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The Influence of a Heterogeneous Surface on the Free Volume Oscillations of an Oblate Gas Bubble

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The Influence of a Heterogeneous Surface on the Free Volume Oscillations of an Oblate Gas Bubble

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Abstract. The natural oscillations of a cylindrical gas bubble surrounded by an incompressible fluid with free interface are considered. The bubble has an equilibrium cylindrical shape and is bounded axially by two parallel solid surfaces. Dynamics of contact lines is taken into account by an effective boundary condition: velocity of the contact line is assumed to be proportional to contact angle deviation from the equilibrium value. The equilibrium contact angle is right. Different Hocking parameters determine individual damping rates, but dissipation in the integral system is determined by their total contribution. The frequency of the volume (breathing) harmonic of free oscillations can vanish in a certain interval of the values of the Hocking parameter for homogeneous plate surface. However, Surface inhomogeneity destroys this monotonic damping effect.

1. Introduction

Vibrations are one of the most popular fluid control methods [1-5]. Interest is caused not only by the great possibilities in the impact on the drops (particles, bubbles), but also by the unusual effects that appear with periodic exposure. Also vibration may either result from the action of external sources or be used for controlling engineering processes.

Motion of triple contact line dynamics is one of the important problems in the field of drops (bubble, particles) control. Note, that the contact line can appear when liquid films are destroyed due to their instability: the formation of dry spots and sessile drops [6-10]. The effective boundary condition is widely used at very fast relaxation processes of a contact line motion [11-15]:

$$\frac{\partial \zeta^*}{\partial t^*} = \Lambda \vec{k} \cdot \vec{\nabla} \zeta^* , \tag{1}$$

where ζ^* is the deviation of the interface from the equilibrium position, $\vec{k}$ is the external normal to the solid surface, Λ is a phenomenological constant (the so-called wetting parameter or Hocking parameter) having the dimension of the velocity. There are two important limit of the boundary condition(1): (a) $\zeta^* = 0$ – the fixed contact line (pinned-end edge condition) [16, 17], (b) $\vec{k} \cdot \vec{\nabla} \zeta^* = 0$ – the constant contact angle [2].

There are the several modifications of the condition (1): (a) Λ is a complex number [18], (b) a hysteresis of a contact angle [19-21], (c) different surfaces of plates for cylindrical symmetry [22, 23], (d) heterogeneous surface of plate [24, 25], (e) electrowetting-on-dielectric (EWOD) [26, 27]. Other models of motion of the line of contact are presented, for example, in [28–31].

In the present article, we consider free oscillations of cylindrical bubble which surrounded by a liquid with non-deformable interface. We apply the modified condition (1) for heterogeneous plates [24, 25], differ in Hocking parameters [22, 23, 32]. Oscillations of cylindrical bubble for are presented in [33, 34] for case of homogeneous plates.

2. Problem formulation

By analogy [32–34] a gas bubble surrounded by an incompressible liquid with a non-deformable external surface are consider (figure 1). The bubble is bounded by two parallel solid plates which separated by a distance h^*. In equilibrium, the bubble and fluid volume have circle cylindrical with a radius r_0^* and R_0^*, respectively, and contact angles γ^* and Γ^* are 90^0. Contact angle γ^* changes during the movement of the contact line, contact angle Γ^* is constant.

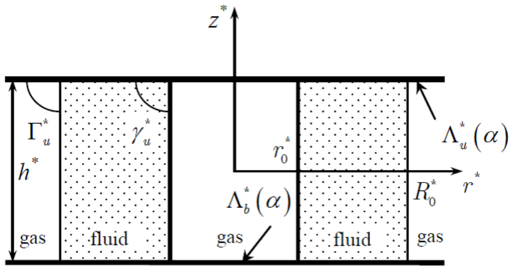


Figure 1. Problem geometry.

The oscillations amplitude A^* is small compared to the equilibrium bubble radius r_0^*. The fluid motion is assumed to be incompressible: $\omega^* r_0^* << c$, where ω^* is fundamental oscillation frequency, c is the sound velocity. However ω^* is large enough for the viscosity could be ignored: $\delta = \sqrt{\nu/\omega^*} << r_0^*$ where δ is the boundary-layer thickness.

Owing to the problem symmetry, it is convenient to introduce cylindrical coordinates r^*, α, z^*. The azimuthal angle α is reckoned from the x axis. Let the lateral surface of the bubble be described by the following equations

$$r^* = r_0^* + \zeta^*\left(\alpha, z^*, t^*\right).$$

Following [13, 32–34], we use $\sqrt{\rho_e^* r_0^{*3}/\sigma^*}$, r_0^*, h^*, A^*, $A^*\sigma^*/r_0^{*2}$, $A^*\sqrt{\sigma^*/\rho_e^* r_0^*}$ as the scales for the time, length, height, deviation of bubble surface and free surface from its equilibrium position, pressure, and velocity potential, respectively (σ^* is the surface tension and ρ_e^* is the liquid density). Thus, the dimensionless boundary value problem is determined by (intermediate steps can be found in [13, 32])

$$p_e = -\varphi_t,\ \Delta\varphi=0,\ p_i = -2n_p P_g^* r_0^*/\sigma^* \langle\zeta\rangle \equiv -P_0\langle\zeta\rangle, \tag{2}$$

$$\Delta = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + b^2\frac{\partial^2}{\partial z^2},$$

$$r=1:\ \zeta_t = \varphi_r,\ [p] = \zeta + \zeta_{\alpha\alpha} + b^2\zeta_{zz}, \tag{3}$$

$$z = \pm\frac{1}{2}:\ \varphi_z = 0, \tag{4}$$

$$r = R_0:\ \varphi = 0, \tag{5}$$

$$r=1,\ z=\pm\frac{1}{2}:\ \zeta_t = \mp\Lambda_{u,b}(\alpha)\zeta_z, \tag{6}$$

where p_e is the liquid pressure, φ is potential of liquid velocity, p_i is the gas pressure in the bubble, n_p is polytropic (e.g., adiabatic) exponent, P_g^* is dimension gas pressure in the bubble, $\Lambda_{u,b}$ are the Hocking parameters at the "upper" and "bottom" plates, respectively, the square brackets denote the jump in the quantity at the interface between the surrounding liquid and the bubble. The boundary-value problem (2)–(6) involves five parameters: the aspect ratio, the radius of free surface, the wetting parameter, the frequency and amplitude

$$b=\frac{r_0^*}{h^*},\ R_0=\frac{R_0^*}{r_0^*},\ \Lambda=\Lambda^* b\sqrt{\frac{\rho_e^* r_0^*}{\sigma^*}},\ \omega=\omega^*\sqrt{\frac{\rho_e^* r_0^{*3}}{\sigma^*}},\ \varepsilon=\frac{A^*}{r_0^*}.$$

3. Free oscillations

Let us consider a particular case of heterogeneous plates: $\Lambda_{u,b}(\alpha)=\lambda_{u,b}\left|\sin\left(k_\lambda\cos(\alpha)\right)\right|$, where k_λ – a real wavenumber of a heterogeneity surface. By the evenness of the natural oscillation modes is meant the evenness of the functions under a change of sign of the axial coordinate z. The solution of the boundary value problem (2)–(6) in the absence of an external force is written as (the eigenfunctions of the Laplace operator (2))

$$\varphi_e(r,z,t)=\left(i\Omega\sum_{m=0}^{\infty}\left(\sum_{n=0}^{\infty}\left(a_{mn}^{(1)}R_{1mn}^{i}(r)+b_{mn}^{(1)}R_{1mn}^{e}(r)\right)\sin\left((2n+1)\pi z\right)+\right.\right.$$
$$\left.\left.+\sum_{n=0}^{\infty}\left(a_{mn}^{(2)}R_{2mn}^{i}(r)+b_{mn}^{(2)}R_{2mn}^{e}(r)\right)\cos\left(2\pi nz\right)\right)e^{im\alpha}e^{i\Omega t}\right), \tag{7}$$

$$\zeta(\alpha,z,t)=\mathrm{Re}\left(\sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\left(c_{1mk}\sin\left((2k+1)\pi z\right)+c_{2mk}\cos\left(2k\pi z\right)\right)e^{im\alpha}+d_{10}\sin\left(\frac{z}{b}\right)+d_{20}\cos\left(\frac{z}{b}\right)+\right.$$
$$\left.+\left(d_{11}z+d_{21}z^2\right)e^{i\alpha}+\sum_{m=2}^{\infty}\left(d_{1m}\,\mathrm{sh}\left(\frac{\sqrt{m^2-1}}{b}z\right)+d_{2m}\,\mathrm{ch}\left(\frac{\sqrt{m^2-1}}{b}z\right)\right)e^{im\alpha}\right)e^{i\Omega t}, \tag{8}$$

where Ω is Eigenfrequency, $R_{1mn}^{i}(r)=\mathrm{I}_m((2n+1)\pi br)$, $R_{1mn}^{e}(r)=\mathrm{K}_m((2n+1)\pi br)$, $R_{2mn}^{i}(r)=r^m$, $R_{2mn}^{i}(r)=\mathrm{I}_m(2\pi nbr)$, $R_{2m0}^{e}(r)=r^{-m}$, $R_{2mn}^{e}(r)=\mathrm{K}_m(2\pi nbr)$, I_m and K_m are modified Bessel functions of the m–th order.

Substituting solutions (7)–(8) into (2)–(6), we obtain a spectral-amplitude problem which eigenvalues are the values of the natural oscillation frequency Ω. These complex algebraic equations have complex solutions, which lead to damping of oscillations. This attenuation is caused only by the condition on the contact line, not by viscosity. We also note that damping times are of the order of magnitude comparable with the period of oscillation, i.e. at a finite value of the wetting parameter, the droplet is able to execute only a few oscillations.

The equations of our spectral-amplitude problem were solved numerically by the two-dimensional secant method. For convenience we will denote the frequencies of the even harmonics as $\Omega_{m,2k}$ ($k=0,1,2,\dots$), and the frequencies of the odd harmonics as $\Omega_{m,2k+1}$ ($k=0,1,2,\dots$). Here, the first index m is a azimuthal number and the second index $2k$ (or $2k+1$) is wavenumber. Thus, the frequencies Ω_{mn} of the natural oscillations with the odd index n will correspond to the odd harmonics and an even index n to the even harmonics. Volume oscillations are more important mode of natural oscillations of a compressible bubble. Below we will focus on this radial (breath) harmonic.

The equations of full spectral-amplitude problem are very cumbersome, so for clarity we give the equation of axisymmetrical mode ($m=1$) for the special case $\Lambda_{u,b}(\alpha)=\lambda_{u,b}$ [32]:

$$\left(M-\lambda_b S\right)\left(N+\lambda_u C\right)+\left(M-\lambda_u S\right)\left(N+\lambda_b C\right)=0, \tag{9}$$
$$M=\sum_{k=0}^{\infty}(-1)^k f_k+\cos\left(\frac{1}{2b}\right),\ N=\sum_{k=0}^{\infty}(-1)^k g_k+\sin\left(\frac{1}{2b}\right),$$
$$C=\frac{1}{ib\Omega}\cos\left(\frac{1}{2b}\right),\ S=\frac{1}{ib\Omega}\sin\left(\frac{1}{2b}\right),$$
$$g_k=\frac{\Omega^2 C_k}{\Omega_{10k}^2-\Omega^2},\ f_0=\frac{\Omega^2-P_0\ln^{-1}\left(R_0\right)}{\Omega_{200}^2-\Omega^2},\ f_k=\frac{\Omega^2 C_k}{\Omega_{20k}^2-\Omega^2},$$

$$\Omega_{10k}^2 = \frac{(2k+1)^2 \pi^2 b^2 - 1}{\dfrac{R_{10k}^e(1)}{R_{10k}^e(R_0)} - \dfrac{R_{10k}^i(1)}{R_{10k}^i(R_0)}} \left(\frac{R_{10kr}^i(1)}{R_{10k}^i(R_0)} - \frac{R_{10kr}^e(1)}{R_{10k}^e(R_0)} \right),\ k = 0,1,2,\dots,$$

$$\Omega_{200}^2 = \frac{P_0 - 1}{\ln(R_0)},\ \Omega_{20k}^2 = \frac{(2k\pi b)^2 - 1}{\dfrac{R_{20k}^e(1)}{R_{20k}^e(R_0)} - \dfrac{R_{10k}^i(1)}{R_{10k}^i(R_0)}} \left(\frac{R_{20kr}^i(1)}{R_{20k}^i(R_0)} - \frac{R_{20kr}^e(1)}{R_{20k}^e(R_0)} \right),\ k = 1,2,3,\dots,$$

where C_k and S_k are the coefficients of the Fourier series expansions of the functions $\cos(b^{-1}z)$ and $\sin(b^{-1}z)$, respectively, Ω_{200} is the volume oscillations frequency of the compressible bubble with freely moving contact line ($\lambda \to \infty$) [32–34], Ω_{j0k} $(j = 1,2)$ are the Eigenfrequencies of the shape harmonics of incompressible drop with the same contact line [27, 35].

If the surfaces are identical, i.e., $\lambda_u = \lambda_b = \lambda$, then then left-hand side of eq. (9) can be represented as a product of two terms:

$$(M - \lambda S)(N + \lambda C) = 0. \tag{10}$$

Each of these terms fields the equation for finding the Eigenfrequencies: the solutions of the first equation are the frequencies of even harmonics, and the solutions of the second equation are the frequencies of odd harmonics.

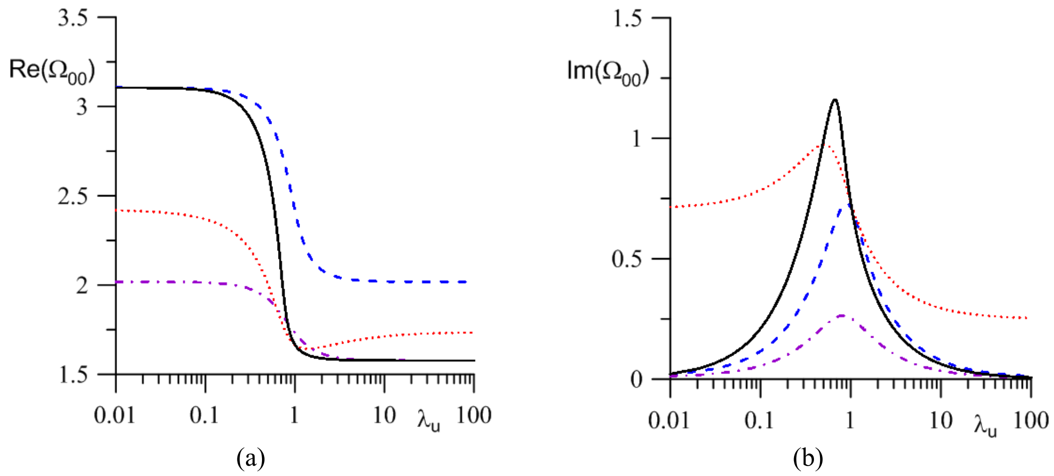
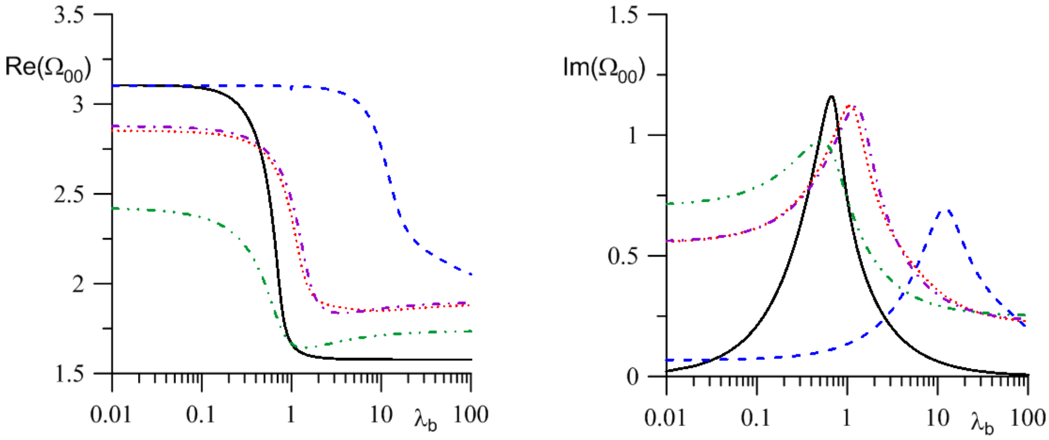


Figure 2. Frequency (a) and damping ratio (b) of volume natural oscillations vs wetting parameter λ_u for Ω_{00} ($R_0 = 5$, $P_0 = 5$, $b = 1$) (9).
$\lambda_u = \lambda_b$ – solid line (10), $\lambda_b = 0.01$ – dashed, $\lambda_b = 1$ – dotted, $\lambda_b = 100$ – dash-dotted.

Figures 2–5 show the real part of $\mathrm{Re}(\Omega)$ (oscillation frequency) and imaginary part $\mathrm{Im}(\Omega)$ (damping ratio) of the complex natural frequency Ω for the volume harmonic Ω_{00} (i.e., $m = 0$, $k = 0$). Typical dependencies are shown in the figure 2: the frequency decreases monotonically with increasing parameter λ_b, and the damping rate is maximum for a finite wetting parameter and decreases in the limiting cases of the free or fixed contact line. Note, that changes in the parameter λ_u (or λ_b) are symmetric relative to each other, i.e. you can change one with a fixed other. The total damping rate is determined by the sum of the individual coefficients for each plate. This fact determines the finite value of the damping parameter at small λ_b (see figure 2b).

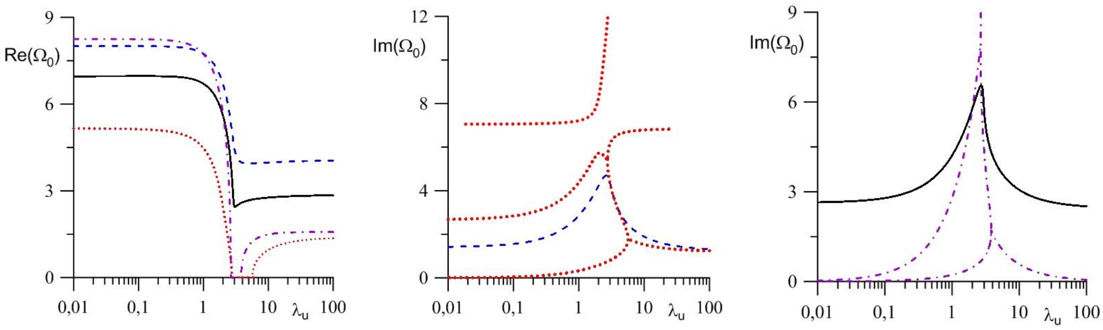
The dependencies for the different heterogeneous surfaces are shown in Figure 3 similar to Figure 2. Wavenumber k_λ changes the effective interaction of contact line with the plate surface, i.e., Hocking parameter. Surface inhomogeneity changes the monotonicity of curves and leads to the appearance of local extrema.



(a) (b)

Figure 3. Frequency (a) and damping ratio (b) of volume natural oscillations vs wetting parameter λ_b for Ω_{00} ($R_0 = 5$, $P_0 = 5$, $b = 1$, $\lambda_u = 1$).
$\lambda_u = \lambda_b$ – solid line (10), $k_\lambda = 0.1$ – dashed, $k_\lambda = 1$ – dotted, $k_\lambda = 10$ – dash-dotted, (9) – 2-dot-dashed.

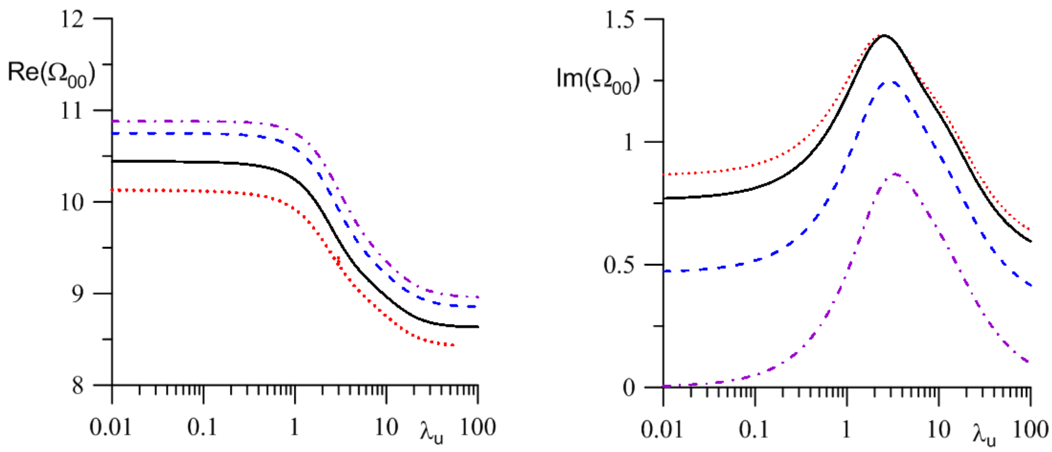
The frequency (figure 2a, figure 4a, figure 5a) and the damping rate (figure 2b, figure 4b,c, figure 5b) increase with increasing the bubble volume, i.e., with the growth of b. In a certain range of λ_u, the real part of the frequency $\mathrm{Re}(\Omega_{00})$ can vanish. It's depending on the value of the ratio b and Hocking parameter λ_b (figure 4a). The vanishing of $\mathrm{Re}(\Omega_{00})$ corresponds to the bifurcation of the branch of the increment $\mathrm{Im}(\Omega_{00})$ (figure 4b,c). The dissipation is proportional to the length of the contact line in this case, because this is just the interaction between the contact line and the solid plate that causes the energy dissipation. Therefore, growing of parameter b increases the length of the contact line at constant drop volume, i.e., it increases the energy dissipation.



(a) (b) (c)

Figure 4. Frequency (a) and damping ratio (b, c) of volume natural oscillations vs wetting parameter λ_u for Ω_{00} ($R_0 = 5$, $P_0 = 5$, $b = 3$) (9).
$\lambda_u = \lambda_b$ – dash-dotted line (10), $\lambda_b = 1$ – dashed, $\lambda_b = 2$ – solid, $\lambda_b = 3$ – dotted.

Surface inhomogeneity destroys this monotonic damping effect (figure 5). It is possible that monotonic damping exists in this case, but it could not be detected.



(a) (b)

Figure 5. Frequency (a) and damping ratio (b) of volume natural oscillations vs wetting parameter λ_u for Ω_{00} ($R_0 = 5$, $P_0 = 5$, $b = 3$, $k_\lambda = 1$).
$\lambda_u = \lambda_b$ – dash-dotted line (10), $\lambda_b = 1$ – dashed, $\lambda_b = 2$ – solid, $\lambda_b = 3$ – dotted.

4. Conclusions

The free oscillations of the cylindrical bubble confined between solid plates have been considered taking into account the dynamics of the contact line. The heterogeneous solid plates have different Hocking parameters. The solid plates have non-uniform surfaces described by the function $\Lambda_{u,b}(\alpha) = \lambda_{u,b} \left|\sin\left(k_\lambda \cos(\alpha)\right)\right|$. The boundary condition imposed on the contact line leads to the damping of oscillations. Firstly, the wavenumber k_λ changes the effective interaction of contact line with the plate surface, i.e., Hocking parameter. Surface inhomogeneity changes the monotonicity of curves and leads to the appearance of local extrema. Secondly, Surface inhomogeneity destroys this monotonic damping effect. It is possible that monotonic damping exists in this case, but it could not be detected. This requires further research.

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