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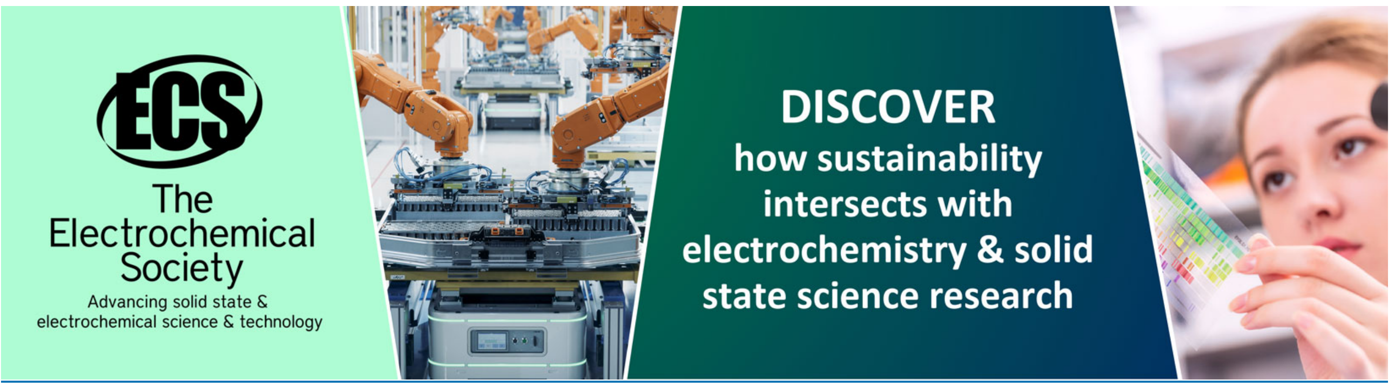

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Soft Graphs of Certain Graphs

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Abstract: Let G^* = (V, E) be a simple graph and A be any nonempty set of parameters. Let subset R of A×V be an arbitrary relation from A to V. A mapping F from A to $\mathcal{P}(V)$ written as F: A → $\mathcal{P}(V)$ can be defined as F(x) = { y ∈ V / x R y} and a mapping K from A to $\mathcal{P}$ (E) written as K:A → $\mathcal{P}(E)$ can be defined as K(x) = {uv ∈ E/{u, v } ⊆ F(x)}. The pair (F, A) is a soft set over V and the pair (K,A) is a soft set over E. Obviously (F(a) , K(a)) is a subgraph of G^* for all a ∈ A. The 4-tuple G = (G^*, F, K, A) is called a soft graph of G. In this paper we discuss different soft graphs of graphs such as Complete graph, Star graph, Complete bipartite graph, Crown graph, Comb graph, Friendship graph, Bistar graph and Wheel graph

Keywords: Soft graph, Soft set, Relations, Parameters, isomorphic

1.Introduction

Molodtsov [5] initiated the novel concept of soft set theory as a new mathematical tool for dealing with uncertainties. This theory provides a parameterized point of view for uncertainty modelling and soft computing. The operations of soft sets are defined by Maji et al. [4]. At present, work on soft set theory is progressing rapidly. A new notion on soft graph using soft sets was introduced by Rajesh K. Thumbakara and Bobin George [6]. The soft graph has also been studied in more detail in many papers.

2. Preliminaries

2.1. *Definition*:

Let U be a universal **set** and E be the set of parameters related to the objects in U. Let $\mathcal{P}$(U) denote the power set of U. Let A be any non-empty sub**set** of E. A pair (F, A) is called **soft set** over U, where F is a set-valued function given by F : A → $\mathcal{P}$ (U). In other words, a **soft set** over U is a parameterized family of subsets of the universe U.

2.2. *Definition*:

Let G^* = (V, E) be a simple graph and A be any nonempty set of parameters. Let subset R of A×V be an arbitrary relation from A to V. A mapping F: A → $\mathcal{P}(V)$ can be defined as F(x) = {y ∈ V/x R y} and a mapping K : A → $\mathcal{P}(E)$ can be defined as K(x) = {uv ∈ E/{u,v } ⊆ F(x)}.

A 4-tuple G = (G*, F, K, A) is called a soft graph of G if it satisfies the following properties:

(i) G* = (V, E) is a simple graph

(ii) A is a nonempty set of parameters

(iii) (F, A) is a soft set over V

(iv) (K, A) is a soft set over E

(v) (F(a), K(a)) is a subgraph of G* for all a ∈A

The subgraph (F(a), K(a)) is denoted by H(a)

A soft graph can also be represented by G = < F, K, A> = {H(x)/ x ∈A}

The set of all soft graphs of G* is denoted by SG(G*)

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3.Results and Discussions:

3.1. *Theorem :*
Let A $\subseteq V(K_n)$ be any m element parameter set. Define $\rho : A \to V$ by x ρ y $\Leftrightarrow$ d (x, y) = k. Then the soft graph (K_n, F, K, A) exists if and only if k =1 and is isomorphic to mK_{n-1} and there does not exist a soft graph if k $\geq$ 2

Proof:

Let $V(K_n)$ = $\{v_1, v_2, \ldots\ldots, v_n\}$

Let A = $\{v_1, v_2, \ldots\ldots, v_m\}$ be any m element parameter set

Case 1: k = 1

Here each H_i is isomorphic to K_{n-1} and hence the soft graph (K_n, F, K, A) is isomorphic to mK_{n-1}

Case 2: k ≥ 2

In this case there does not exist a soft graph.

3.2. *Observation*:
In the above theorem, define $\rho : A \to V$ by x ρ y $\Leftrightarrow$ d (x, y) ≤ k. Then the soft graph (K_n, F, K, A) is isomorphic to mK_n

3.3. *Theorem:*
Let $K_{1,n}$ = $\{ v, v_1, v_2, \ldots\ldots, v_n\}$ where v is the central vertex and A$\subseteq V(K_{1,n})$ be any m element parameter set. Define $\rho : A \to V$ by x ρ y $\Leftrightarrow$ d (x, y) = k where k = 1,2. Then the soft graph ($K_{1,n}$, F, K, A) is totally disconnected if exist

Proof:

Let V($K_{1,n}$) = $\{ v, v_1, v_2, \ldots\ldots, v_n\}$where v is the central vertex.

Let A be any m element parameter set.

Case 1: k =1

Case 1 a: A contains v

Corresponding to v, H(v) is isomorphic to $\overline{K_n}$

For all other vertices , $each\ H_i(v_i)$ is isomorphic to K_1

Then the soft graph ($K_{1,n}$, F, K, A) is isomorphic to $\overline{K_{n+(m-1)}}$ and hence totally disconnected.

Case 1 b: A does not contain v

$Here\ each\ H_i\,(v_i)$ is isomorphic to K_1

Hence the soft graph ($K_{1,n}$, F, K, A) is isomorphic to $\overline{K_m}$ and hence totally disconnected

Case 2: k = 2

Case 2 a: A contains v

Corresponding to v , $H(v)$ doesn't exist.

For all other vertices , $each\ H_i(v_i)$ is isomorphic to $\overline{K_{n-1}}$

Hence the soft graph ($K_{1,n}$,F,K, A) is isomorphic to union of $n(m-1)\ times$ $\overline{K_{n-1}}$

Case 2 b: A does not contain v

$Here\ each\ H_i(v_i)$ is isomorphic to $\overline{K_{n-1}}$

Hence the soft graph ($K_{1,n}$,F,K, A) is isomorphic to $union\ of\ m\ times\ \overline{K_{n-1}}$

Hence in general, the soft graph ($K_{1,n}$,F,K, A) is totally disconnected

3.4. *Remark:*
In the above theorem, if k > 2 and x ρ y $\Leftrightarrow$d (x, y) =k, then there does not exist a soft graph for $K_{1,n}$

3.5. *Theorem:*
Let $K_{1,n}$ = $\{ v, v_1, v_2, \ldots\ldots, v_n\}$ where v is the central vertex and A$\subseteq V(K_{1,n})$ be any m element parameter set. Define $\rho : A \to V$ by x ρ y $\Leftrightarrow$ d (x, y) ≤ 1. Then the soft graph

$(K_{1,n}$,F, K, A) is $\begin{cases} K_{1,n} \cup (m-1) K_2 & \text{if } v \in A \\ mK_2 & \text{if } v \notin A \end{cases}$

Proof:

Let V($K_{1,n}$) = { v, v_1,v_2,……,v_n}where v is the central vertex.

Case 1: A contains v

Corresponding to v, H(v) is isomorphic to $K_{1,n}$

For all other vertices , $each\ H_i(v_i)$ is isomorphic to K_2

Then the soft graph ($K_{1,n}$,F,K, A) is isomorphic to $K_{1,n}$ ∪ (m-1) K_2

Case 2: A does not contain v

Here $each\ H_i(v_i)$ is isomorphic to K_2

Then the soft graph ($K_{1,n}$,F,K, A) is isomorphic to mK_2

3.6. *Theorem:*

Let $K_{1,n}$ = { v, v_1,v_2,……,v_n} where v is the central vertex and A⊆ $V(K_{1,n})$ be any m element parameter set. Define $\rho : A \to V$ by x ρ y ⟺d (x, y) ≤ k where k = 2. Then the soft graph ($K_{1,n}$,F, K, A) is isomorphic to $m\ K_{1,n}$

Proof:

Let V($K_{1,n}$) = { v, v_1,v_2,……,v_n}where v is the central vertex.

Case 1: A contains v

Corresponding to v, H(v) is isomorphic to $K_{1,n}$

And for all other vertices , $each\ H_i(v_i)$ is isomorphic to $K_{1,n}$

Then the soft graph ($K_{1,n}$,F,K, A) is isomorphic to $m\ K_{1,n}$

Case 2: A does not contain v

Here , $each\ H_i(v_i)$ is isomorphic to $K_{1,n}$

Then the soft graph ($K_{1,n}$,F,K, A) is isomorphic to $m\ K_{1,n}$

3.7. *Remark:*

In the above theorem, the soft graph ($K_{1,n}$,F, K, A) is isomorphic to $m\ K_{1,n}$ if k > 2

3.8. *Theorem:*

Let A ⊆ V ($K_{m,n}$) be any t element parameter set. Define $\rho : A \to V$ by x ρ y ⟺ d (x, y) = k. Then the soft graph ($K_{m,n}$,F, K, A) is totally disconnected if k = 1, 2 and there is no soft graph for k > 2

Proof:

Consider V = {U, W} where U = {u_1,u_2,……,u_m} and W = {w_1,w_2,……,w_n}

Let A be any t element parameter set where r elements from U and s elements from W such that r + s = t

Case 1: k = 1

If the vertex belongs to U, then $each\ H_i(u_i)$ is isomorphic to $\overline{K_n}$

If the vertex belongs to W, then $each\ H_i(w_i)$ is isomorphic to $\overline{K_m}$

Then the soft graph ($K_{m,n}$,F,K, A) is isomorphic to r $\overline{K_n}$ ∪ s $\overline{K_m}$ and hence totally disconnected

Case 2: k = 2

If the vertex belongs to U, then $each\ H_i(u_i)$ is isomorphic to $\overline{K_{m-1}}$

If the vertex belongs to W, then $each\ H_i(w_i)$ is isomorphic to $\overline{K_{n-1}}$

Then the soft graph ($K_{m,n}$, F, K, A) is isomorphic to r $\overline{K_{m-1}}$ ∪ s $\overline{K_{n-1}}$ and hence totally disconnected

Case 3: k > 2

Then there does not exist a soft graph.

3.9. *Theorem:*
Let A ⊆ $V(K_{m,n})$ be any t element parameter set. Define $\rho : A \to V$ by $x \rho y \Leftrightarrow$ d (x, y) ≤ k .Then the soft graph ($K_{m,n}$,F, K, A) is isomorphic to r $K_{1,n}$ ∪ s $K_{1,m}$ if k = 1 and t $K_{m,n}$ if k > 1 where 1 ≤ r ≤ m and $1 \le s \le n$

Proof:

Let V = {U, W} where U = $\{u_1, u_2, \ldots\ldots, u_m\}$ and W = $\{w_1, w_2, \ldots\ldots, w_n\}$
Let A be any t element parameter set where r elements from U and s elements from W such that r + s = t

Case 1: **k** = 1

If the vertex belongs to U, then $each\ H_i(u_i)$ is isomorphic to $K_{1,n}$
If the vertex belongs to W, then $each\ H_i(w_i)$ is isomorphic to $K_{1,m}$
Then the soft graph ($K_{m,n}$,F,K, A) is isomorphic to r $K_{1,n}$ ∪ s $K_{1,m}$

Case 2: k > 1

If the vertex belongs to U, then $each\ H_i(u_i)$ is isomorphic to $K_{m,n}$
If the vertex belongs to W, then $each\ H_i(w_i)$ is isomorphic to $K_{m,n}$
The soft graph ($K_{m,n}$,F,K, A) is isomorphic to $(r + s)K_{m,n}$ = t $K_{m,n}$

3.10. *Observation:*
(i)Let A ⊆ $V(P_n \odot K_1)$ be one element parameter set. Define $\rho : A \to V$ by $x \rho y \Leftrightarrow$ d (x, y) = k. Then the soft graph ($P_n \odot K_1$,F, K, A) is totally disconnected
(ii)Let A ⊆ $V(P_n \odot K_1)$ be singleton parameter set. Define $\rho : A \to V$ by $x \rho y \Leftrightarrow$ d (x, y) ≤ k. Then in the soft graph ($P_n \odot K_1$,F, K, A),each H_i is Path or Star or Caterpillar

3.11. *Observation:*
Let $C_n \odot K_1 = \{u_1, u_2, \ldots\ldots, u_n, w_1, w_2, \ldots\ldots, w_n\}$ where u_i are the vertices of cycle and w_i are the end vertices and A ⊆ $V(C_n \odot K_1)$ be any one element parameter set. Define $\rho : A \to V$ by $x \rho y \Leftrightarrow$d (x, y) =k. The soft graph ($C_n \odot K_1$,F, K, A) is depicted in the following table for 3 ≤ n ≤ 20 and 1 ≤ k ≤ 12

k	d(x,y) = 1		d(x,y) = 2		d(x,y) = 3		d(x,y) = 4		d(x,y) = 5		d(x,y) = 6		d(x,y) = 7		d(x,y) = 8		d(x,y) = 9		d(x,y) =10		d(x,y)= 11		d(x,y) =12	
$C_n \odot K_1$	Soft graphs of $C_n \odot K_1$ corresponding to the cycle and pendant(end)vertices for different values of k																							
	cycle	pendant	Cycle	pendant	cycle	pendant	cycle	pendant	cycle	pendant	cycle	pendant	cycle	pendant	cycle	pendant	cycle	pendant	cycle	Pendant	cycle	pendant	cycle	Pendant
$C_3 \odot K_1$	$K_2 \cup K_1$	K_1	$\overline{K_2}$	K_2																				
$C_4 \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_3}$	$\overline{K_2}$	K_1	$\overline{K_3}$	-	K_1																
$C_5 \odot K_1$	$\overline{K_3}$	K_1	$K_2 \cup \overline{K_2}$	$\overline{K_2}$	$\overline{K_2}$	$K_2 \cup \overline{K_2}$	-	$\overline{K_2}$																

$C_6 \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_3}$	$\overline{K_4}$	K_1	$\overline{K_3}$	-	K_1														
$C_7 \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$K_2 \cup \overline{K_2}$	$\overline{K_4}$	$\overline{K_2}$	$K_2 \cup \overline{K_2}$	-	$\overline{K_2}$														
$C_8 \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\boldsymbol{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_3}$	$\overline{K_4}$	K_1	$\overline{K_3}$	-	K_1												
$C_9 \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$K_2 \cup \overline{K_2}$	$\overline{K_4}$	$\overline{K_2}$	$K_2 \cup \overline{K_2}$	-	$\overline{K_2}$												
$C_{10} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_3}$	$\overline{K_4}$	K_1	$\overline{K_3}$	-	K_1										
$C_{11} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$K_2 \cup \overline{K_2}$	$\overline{K_4}$	$\overline{K_2}$	$K_2 \cup \overline{K_2}$	-	$\overline{K_2}$										
$C_{12} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_3}$	$\overline{K_4}$	K_1	$\overline{K_3}$	-	K_1								
$C_{13} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$K_2 \cup \overline{K_2}$	$\overline{K_4}$	$\overline{K_2}$	$K_2 \cup \overline{K_2}$	-	$\overline{K_2}$								
$C_{14} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_3}$	$\overline{K_4}$	K_1	$\overline{K_3}$	-	K_1						
$C_{15} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$K_2 \cup \overline{K_2}$	$\overline{K_4}$	$\overline{K_2}$	$K_2 \cup \overline{K_2}$	-	$\overline{K_2}$						
$C_{16} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_3}$	$\overline{K_4}$	K_1	$\overline{K_3}$	-	K_1				
$C_{17} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$K_2 \cup \overline{K_2}$	$\overline{K_4}$	$\overline{K_2}$	$K_2 \cup \overline{K_2}$	-	$\overline{K_2}$				
$C_{18} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_3}$	$\overline{K_4}$	K_1	$\overline{K_3}$	-	K_1		
$C_{19} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$K_2 \cup \overline{K_2}$	$\overline{K_4}$	$\overline{K_2}$	$K_2 \cup \overline{K_2}$	-	$\overline{K_2}$		
$C_{20} \odot K_1$	$\overline{K_3}$	K_1	$\overline{K_4}$	$\overline{K_2}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_4}$	$\overline{K_3}$	$\overline{K_4}$	K_1	$\overline{K_3}$	-	K_1

From the above table,

we infer that the soft graph is isomorphic to $\overline{K_4}$ for n ≥ 2k + 2 and k > 2

3.12. ***Theorem:***
Let A⊆ $C_n \odot K_1$ be any t element parameter set. Define $\rho : A \to V$ by x ρ y ⟺d (x, y) ≤ k where k =1.Then the soft graph ($C_n \odot K_1$, F, K, A) is isomorphic to $rK_{1,3} \cup s\, K_2$ where 1 ≤ r ≤ m and $1 \leq s \leq n$

Proof:

Let V = {U, W} where U = {u_1,u_2,……,u_n} be the vertices of cycle and

W = $\{w_1, w_2, \ldots\ldots, w_n\}$ be the end vertices .Let A be any t element parameter set where r elements from U and s elements from W such that r + s = t

Let k = 1

Case 1: If $v = u_i$ $for\ some\ i$, then each $H_i(u_i)$ is isomorphic to $K_{1,3}$

Case 2: If $v = w_i$ $for\ some\ i$,, each $H_i(w_i)$ is isomorphic to K_2

Hence the soft graph ($C_n \odot K_1$,F, K, A) is isomorphic to $rK_{1,3} \cup sK_2$

3.13. ***Theorem:***

Let A ⊆ $C_n \odot K_1$be singleton parameter set. Define $\rho : A \to V$ by x ρ y ⟺d (x, y) ≤ k where k=2. Then in the soft graph ($C_n \odot K_1$, F, K, A), each H_i is $K_{1,3}$ if the vertex is an end vertex and any one of $C_n \odot K_1$ or $(C_n \odot K_1) - \{e_1\}$ or $(C_n \odot K_1) - \{e_1, e_2\}$ or caterpillar graph if the vertex is vertex on cycle

Proof:

Let V = {U, W} where U = $\{u_1, u_2, \ldots\ldots, u_n\}$ be the vertices of cycle and W = $\{w_1, w_2, \ldots\ldots, w_n\}$ be the end vertices

Let A be any singleton parameter set

Let k=2

Case 1: If $v = w_i$ $for\ some\ i$,each $H_i(w_i)$ is $K_{1,3}$

Case 2: If $v = u_i$ $for\ some\ i$, then the soft graph is

(i) $C_3 \odot K_1$ for $n = 3$

(ii) a crown graph with one pendant edge removed for $n = 4$

(iii) a crown graph with consecutive pendant edges removed for $n = 5$

(iv) a caterpillar for $n > 5$

3.14. ***Theorem:***

Let A ⊆ $C_n \odot K_1$be any singleton parameter set. Define $\rho : A \to V$ by x ρ y ⟺ d (x, y) ≤ k where k > 2.Then in the soft graph ($C_n \odot K_1$,F, K, A), each H_i is $C_n \odot K_1$ or $(C_n \odot K_1) - \{e_1\}$ or $(C_n \odot K_1) - \{e_1, e_2\}$ or caterpillar graph where e_1, e_2 are the consecutive pendant edges.

Proof:

Let V = {U, W} where U = $\{u_1, u_2, \ldots\ldots, u_n\}$ be the vertices of cycle and W = $\{w_1, w_2, \ldots\ldots, w_n\}$ be the end vertices

Let A be singleton parameter set

Case 1: k = 3

Case 1 a: If $v = u_i$ $for\ some\ i$, then the soft graph is

(i) $C_n \odot K_1$ for $n \leq 5$

(ii) a crown graph with one pendant edge removed for $n = 6$

(iii) a crown graph with two consecutive pendant edges removed for $n = 7$

(iv) a caterpillar graph for n >7

Case 1 b: If $v = w_i$ $for\ some\ i$, each $H_i(w_i)$ is

(i) $C_3 \odot K_1$ for $n = 3$

(ii) a crown graph with one pendant edge removed for $n = 4$

(iii) a crown graph with two pendant edges removed for $n = 5$

(iv)a caterpillar graph for n > 5

Case 2: k = 4

Case 2 a: If $v = u_i$ $for\ some\ i$, then each $H_i(u_i)$ is

(i) $C_n \odot K_1$ for $n \leq 7$

(ii) a crown graph with one pendant edge removed for $n = 8$

(iii) a crown graph with two consecutive pendant edges removed for $n = 9$

(iv) caterpillar graph for n >9

Case 2 b: If $v = w_i$ $for\ some\ i$, each $H_i(w_i)$ is

(i) $C_n \odot K_1$ for $n \leq 5$

(ii) a crown graph with one pendant edge removed for $n = 6$

(iii) a crown graph with two pendant edges removed for $n = 7$

(iv) a caterpillar graph for n> 7

As analysing above for any k > 2, in the soft graph ($C_n \odot K_1$, F, K, A), each H_i is $C_n \odot K_1$ *or* $(C_n \odot K_1) - \{e_1\}$ *or* $(C_n \odot K_1) - \{e_1, e_2\}$ or caterpillar graph where e_1, e_2 are the consecutive pendant edges.

3.15. *Observation:*
Consider a **friendship** graph F_n . Let A ⊂ V(F_n) be any one element parameter set.

(i) Suppose $\rho : A \to V$ defined by x ρ y ⇔ d (x, y) = 1. Then in the soft graph (F_n , F, K, A) *each* H_i is either isomorphic to nK_2 or K_2
(ii) Suppose $\rho : A \to V$ defined by x ρ y ⇔ d (x, y) = 2. Then in the soft graph (F_n , F, K, A) , *each* H_i is isomorphic to (n-1) K_2 if exist
(iii) Suppose $\rho : A \to V$ defined by x ρ y ⇔ d (x, y) = k where k > 2. Then there does not exist a soft graph (F_n , F, K, A)
(iv) Suppose $\rho : A \to V$ defined by x ρ y ⇔ d (x, y) ≤ 1.Then in the soft graph (F_n , F, K, A), *each* H_i is either isomorphic to F_n or C_3
(v) Suppose $\rho : A \to V$ defined by x ρ y ⇔ d (x, y) ≤ k where k > 2. Then in the soft graph (F_n , F, K, A) , *each* H_i is isomorphic to F_n

3.16. *Observation:*
Consider a **bistar** graph $B_{n,n}$. Let A ⊂ V($B_{n,n}$) be any one element parameter set.

(i) Define $\rho : A \to V$ by x ρ y ⇔ d (x, y) = k. Then the soft graph ($B_{n,n}$, F, K, A) is totally disconnected.
(ii) Define $\rho : A \to V$ by x ρ y ⇔ d (x, y) ≤ 1. Then in the soft graph ($B_{n,n}$, F, K, A) ,each H_i is isomorphic to K_2 or $K_{1,n+1}$
(iii) Define $\rho : A \to V$ by x ρ y ⇔ d (x, y) ≤ 2. Then in the soft graph ($B_{n,n}$, F, K, A) ,each H_i is isomorphic to $K_{1,n+1}$ *or* $B_{n,n}$
(iv) Define $\rho : A \to V$ by x ρ y ⇔ d (x, y) ≤ k where k>2.Then in the soft graph ($B_{n,n}$, F, K, A) each H_i is isomorphic to $B_{n,n}$

3.17. *Observation:*
Consider a wheel graph W_n. Let A ⊂ V(W_n) be any one element parameter set.

(i) Define $\rho : A \to V$ by x ρ y ⇔ d (x, y) = 1. Then in the soft graph (W_n, F, K, A), *each* H_i is isomorphic to C_{n-1} or P_3 if n > 4
(ii) Define $\rho : A \to V$ by x ρ y ⇔ d (x, y) = 2. Then in the soft graph (W_n, F, K, A) , *each* H_i *is* P_{n-4} *if exist* where n >4
(iii) Define $\rho : A \to V$ by x ρ y ⇔ d (x, y) = k where k > 2. Then there does not exist a soft graph (W_n, F, K, A)
(iv) Define $\rho : A \to V$ by x ρ y ⇔ d (x, y) ≤ 1. Then in the soft graph (W_n, F, K, A), , *each* H_i is isomorphic to W_n or K_4 –{e} where e is any edge.
(v) Define $\rho : A \to V$ by x ρ y ⇔ d (x, y)≤ k. Then in the soft graph(W_n,F,K,A) *each* H_i, is isomorphic to W_n *where* $k > 1$

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