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Correcting for risk premium on Extended Generalised Leland Models: an empirical study on Dow Jones Industrial Average (DJIA) index options

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Abstract. The relative option pricing performance of Extended Generalised Leland models is examined in this study. We generalise the extended Leland models based on the implied adjusted volatility introduced in the models. Non-parametric framework is fitted into parametric option-pricing framework based on the Leland models to achieve a more realistic option pricing. To reflect the real probability measure, the implied adjusted information is corrected in terms of risk premium. This study concentrates mainly in examining the option-implied information produced by the models after correcting for risk-premium. Data extracted from DJIA index options are employed in this study, which covers the period from January 2009 until the end of 2015. We discovered that the option-implied volatility, which is priced using the Extended Generalised Leland models, especially after being corrected for risk-premium factor improves the option valuation accuracy significantly.

1. Introduction

Improving asset allocation strategies has become an epitome problem to any financial practitioners. Option information has demonstrated to efficiently encapsulate derivative market perception. This has triggered many researchers to study the optimal selection of asset allocation by exploiting the option moments in developing a portfolio. The studies on this field have amplified these recent years. Most of which were intrigued by the groundbreaking study of [1]. Estimating option moments are heavily depended on historical data back then. On that sense, [2] and [3] argued that portfolio that is based on historical-data estimation has been found to be poorly performed out-of-sample. Studies [4] and [5] have used option-implied information for risk management purposes while studies [6] and [7] employed option-implied information to forecast volatility. Most of the existing studies have recorded the use of risk-premium factor in revising the option-implied information (refer table 1). However, most of them focused on using the corrected option-implied information in forecasting and risk management purposes. Different from other studies, this research aims on utilising adjusted option moments implied by option prices, revised using simple non-parametric risk-premium factor in constructing an optimal asset allocation strategy.

The primary focus of this study is to empirically investigate whether the information offered in Modified Generalised Leland (MGL) models allows for improvement on asset allocation strategies.

Table 1. Existing Studies

Current Method	Existing Literature	Findings	Gap
simple non-parametric adjustments	[3], [7], [12], [13]	• an implied volatility model that corrects for the volatility risk premium is superior at the monthly horizon and further improves portfolio performance. • implied volatility offers significant improvements against historical methods for international portfolio diversification	• Focused on international equity markets forecasting • does not hold for extreme quantile prediction •
Risk-neutral skewness	[4]	• documented a positive relationship between the option-implied risk-neutral skewness (RNS) of individual stock returns distribution and future realized stock returns during the period 1996-2012 • employed MFBKM and short-selling constraint	• use the option-implied information for risk management purposes
non-parametric and parametric volatility risk adjustments	[5]	• this adjustment is effective in reducing the bias • it still does not allow the implied volatility to outperform the historical volatility models. • results contrast with the volatility forecasting literature, which favours implied volatilities over the historical volatility model due to the non-linear and regime changing dynamics of the volatility risk premium	• assume the relationships between the volatility risk premium, volatility, returns and innovations are highly non-linear around extreme events • use the option-implied information for risk management purposes
risk-adjusted using average standard deviation	[6]	• Model-based estimates result in out-performance of the basic mean–variance optimization strategy after transaction costs.	• Employ basic mean-variance strategy • use the option-implied information for volatility forecasting purposes
skewness risk premium correction estimator	[14]	• The estimator has the highest information content on future skewness while it consistently leads to the lowest out-of-sample forecast errors, compared to the remaining models. • A portfolio strategy that employs this estimator is superior to the 1/N portfolio and to strategies based on the rest of the skewness models considered.	• Focused on modelling and predicting skewness
real-world densities	[15]	• a simple behavioural correction generates substantial forecast gains. • the improvement delivered by real-world densities is robust across all evaluation methods, risk-preference hypotheses, and sentiment calibrations	• Focused on distribution forecasting • use the option-implied information for risk management purposes

Echoing to that concern, this study considers 36 different portfolio strategies, which are built distinguishingly in relation to the MGL models and the Model-Free-Bakshi-Kapadia-Madan (MFBKM) model (refer table 2). Whether the use of option-implied information, which is projected by the MGL model, is able to improve a portfolio performance will be investigated in this study. The naïve portfolio is taken as the benchmark portfolio in order to assess the portfolio performance at the end of this study.

Table 2. Asset allocation strategies

No.	Model	Abbreviation
	Naïve Portfolio	
1	1/N	1N
	Classical portfolios	
2	Mean-variance	MV
3	Minimum-variance	Min
4	Median-variance	Med
	Optimal Combinations of Portfolios	
5	Mixture of mean-variance and minimum-variance	MV-Min
6	Mixture of mean-variance and median-variance	MV-Med
7	Mixture of minimum-variance and median-variance	Min-Med
8	Mixture of 1/N and mean-variance	1N-MV
9	Mixture of 1/N and minimum-variance	1N-Min
10	Mixture of 1/N and median-variance	1N-Med
11	Mixture of 1/N, mean-variance, and median-variance	1N-MV-Med
12	Mixture of 1/N, minimum-variance, and median-variance	1N-Min-Med
13	Mixture of 1/N, mean-variance, minimum-variance, and median-variance	1N-MV-Min-Med
	Short-Selling Portfolios	
14	Mean-variance with short-selling assumption	MV-C
15	Minimum-variance with short-selling assumption	Min-C
16	Median-variance with short-selling assumption	Med-C
17	Mixture of mean-variance and minimum-variance with short-selling assumption	MV-Min-C
18	Mixture of mean-variance and median-variance with short-selling assumption	MV-Med-C
19	Mixture of minimum-variance and median-variance with short-selling assumption	Min-Med-C
20	Mixture of 1/N and mean-variance with short-selling assumption	1N-MV-C
21	Mixture of 1/N and minimum-variance with short-selling assumption	1N-Min-C
22	Mixture of 1/N and median-variance with short-selling assumption	1N-Med-C
23	Mixture of 1/N, mean-variance, and median-variance with short-selling assumption	1N-MV-Med-C
24	Mixture of 1/N, minimum-variance, and median-variance with short-selling assumption	1N-Min-Med-C
25	Mixture of 1/N, mean-variance, minimum-variance, and median-variance with short-selling assumption	1N-MV-Min-Med-C

Table 2. – *Continued.*

No.	Model	Abbreviation
	Zero-Correlation Portfolio	
26	Mean-variance with zero-correlation assumption	MV-ZC
27	Minimum-variance with zero-correlation assumption	Min-ZC
28	Median-variance with zero-correlation assumption	Med-ZC
29	Mixture of mean-variance and minimum-variance with zero-correlation assumption	MV-Min-ZC
30	Mixture of mean-variance and median-variance with zero-correlation assumption	MV-Med-ZC
31	Mixture of minimum-variance and median-variance with zero-correlation assumption	Min-Med-ZC
32	Mixture of 1/N and mean-variance with zero-correlation assumption	1N-MV-ZC
33	Mixture of 1/N and minimum-variance with zero-correlation assumption	1N-Min-ZC
34	Mixture of 1/N and median-variance with zero-correlation assumption	1N-Med-ZC
35	Mixture of 1/N, mean-variance, and median-variance with zero-correlation assumption	1N-MV-Med-ZC
36	Mixture of 1/N, minimum-variance, and median-variance with zero-correlation assumption	1N-Min-Med-ZC
37	Mixture of 1/N, mean-variance, minimum-variance, and median-variance with zero-correlation assumption	1N-MV-Min-Med-ZC

The main model anchoring this study is the Modified Generalised Leland (MGL) models, which are developed based on the implied adjusted volatility introduced in Leland models. The option prices are found using the Black-Scholes-Merton (BSM) model in the first place. New option-implied adjusted moments are the generated based on the MGL model. In other words, this model attempts to integrate the Leland models into model-free framework, developed by [8]. The higher moments investigated in this study are realised from the Model-Free Bakshi-Kapadia-Madan (MFBKM). The integration considered in the model framework is to reduce the model misspecification error introduced by [9] and [10]. According to [11], there is a gap in studying the hybrid portfolio made of both fully-implied and option-implied information. Thus, this study endeavours in fulfilling this gap.

Four sections are formed in this study. The first section presents the brief background information of this study. Section 2 describes the data used. The research methodology involved in investigating each asset allocation strategy is explained in Section 3. The main findings of this part of study are recorded in Section 4. Finally, we conclude in Section 5.

2. Data

This paper employs all call and put options on the Dow Jones Industrial Index (DJIA) traded daily during the period of January 2009 until December 2015 on the Chicago Board Options Exchange (CBOE) market. The daily index data retrieved from the DJIA are constituted of trading date, expiration date, closing price, exercise price and trading volume for each trading option. The closing price of the DJIA index will be used as the underlying price in this analysis, while the real option price will be taken from the option price's closing price. The Dow Jones Industrial Average (DJIA) index options data is used in this analysis. The options include index and stock options for 30 blue chipped firms, which represent the most actively traded and listed in the United States.

3. Research Methodology

To reflect the real probability measure, the implied adjusted information is corrected in terms of risk premium. The derivation of the MGL models is presented first in Section 3.1. We concentrate our study only on the option-implied adjusted volatility, rebalanced on daily basis. In order to utilise the option-implied adjusted moment in selecting portfolio, an adjustment to the option-implied adjusted moments to be under objective measure is considered, instead. The volatility risk premium is included in the option-implied moments adjusted to reflect the true probability measure. It is hypothesised that the risk-premium-corrected implied adjusted volatility should outperform those realised from historical volatility, at forecasting the realised volatility. The results are based on the pricing error measures, i.e. root-mean-square-error (RMSE), mean value of the relative pricing error (MRPE), and mean value of the absolute relative pricing error (MARPE).

3.1. The Modified Generalised Leland (MGL) Function

This research uses the MGL models, which include both the Generalized Leland-Infused (GLI) model and the model-free Leland model (MFIL), inferred from the original Leland models. In the hybrid model-free setting, the proposed models are designed to integrate the transaction costs rate. The MGL models are constructed by combining the Leland models and the model-free models as defined in [8]. The Leland models ([9]-[10]) include Leland (1985), Leland All-Cash and Leland All-Stock models. The Leland models are: 1) Leland (1985) was the first work on option pricing in the presence of transaction costs; 2) Leland All-Cash model is [10] formula with the assumption that the initial portfolio consists of all cash positions; and 3) Leland All-Stock model is [10] formula with the assumption that the initial portfolio consist of all stock position. Only the transaction costs rate and the rebalancing period factors are addressed in the MGL model. The GLI model, on the other hand, did not directly account for the initial cost of trading. On top of the transaction costs rate and the rebalancing interval variables, the MFIL model also accounts for the initial cost of trading. The MFIL models take into account the initial cost of trading while assuming that the initial portfolio is made up entirely of cash and stock positions. The two models are referred to as the MFIL All-Cash model and the MFIL All-Stock model in this study.

The cross-section of the option prices of both call and put are first extracted from the Leland models in order to obtain the option-implied adjusted volatility values using the MFIL model. The hybrid model is then revised with the option-implied adjusted volatilities. In the GLI model, however, the option prices of both call and put provided by the wavelet transform are used beforehand to obtain option-implied volatilities. The Leland's modified function is used to produce the new option-implied adjusted volatilities. Based on the design, a new function of revised generalised model-free implied volatility is developed in this study.

The new MGL model is derived from the fact that the model-free option-implied volatility is only the square-root of the [8]'s variance contract. In [8], they defined the variance contract as

$$VAR\,(t,\tau) \equiv E^q\{(R_{t,\tau} - E^q[R_{t,\tau}])^2\} \tag{1}$$

or

$$VAR\,(t,\tau) = e^{r\tau}V(t,\tau) - \mu(t,\tau)^2 \tag{2}$$

By equating the variance contract with the square of the adjusted volatility introduced in [3], we obtain

$$e^{r\tau}V(t,\tau) - \mu(t,\tau)^2 = \sigma^2\left(1 + \frac{k\sqrt{\frac{2}{\pi}}}{\sigma\sqrt{\Delta t}}\right) \tag{3}$$

A quadratic equation can be created out of the above equation.

$$\sigma^2 + \sigma \cdot \frac{k}{\sqrt{\sqrt{\frac{2}{\pi}}}} \cdot \sqrt{\frac{2}{\pi}} - e^{r\tau}V(t,\tau) + \mu(t,\tau)^2 = 0 \tag{4}$$

Based on the equation, we propose a new modified generalised (MG) function to be:

$$MG = \frac{-k}{\sqrt{2\pi \cdot \Delta t}} + \sqrt{\frac{k^2}{\pi \cdot \Delta t} - 2(\mu^2 - e^{r\tau} \cdot V)} \tag{5}$$

where *V* represents the variance contract, *k* is the round-trip transaction cost rate per unit dollar of transaction and Δt is the time between hedging adjustment, i.e. the rebalancing interval. The MGL implied volatility is adjusted as above to account for several extra parameters which are not considered in the original Bakshi-Kapadian-Madan (BKM) model, i.e. the transaction cost rate and the time between hedging adjustment. Particularly this model adopts the transaction cost function introduced by Leland models.

3.2 Historical Volatility Risk Premium (HVRP)

We employ Historical Volatility Risk Premium (HVRP) to estimate the volatility risk premium. The estimation is performed on monthly basis, following the 30-days of options maturity fixed in the beginning of this research. The selection of 30-day maturity options is anchored by [12]. Under the objective measure, it is first assumed that the volatility risk premium magnitude to be proportional to the volatility level. The HVRP is then estimated as the ratio of average implied volatilities to the realised volatilities for a particular stock. The HVRP is estimated over the $\boldsymbol{T} + \Delta t$ trading days, in which:

$$HVRP_t = \frac{\sum_{t-T-\Delta t+1}^{T-\Delta t} MFIAV_{i,i+\Delta t}}{\sum_{t-T-\Delta t+1}^{T-\Delta t} RV_{i,i+\Delta t}}. \tag{6}$$

MFIAV stands for model-free implied adjusted volatility which indicates the option-implied volatility adjusted from the MGL models, whereas the realised volatility is denoted by RV.

From that, we correct the adjusted volatility implied by the MGL models based on the risk-premium by inducing that the existing volatility risk premium is best estimated using the HVRP. The successive realised volatility can be best represented by the risk-premium-corrected implied adjusted volatility as follows:

$$\widehat{RV}_{t,t+\Delta t} = \frac{MFIAV_{t,t+\Delta t}}{HVRP_t}. \tag{7}$$

4. Results and Discussion

In this section, we consider an adjustment to the option-implied moments under objective measure. Previous study only concerns the risk-neutral implied information. In order to expand this study to include a portfolio selection, option-implied adjusted moments under real-world or objective distribution should be considered instead. The volatility risk premium is included in the option-implied moments adjusted to reflect the true probability measure.

By hypothesis, the risk-premium-corrected implied adjusted volatility should outperform those realised from historical volatility at forecasting the realised volatility. Each predictor is compared based on how well they approximate the realised volatility. We grounded our conclusion based on the pricing error measures, i.e. RMSE, MRPE and MARPE.

The pricing performance between the risk-premium-corrected implied adjusted volatility is compared to that of the adjusted volatility implied by the MGL models per se without considering the risk-premium. We take into account only the estimation performed with daily rebalancing based on the robust results recorded in the prior sections. The results are simplified in the following table 3. This study employs three integrals approximation approaches – basic, adapted and advanced method. Basic approach [16] is based on the approximation of the integral is performed using the summation equations. Adapted approach [8] considers the integrals of calls and puts as one integral. The two separate integrals are combined and are treated as one. The advanced approach is the addition to the adapted approach, in which on top of treating the contract separated integrals as one, it involves the use of smoothing method [17].

We found that the RMSE in GLI model for the risk-premium corrected implied adjusted volatility is 0.1049 when the estimation using the advanced approach is considered. The RMSE is much lesser compared to those without correcting for risk-premium, which is 0.2016. Aligning with the findings in GLI model, the MFIL models documented similar results. Smaller RMSE is recorded when the correction of risk-premium is considered, across all three model variants. As a matter of fact, the pricing error of the MGL models is halved across all approaches when we correct for the risk-premium. This finding is expected due to the initial trading assumption which has already asserted the risk premium; hence, has double corrected the risk premium in the model.

Table 3. Summary of Error Analysis of Volatility Implied from the MGL models considering the Risk-Premium Correction with daily rebalancing.

Model		Approach	RMSE (pts)	S.D.	MRPE (%)	S.D. (%)	MARPE (%)	S.D. (%)
PANEL A: With Risk-Premium Correction								
MFIL	L85	Basic	0.0994	0.0360	5.3796	22.9193	17.2129	15.9192
		Adapted	0.0996	0.0359	5.4118	22.9648	17.2988	15.9012
		Advanced	0.0980	0.0283	4.5561	26.3886	17.4912	20.1579
	LCASH	Basic	0.1004	0.0384	5.6973	23.0675	17.2074	16.2482
		Adapted	0.1008	0.0382	6.7372	22.9946	17.7725	15.9278
		Advanced	0.0973	0.0307	5.1016	24.9465	16.9404	18.8932
	LSTOCK	Basic	0.1013	0.0377	5.6561	23.9660	17.5507	17.1364
		Adapted	0.1015	0.0375	5.6820	23.9422	17.6193	17.0399
		Advanced	0.0996	0.0298	5.0040	26.7061	18.1148	20.1239
GLI		Basic	0.1048	0.0398	5.1563	24.3849	17.40951	17.71264
		Adapted	0.1034	0.0390	5.1563	23.9513	17.07056	17.44595
		Advanced	0.1049	0.0373	4.9676	25.0823	18.2547	17.7577

Note: L85 is Leland (1985) model, LCASH is Leland All-Cash model, LSTOCK is Leland All-Stock model, MFIL is model-free implied Leland model and GLI is Generalised Leland-Infused model.

Table 3. – *Continued.*

Model		**Approach**	**RMSE (pts)**	**S.D.**	**MRPE (%)**	**S.D. (%)**	**MARPE (%)**	**S.D. (%)**
PANEL B: Without Risk-Premium Correction								
MFIL	L85	Basic	0.1936	0.0825	-48.4824	11.9707	48.4824	11.9707
		Adapted	0.1908	0.0820	-47.3516	12.1761	47.3516	12.1761
		Advanced	0.1697	0.0671	-39.2475	16.9797	39.5665	16.2094
	LCASH	Basic	0.1784	0.0795	-42.2676	12.2848	42.2676	12.2848
		Adapted	0.1756	0.0790	-41.0735	12.5324	41.0735	12.5324
		Advanced	0.1513	0.0640	-31.1214	16.9298	31.7973	15.6004
	LSTOCK	Basic	0.1875	0.0814	-45.7967	12.4098	45.7967	12.4098
		Adapted	0.1846	0.0808	-44.5736	12.6233	44.5736	12.6233
		Advanced	0.1629	0.0658	-35.8751	17.4030	36.3461	16.3790
GLI		Basic	0.2160	0.0894	-57.5798	9.8128	57.5798	9.8128
		Adapted	0.2127	0.0884	-56.3616	9.9493	56.3616	9.9493
		Advanced	0.2016	0.0853	-51.5572	12.3317	51.5572	12.3317

Note: L85 is Leland (1985) model, LCASH is Leland All-Cash model, LSTOCK is Leland All-Stock model, MFIL is model-free implied Leland model and GLI is Generalised Leland-Infused model

5. Conclusions

The pricing performance of extended Leland models, i.e., MGL models, is investigated in this study, especially after the risk premium factor is addressed. The pricing performance of risk-premium-corrected implied adjusted volatility is compared to the adjusted volatility implied by MGL models without taking the risk-premium into account. This study considers the different Leland models, namely the Leland (1985), Leland All-Cash, and Leland All-Stock, in order to assess model relative pricing performance.

In evaluating the option pricing efficiency of the models, this study aims to analyse the manipulation variables, such as estimation approach and risk premium. The relative performance of the MGL models, i.e. the Generalized Leland-Infused (GLI) model and the model-free implied Leland (MFIL) model, is compared. Based on our research, we were able to demonstrate that the risk-premium-corrected pricing error obtained from Modified Generalized Leland (MGL) models is halved across all approaches. This result is most indicative of the fact that the risk premium was already factored into the initial trading assumption. The risk premium in the model has been corrected twice. Based on our empirical findings, we conclude that after the risk-premium adjustment, stock option prices provide valuable information that can help improve portfolio efficiency.

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