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Higher moment of portfolio optimization with Polynomial Goal Programming approach

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Abstract. The mean-variance (MV) model has been introduced in portfolio optimization to minimize the risk and achieve the target rate of return in the investment. However, the higher moment skewness and kurtosis are not considered in this model. The investors prefer portfolio with high skewness value and low kurtosis value so that the probability of getting extreme negative rates of return will be reduced. Therefore, the MV model has been extended to the mean-variance-skewness-kurtosis (MVSK) model by incorporating the skewness and kurtosis factor. The objective of this study is to construct the optimal portfolio of the MVSK model by using the polynomial goal programming (GP) approach. The data of this study comprises technology companies that listed in Malaysian stock market. In the fourth industrial revolution, technology companies play an important role in the development of a country. The results of this study show that the optimal portfolio of MVSK model outperforms the MV model by giving higher portfolio skewness value and lower portfolio kurtosis value. This study is significant because the investors can maximize the portfolio skewness value and minimize the portfolio kurtosis value with the MVSK model.

1. Introduction

Markowitz [1] has introduced the mean-variance (MV) model to minimize the risk of the portfolio in achieving the mean return in the investment. In this model, the risk of the portfolio is measured with variance. Investors wish to find the trade-off between the risk and return in their investment. According to the past studies, the MV model has been employed by various researchers [2-6]. The MV model has been used by the past researchers to construct the optimal portfolio (OP) that will minimize the portfolio risk and can achieve the expected rate of return. However, the higher moment skewness and kurtosis are not considered in this model. The investors prefer portfolio with larger skewness and smaller kurtosis value in order to reduce the chances of getting extreme negative return [7-10]. The investors will be exposed to the extreme loss with lower portfolio skewness value and higher portfolio kurtosis value. Therefore, the MV model has been extended to the MVSK model by incorporating the skewness and kurtosis factor to improve the MV model in portfolio optimization [11-13]. In portfolio optimization, selection of stocks as well as determination of stocks' weights are two important elements for constructing the OP [14-27]. This research aims to construct the OP of the MVSK model by using the polynomial goal programming (GP) approach to maximize the mean return and skewness

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value of the portfolio as well as minimize the variance and kurtosis value of the portfolio. GP approach has been used in financial management based on the past studies [28-33].

2. Data and Methodology

The data comprises the monthly returns of listed technology companies in Malaysia from January 2011 until December 2017. The technology companies are important in the development of a country in the fourth industrial revolution. The accomplishment of Vision 2050 in Malaysia which is to transform Malaysian into smart communities with sustainable national economic growth will be contributed by the technology companies [34]. In this study, the MVSK model and MV model are employed in constructing the optimal portfolio. The objective function of the MV model is to minimize the portfolio risk. The portfolio risk is represented by the portfolio variance while the expected rate of return is represented by the portfolio mean return. The MV model can be formulated as follows:

$$\text{Minimize } \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j \tag{1}$$

$$\text{subject to } \sum_{j=1}^{n} r_j x_j \geq \rho, \tag{2}$$

$$\sum_{j=1}^{n} x_j = 1, \tag{3}$$

$$x_j \geq 0,\ j = 1,\ldots,n. \tag{4}$$

where

r_j : mean return of asset j per period,

ρ : parameter denoting the minimum return determined by an investor,

x_i : weight of asset i,

x_j : weight of asset j,

σ_{ij} : covariance between assets i and j.

MVSK model aims to maximize the mean return and skewness value as well as minimize the variance and kurtosis value of OP. The OP composition and performance of the MVSK model are compared with the MV model. The target levels of four moments are also computed in this study. The MVSK model using the polynomial GP approach is shown as follows:

$$\text{Maximize } R(x) = E\left(X^T \bar{R}\right), \tag{5}$$

$$\text{Minimize } V(X) = X^T V X, \tag{6}$$

$$\text{Maximize } S(X) = E\left[X^T\left(R - \bar{R}\right)\right]^3 \Big/ \sigma_x^3, \tag{7}$$

$$\text{Minimize } K(X) = E\left[X^T(R - \bar{R})\right]^4 \Big/ \sigma_x^4, \tag{8}$$

subject to

$$X^T I = 1, \tag{9}$$

$$X \geq 0. \tag{10}$$

where
$R(x)$: portfolio mean return,
$V(x)$: portfolio variance,
$S(X)$: portfolio skewness,
$K(X)$: portfolio kurtosis
X^T : transposed of X, $X = (x_1, x_2, ..., x_n)$,
x_i : wealth invested in the ith risky asset (%),
$\bar{R}$: mean return of the assets,
R : rate of return of the assets,
V: covariance matrix of rates of return of the assets,

There are two steps to solve this model. Firstly, the target levels of $R^*(x)$, $V^*(x)$, $S^*(x)$ and $K^*(x)$ are determined by solving each objective individually subject to the constraints in equations (9)-(10). Secondly, the optimal values of $R^*(x)$, $V^*(x)$, $S^*(x)$ and $K^*(x)$ are substituted into the model as shown below:

$$\text{Minimize } Z = \left|\frac{d_1}{R*}\right|^{\lambda_1} + \left|\frac{d_2}{V*}\right|^{\lambda_2} + \left|\frac{d_3}{S*}\right|^{\lambda_3} + \left|\frac{d_4}{K*}\right|^{\lambda_4} \tag{11}$$

$$\text{subject to } X^T\bar{R} + d_1 = R^* \tag{12}$$

$$X^TVX - d_2 = V^* \tag{13}$$

$$E[X^T(R-\bar{R})]^3 / \sigma_x{}^3 + d_3 = S^* \tag{14}$$

$$E[X^T(R-\bar{R})]^4 / \sigma_x{}^4 - d_4 = K^* \tag{15}$$

$$X^TI = 1 \tag{16}$$

$$X \geq 0 \tag{17}$$

$$d_i \geq 0, i = 1,...,4 \tag{18}$$

where
d_1, d_2, d_3 and d_4 : non-negative variables which denote the deviations of each moment from the optimal values,
λ_1, λ_2, λ_3 and λ_4 : non-negative parameters which denote the investor's degree of preferences on the four moments.

3. Results
Table 1 presents the OP composition of the MVSK model and the MV model in percentage (%).

Table 1. OP composition of the MVSK model and the MV model in percentage

Companies	MVSK model	MV model
CENSOF	0.00	0.00
CUSCAPI	0.00	0.00
D&O	0.00	0.00

DATAPRP	0.00	0.00
DIGISTA	0.00	0.66
DNEX	0.00	0.00
EFORCE	0.00	0.00
ELSOFT	0.00	3.00
FRONTKN	0.00	0.00
GHLSYS	0.00	0.00
GRANFLO	20.34	27.56
GTRONIC	11.45	4.75
HTPADU	0.00	0.00
ITRONIC	0.00	1.11
JCY	0.00	0.00
KESM	9.89	10.76
KEYASIC	1.58	0.23
MMSV	0.00	0.00
MPI	4.59	4.51
MSNIAGA	32.04	30.23
MYEG	5.40	4.03
NOTION	0.00	0.00
OMESTI	11.73	8.77
PENTA	1.99	1.01
THETA	1.00	2.45
TRIVE	0.00	0.00
TURIYA	0.00	0.01
UNISEM	0.00	0.00
VITROX	0.00	0.91
WILLOW	0.00	0.00

As presented in table 1, the OP of the MVSK model comprises GRANFLO (20.34%), GTRONIC (11.45%), KESM (9.89%), KEYASIC (1.58%), MPI (4.59%), MSNIAGA (32.04%), MYEG (5.40%), OMESTI (11.73%), PENTA (1.99%) and THETA (1.00%). In contrast, the optimal portfolio of the MV model comprises DIGISTA (0.66%), ELSOFT (3.00%), GRANFLO (27.56%), GTRONIC (4.75%), ITRONIC (1.11%), KESM (10.76%), KEYASIC (0.23%), MPI (4.51%), MSNIAGA (30.23%), MYEG (4.03%), OMESTI (8.77%), PENTA (1.01%), THETA (2.45%), TURIYA (0.01%) and VITROX (0.91%). MSNIAGA is the biggest component in both OP of MVSK model and MV model. THETA is the smallest component in the OP of the MVSK model whereas TURIYA is the smallest component in the OP of the MV model. Besides that, 0.00% implies that the companies are not selected in the OP. CENSOF, CUSCAPI, D&O, DATAPRP, DIGISTA, DNEX, EFORCE, ELSOFT, FRONTKN, GHLSYS, HTPADU, ITRONIC, JCY, MMSV, NOTION, TRIVE, TURIYA, UNISEM, VITROX and WILLOW are not invested in the OP of the MVSK model because the weights are 0.00%. On the other hand, CENSOF, CUSCAPI, D&O, DATAPRP, DNEX, EFORCE, FRONTKN, GHLSYS, HTPADU, JCY, MMSV, NOTION, TRIVE, UNISEM and WILLOW are not invested in the OP of the MV model because the weights are 0.00%. The results show that the OP of the MVSK model shows different portfolio composition with the MV model.

Besides that, the target levels of four moments are presented in table 2.

Table 2. Target levels of four moments

Sub-objectives	$R^*(X)$	$V^*(X)$	$S^*(X)$	$K^*(X)$
Optimal values	0.079	0.002	3.016	2.239

Based on table 2, the target levels of four moments are 0.079, 0.002, 3.016 and 2.239 for $R^*(x)$, $V^*(x)$, $S^*(x)$ and $K^*(x)$, respectively. It implies that these are the optimal values of the four moments.

Moreover, table 3 displays the summary statistics of the OP of the MVSK model and the MV model.

Table 3. Summary statistics of the OP of the MVSK model and the MV model

Summary statistics	MVSK model	MV model
Portfolio mean return	0.011	0.010
Portfolio variance	0.002	0.002
Portfolio skewness	0.471	0.310
Portfolio kurtosis	3.195	3.359
Total deviations from the optimal values	3.571	3.897

As shown in table 3, the mean return, variance, skewness value, kurtosis value and total deviations from the optimal values of the optimal portfolio of the MVSK model are 0.011, 0.002, 0.471, 3.195 and 3.571 respectively. In contrast, the return, variance, skewness value, kurtosis value and total deviations from the optimal values of the optimal portfolio of the MV model are 0.010, 0.002, 0.310, 3.359 and 3.897 respectively. It indicates that the optimal portfolio of MVSK model outperforms the MV model by giving higher portfolio mean return, higher portfolio skewness value, lower portfolio kurtosis value and lower total deviations from the optimal values.

4. Conclusion

As a conclusion, this research aims to construct the OP that consists of the technology companies with the MVSK model by using the polynomial GP approach to optimize the four moments of the portfolio. It is a pioneer study in Malaysia by employing the MVSK model on technology companies. The OP of the MVSK model shows different portfolio composition with the MV model. Besides that, the OP of MVSK model outperforms the MV model by giving higher portfolio mean return, higher portfolio skewness value, lower portfolio kurtosis value and lower total deviations from the optimal values. This research is significant because the investors will be able to optimize the four moments of the portfolio with the MVSK model in their investment. For future research, it is recommended to study the portfolio optimization of technology companies in other countries with the MVSK model.

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