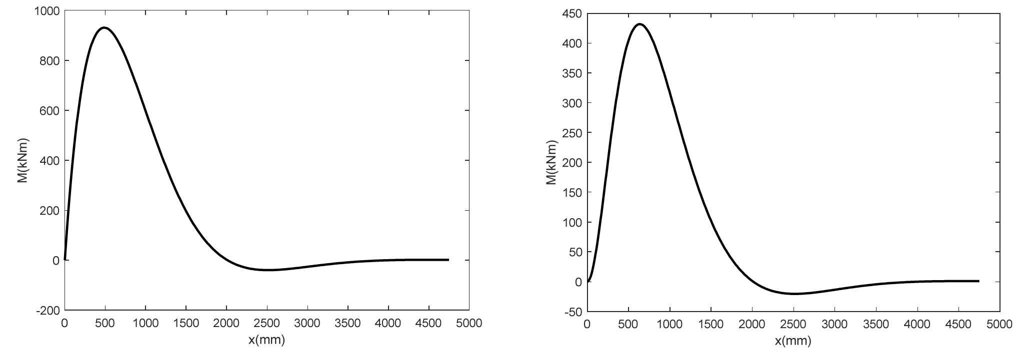
Figure 12 illustrates the distributions of the internal bending moment $M_{r1}(x)$ of the rack and the internal bending moment $M_{r2}(x)$ of the adjusting beam. The maximum internal bending moment of the rack is 9.307×10^5Nm occurring at x=488.1mm. The maximum internal bending moment of adjusting beam is 4.318×10^7Nm occurring at x=636.2mm.

Figure 13 illustrates the distributions of the internal shear force $V_{r1}(x)$ of the rack and the internal shear force $V_{r2}(x)$ of the adjusting beam. The maximum shear force of the rack occurs at the left end which equals to the concentrated force P_0. The maximum shear force of the adjusting beam is -1.116×10^5N occurring at x=221.7mm.



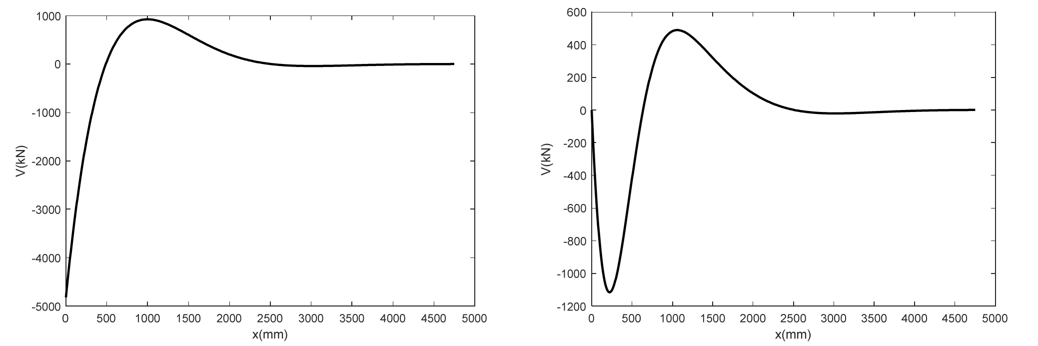
(a) Deflection of the rack (b) Deflection of the adjusting beam

Figure 11 Deflections functions of the RLTS of the Three Gorges shiplift



(a) Moment of the rack (b) Moment of the adjusting beam

Figure 12 Bending moment functions of the RLTS of the Three Gorges shiplift



(a) Shear force of the rack (b) Shear force of the adjusting beam

Figure 13 Shear force functions of the RLTS of the Three Gorges shiplift

5. Conclusion

Based on the structures and design conditions of the nut column load transfer systems and the rack load transfer systems of the rack and pinion vertical shiplift, this paper proposes a so-called the semi-infinite double elastic foundation beam model shared by both systems, and performs analytical research on the deflections and internal forces of the nut columns, racks and the second stage embedded parts attached, and provides a set of design-oriented formulas for shiplift designers to calculate the deflections, internal bending moments and shear forces of nut columns, racks and attached adjusting beam in primary design stage. The application of the derived formulas to the Three-Gorges shiplift is demonstrated, and the calculation results show that the semi-infinite assumption is rational.

Acknowledgments
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