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Parameter Determination and Model Modification of Sherwood-Frost Constitutive Model

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Parameter Determination and Model Modification of Sherwood-Frost Constitutive Model

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Abstract. The Sherwood-Frost constitutive model is widely used to predict the mechanical behavior of polymer materials, and it is a very important material model. In this study, the stress-strain curve of high-density polyethylene (HDPE) with a strain rate ranging from 935 to 5450 s^{-1} was obtained through the split Hopkinson bar test device. Based on the HDPE dynamic mechanical performance test, the method for determining the strain-rate parameters of the Sherwood-Frost constitutive model was analysed and the strain-rate term in the constitutive model revised. Using the experimental results of Sherwood and Frost, the method to determine the parameters of the density and temperature terms in the model was introduced. This study provides a reference for parameter determination and model modification of the Sherwood-Frost constitutive model.
Keywords: Sherwood-Frost constitutive model; SHPB; Strain rate; HDPE

1. Introduction

To describe the compressive stress-strain response of polyurethane foam under uniaxial compression impact load, Sherwood and Frost [1] added density and temperature functions to the integral power form strain-rate term proposed by Nagy *et al*. [2], and they established the Sherwood-Frost constitutive model of polyurethane foam. The test results of Sherwood and Frost showed that the model predicts the impact response of polyurethane-absorbing materials under uniaxial compression loads [1].

The Sherwood-Frost constitutive model has the advantages of simple form and comprehensive considerations [3]. In this study, high-density polyethylene (HDPE) was used as the research object, the uniaxial compression test was carried out on HDPE through the separated Hopkinson pressure bar test device (SHPB), and the stress-strain curve of HDPE strain-rate range of 935–5450 s^{-1} was obtained. The Sherwood-Frost constitutive equation was used to fit the dynamic stress-strain curve of HDPE, the strain-rate term in the constitutive model modified, and the parameter values in the model obtained. Using the experimental results of Sherwood and Frost, the method to determine the parameters of the density and temperature terms in the model was introduced. This study provides a reference for parameter determination and model modification of the Sherwood-Frost constitutive model.

2. HDPE Dynamic Compression Test

2.1. Experimental Design

Figure 1 shows a schematic of the SHPB test device. The device is mainly composed of a striker, incident bar, transmission bar, and damping device. The specific parameters of the bars are shown in table 1 [4].

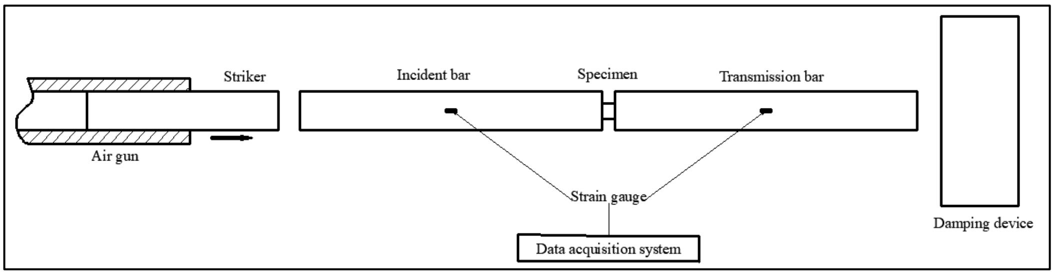

Figure 1. Schematic of SHPB test device.

Table 1. Parameters of SHPB bars [4].

E_0 (GPa)	Elastic wave velocity C_0 (m/s)	Bar diameter (mm)	Striker length (mm)	Incident bar length (mm)	Transmission bar length (mm)
70	4991	14.5	400	1500	2000

The HDPE used in this study is a commercially produced ordinary high-density polyethylene. The sizes of the HDPE specimens are Φ10 mm × 5 mm and Φ7 mm × 3.5 mm. The HDPE specimen was placed between the incident and transmission bars. The striker hit the incident bar at a certain speed, and an incident wave ε_i propagating to the right was generated on the incident bar. The specimen was deformed under the action of the incident wave, and the reflected wave ε_r was generated on the contact end of the incident bar and specimen, and the transmitted wave ε_t was generated on the transmission bar. These signals were measured by the strain gauges on the incident and transmission bars. The true stress-strain curve of HDPE was obtained through the following equations:

$$\dot{\varepsilon} = -\frac{2C_0}{L_S}\varepsilon_r(t), \tag{1}$$

$$\varepsilon_E = \int_0^t \dot{\varepsilon}\, dt, \tag{2}$$

$$\sigma_E = \frac{A_0 E_0}{A_S}\varepsilon_t(t), \tag{3}$$

$$\sigma_T = \sigma_E(1-\varepsilon_E), \tag{4}$$

$$\varepsilon_T = -\ln(1-\varepsilon_E). \tag{5}$$

2.2. Test Results

Figure 2 shows the dynamic stress-strain curve of HDPE. It can be seen from the test results that HDPE exhibits an obvious strain effect. Some scholars think that the strain-rate effect of a polymer is related to the secondary molecular process of polymers. The increase in strain rate will make the polymer chain hard, thus reducing its molecular mobility and resulting in the increase of material stress with increasing strain rate [5, 6].

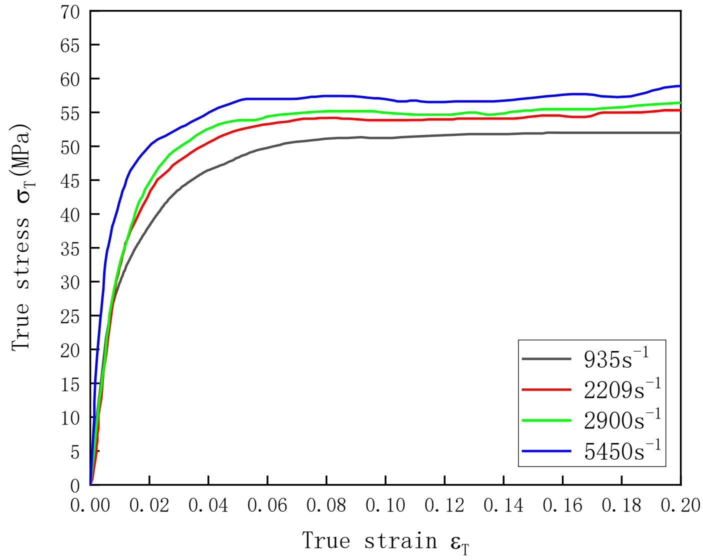

Figure 2. Stress-strain curves of HDPE under different strain rates.

3. Sherwood-Frost Constitutive Model and Parameter Determination

The Sherwood-Frost constitutive model expresses the true stress of the material as a combination of the material's shape function $f(\varepsilon)$, temperature function $H(T)$, density function $G(\rho)$, and strain-rate function $M(\varepsilon,\dot{\varepsilon})$ product, namely, [1]

$$\sigma_c = H(T)G(\rho)M(\varepsilon,\dot{\varepsilon})f(\varepsilon), \tag{6}$$

where $f(\varepsilon)$ is expressed as a polynomial function describing the shape of the stress-strain curve:

$$f(\varepsilon) = \sum_{n=1}^{10} A_n \varepsilon^n \tag{7}$$

$M(\varepsilon,\dot{\varepsilon})$ is the exponential strain-rate term proposed by Nagy *et al*. [2], expressed as

$$M(\varepsilon,\dot{\varepsilon}) = (\dot{\varepsilon}/\dot{\varepsilon}_0)^{n(\varepsilon)}, \tag{8}$$

$$n(\varepsilon) = b_1 + b_2\varepsilon. \tag{9}$$

Among these, A_n is a parameter describing the stress-strain shape of the material; $\dot{\varepsilon}_0$ is the lowest possible strain rate in the experiment; and b_1 and b_1 are the material parameters determined by experiments. The Sherwood-Frost constitutive model did not give specific functional forms of the temperature function $H(T)$ and density function $G(\rho)$.

3.1. Determination of Shape Function

The shape function in the constitutive model is determined by the compression test of the lowest possible strain rate $\dot{\varepsilon}_0$, the midpoint of the density range $\rho_{0.5}$, and the midpoint of the temperature range, $T_{0.5}$. The effect of density and temperature on the stress of HDPE was not investigated in this study. Therefore, it is considered that, when $\dot{\varepsilon} = \dot{\varepsilon}_0$ and $M(\varepsilon,\dot{\varepsilon}) = H(T) = G(\rho) = 1$, Eq. (6) can be described as follows:

$$\sigma_c = 1f(\varepsilon) = \textstyle\sum_{n=1}^{10} A_n \varepsilon^n. \tag{10}$$

The lowest possible strain rate in the HDPE dynamic test was $\dot{\varepsilon}_0 = 935\ \mathrm{s}^{-1}$. Fitting equation (10) with the true stress-strain curve under the condition of $\dot{\varepsilon} = \dot{\varepsilon}_0 = 935\ \mathrm{s}^{-1}$, the parameter values of A_n in f(ε) were obtained, as shown in table 2. Figure 3 shows the fitting result of Eq. (10).

Table 2. Parameter values of A_n in the polynomial $f(\varepsilon)$ [1].

n	1	2	3	4	5	6	7	8	9	10
A_n	5100.37	-299326.21	1.0559×10^{7}	-2.26268×10^{8}	3.0366×10^{9}	-2.60253×10^{10}	1.4214×10^{11}	-4.78187×10^{11}	9.02466×10^{11}	-7.30736×10^{11}

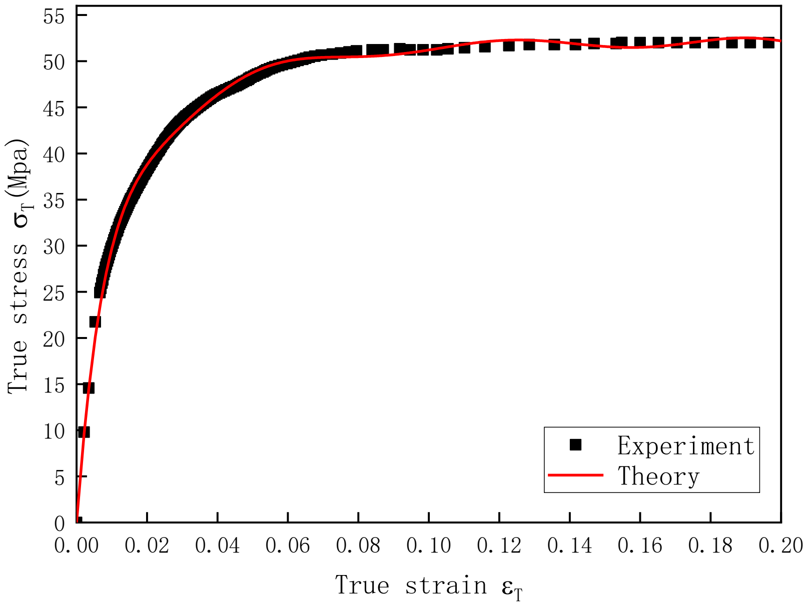

Figure 3. Fitting result of stress-strain curve of strain rate $935\ \mathrm{s}^{-1}$.

3.2. Determination of Strain-Rate Term

The strain-rate term was determined by the compression test under the condition of the midpoint of the density range, $\rho_{0.5}$; the midpoint of the temperature range, $T_{0.5}$; and different strain rates. The effect of density and temperature on the stress of HDPE was not researched in this study. Therefore, it is considered that $H(T) = G(\rho) = 1$, and Eq. (6) can be described as follows:

$$\sigma_c = M(\varepsilon, \dot{\varepsilon}) f(\varepsilon) \tag{11}$$

Therefore, the strain-rate term can be obtained by

$$M(\varepsilon, \dot{\varepsilon}) = \frac{\sigma_c}{f(\varepsilon)} \tag{12}$$

Figure 4 shows the results obtained by fitting the true stress-strain curves of HDPE at different strain rates with Eq. (12).

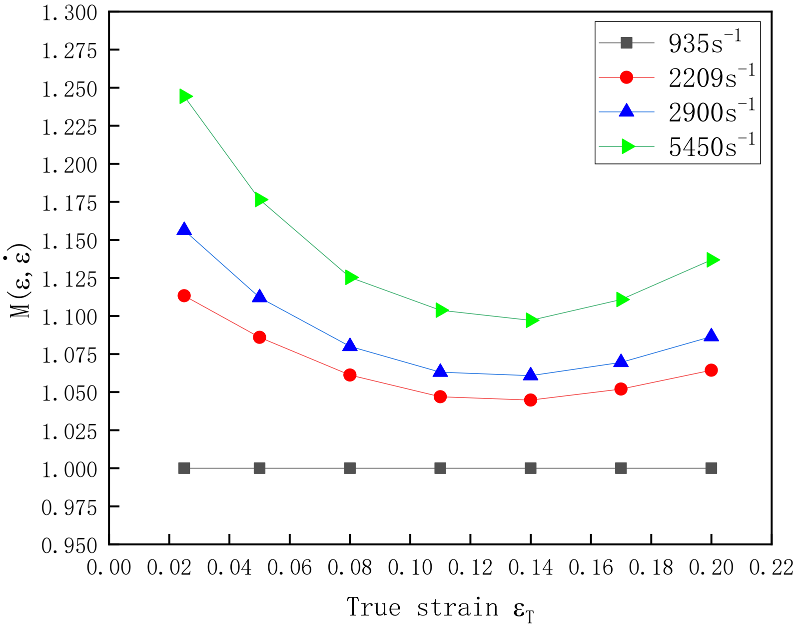

Figure 4 Value of $M(\varepsilon,\dot{\varepsilon})$ obtained by Eq. (12).

To obtain the specific form of $M(\varepsilon,\dot{\varepsilon})$, one takes the logarithm of both sides of Eq. (8) to obtain

$$n(\varepsilon) = \frac{\ln M(\varepsilon,\dot{\varepsilon})}{\ln \dot{\varepsilon}/\dot{\varepsilon}_0}. \tag{13}$$

Figure 5 shows the results obtained by fitting the true stress-strain curves of HDPE under different strain-rate conditions by Eq. (13). It can be found from figure 5 that $n(\varepsilon)$ and ε no longer have a simple linear relationship, and the experimental data points are in a non-linear relationship. The form of $n(\varepsilon)$ was obtained by fitting the data in figure 5, as shown in the following equation:

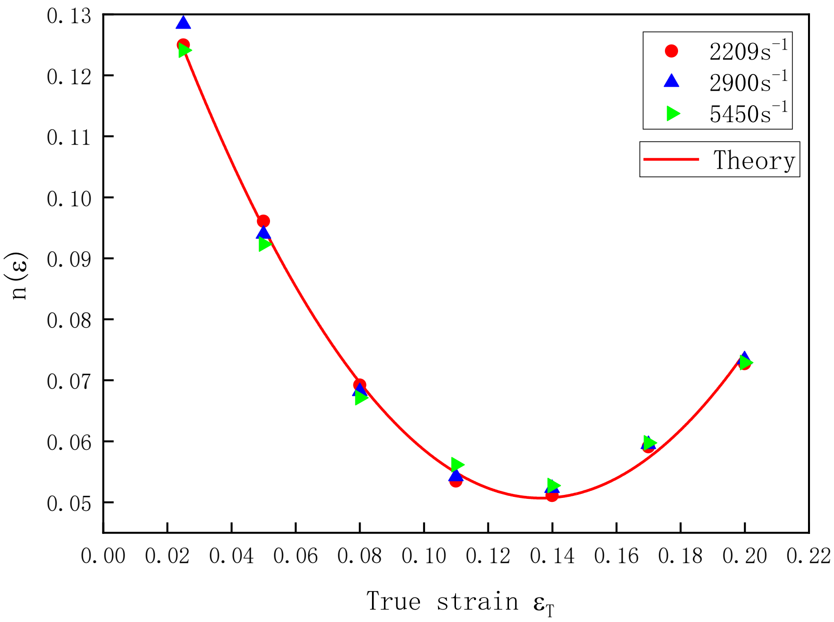

Figure 5. Value of $n(\varepsilon)$ obtained by Eq. (13).

$$n(\varepsilon) = 0.16106 - 1.61665\varepsilon + 5.91974\varepsilon^2. \tag{14}$$

Figure 6 shows the comparison between the model prediction and experimental results. It can be seen that the modified Sherwood-Frost constitutive model can better fit the stress-strain curve of HDPE under different strain rates.

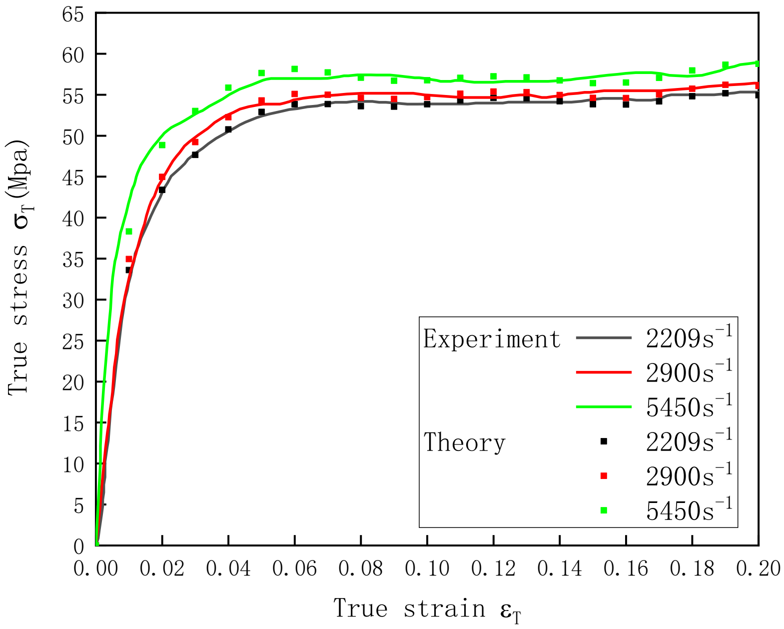

Figure 6. Comparison of prediction results of constitutive model with test results.

3.3. Determination of Density and Temperature Terms

In the study of HDPE compression mechanical properties, the influence of density and temperature was not considered. Therefore, in this study, Sherwood and Frost's research results on polyurethane foam were used to discuss the determination of the density and temperature terms in the Sherwood-Frost constitutive model.

3.3.1. Determination of Density Term. The density term was determined by the compression test under the condition of the lowest possible strain rate $\dot{\varepsilon}_0$; the midpoint of the temperature range, $T_{0.5}$, and different densities; that is, when $\dot{\varepsilon} = \dot{\varepsilon}_0$, $T = T_{0.5}$, and $M(\varepsilon, \dot{\varepsilon}) = H(T) = 1$, Eq. (6) can be described as follows:

$$\sigma_c = G(\rho) f(\varepsilon). \tag{15}$$

Therefore, the density term can be obtained by Eq. (15) as follows:

$$G(\rho) = \frac{\sigma_{c(\dot{\varepsilon}=\dot{\varepsilon}_0, T=T_{0.5})}}{f(\varepsilon)}. \tag{16}$$

Sherwood and Frost fitted the true stress-strain curve under the condition of the lowest possible strain rate $\dot{\varepsilon} = \dot{\varepsilon}_0 = 4.233 \times 10^{-3}\ \text{s}^{-1}$; the midpoint of the temperature range, T=20 ℃; and different densities through Eq. (16). The results are shown in figure 7. Equation (17) [1] is the form of $G(\rho)$ obtained by piecewise linear fitting:

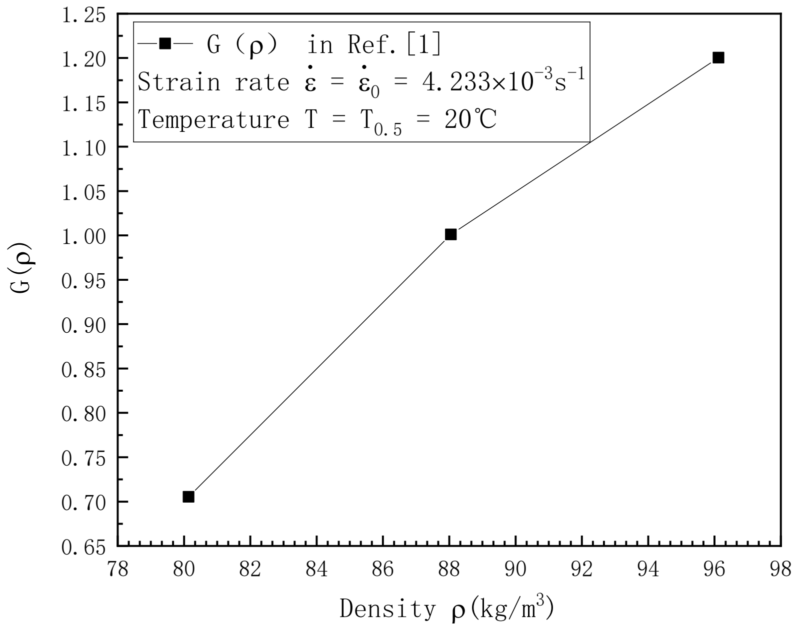

Figure 7. Value of $G(\rho)$ obtained by Eq. (16).

$$\begin{cases} G(\rho) = 0.03688\rho - 2.24 & \rho\epsilon[80{,}88)\ kg/m^3 \\ G(\rho) = 0.02500\rho - 1.20 & \rho\epsilon[88{,}96)\ kg/m^3 \end{cases}. \tag{17}$$

3.3.2. Determination of Temperature Term. The temperature term was determined by the compression test under the condition of the lowest possible strain rate $\dot{\varepsilon}_0$; the midpoint of the density range, $\rho_{0.5}$; and different temperatures; that is, when $\dot{\varepsilon} = \dot{\varepsilon}_0$, $\rho = \rho_{0.5}$, and $M(\varepsilon, \dot{\varepsilon}) = G(\rho) = 1$, Eq. (6) can be described as follows:

$$\sigma_c = H(T)f(\varepsilon). \tag{18}$$

Therefore, the temperature term can be obtained by Eq. (18) as follows:

$$H(T) = \frac{\sigma_{c(\dot{\varepsilon}=\dot{\varepsilon}_0,\rho=\rho_{0.5})}}{f(\varepsilon)}. \tag{19}$$

Sherwood and Frost fitted the true stress-strain curve under the condition of the lowest possible strain rate $\dot{\varepsilon} = \dot{\varepsilon}_0 = 4.233 \times 10^{-3}\ \mathrm{s}^{-1}$; the midpoint of the density range, $\rho = 88\,\mathrm{kg/m}^3$; and different temperatures through Eq. (19). The results are shown in figure 8. Equation (20) [1] is the form of $H(T)$ obtained by piecewise linear fitting:

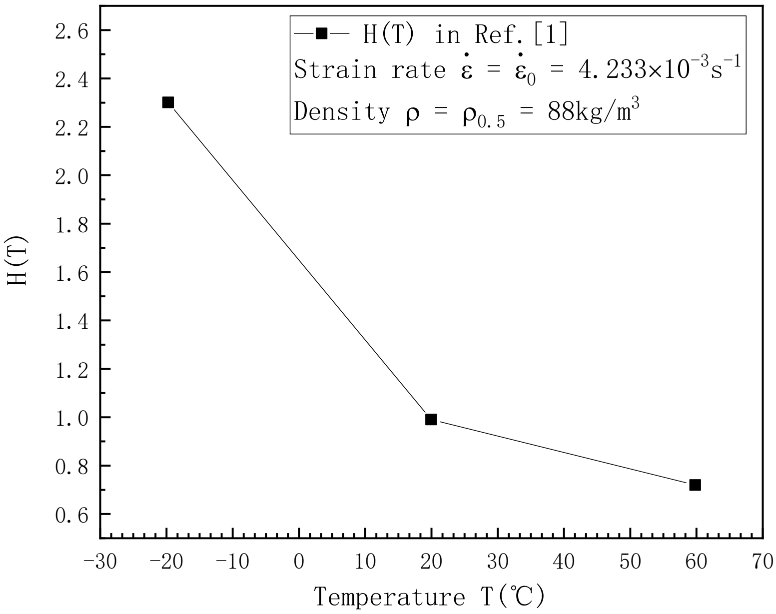

Figure 8. Value of $H(T)$ obtained by Eq. (19).

$$\begin{cases} H(T) = 1.6500 - 0.03250T & T\epsilon[-20,20)°C \\ H(T) = 1.1325 - 0.00663T & T\epsilon[20,60)°C \end{cases} \tag{20}$$

4. Conclusions

In this study, the strain-rate term in the Sherwood-Frost constitutive model was fitted based on dynamic compression tests on HDPE. Results show that the strain-rate exponential term $n(\varepsilon)$ of HDPE had a quadratic nonlinear relationship with strain. The constitutive model was modified using the HDPE test results, and the calculated results of the revised model were in good agreement with test results. Using the experimental results of Sherwood and Frost, the method to determine the parameters of the density and temperature terms in the model was introduced. This study provides a reference for parameter determination and model modification of the Sherwood-Frost constitutive model.

Declaration of Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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