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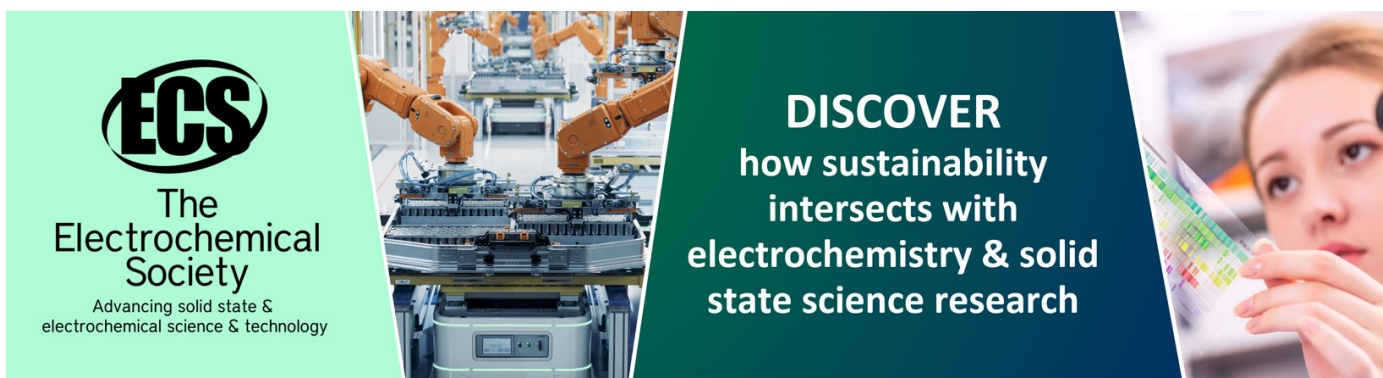
An Efficient Shrinkage Estimators For Generalized Inverse Rayleigh Distribution Based On Bounded And Series Stress-Strength Models

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An Efficient Shrinkage Estimators For Generalized Inverse Rayleigh Distribution Based On Bounded And Series Stress-Strength Models

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ABSTRACT

In this paper, we investigate two stress-strength models (Bounded and Series) in systems reliability based on Generalized Inverse Rayleigh distribution. To obtain some estimates of shrinkage estimators, Bayesian methods under informative and non-informative assumptions are used. For comparison of the presented methods, Monte Carlo simulations based on the Mean squared Error criteria are applied.

1. Introduction

Reliability (R) is a broad term that focuses on the ability of a product to perform its intended function. for the reliability (R) in the stress- strength (S-S) model was attracted many statisticians for several years owing to their applicability in different and diverse parts such as engineering, quality control, economics. In addition, in the previous thirty years, there have been many applications to medical problems and clinical trials [1,2].

The term stress-strength (S-S) refers to a component which has a random strength X subject to a random stress Y to evaluate the reliability. The component fails if the stress applied to it exceeds the strength, while the component works whenever Y less than X ($Y < X$). Several researchers assuming various lifetime distributions for the stress-strength random variates [3,4,5].

However, because modern engineering systems may have more than two components [6]. For instant, bridges, car engines, air-conditioning systems, biological and ecological systems, quality control systems in manufacturing plants, etc.) may be viewed as assemblies of many interacting elements. The elements are often arranged in mechanical or logical series or parallel configuration. Similarly, the blood pressure for each person, there are two border diastolic pressure and systolic pressure should be in these limits. The stress-strength models of $P(Y_1 < X < Y_2)$ were studied in many branches of science such as psychology, medicine, pedagogy, etc.[7]. The probability equation: $R = P(Y_1 < X < Y_2)$ of "stress- strength "reliability "describes that if the random variables X that represent the " pressure "of the components exceed the random variables that represent the " strength "of the component, the component. statistical studies of stress-strength as the main part of the reliability system started after it was introduced [8], after that model system $P(Y_1 < X < Y_2)$ gets great space of authors studies since the seventies of the last century till now. In addition, for series stress–strength models $P(Y < \min(X_1, X_2, \dots, X_n))$,



[9] proved the sufficient and necessary condition for the existence of MVUE of $R = P(X_1 \geq Y, \dots, X_n \geq Y)$ is $m \geq n$, also, obtains R 's MVUE, where $X_1, \dots, X_n$ iid. $\sim N(\mu, \sigma^2)$, $Y \sim N(0, 1)$, Y is independent of $X_1, \dots, X_n$, μ, σ^2 are unknown, $Z_1, \dots, Z_m$ ($m \geq 2$) are iid. sample from $N(\mu, \sigma^2)$. [10] obtained the estimation of system reliability in multi-component series stress-strength models. He considered the estimation of $R = P(X_{k+1} < \min(X_1, X_2, \dots, X_k))$ when X_i , $i = 1, 2, \dots, k+1$, all follow independent Gamma, Weibull, and Pareto distributions. [11] discussed series stress-strength models having bivariate Marshall–Olkin exponential strengths subjected to q stresses. The stresses are independent and exponentially distributed.

On the other hand, Statistical distributions have long been employed in the assessment of semiconductor device and product reliability. Generalized Inverted Rayleigh distribution (GIRD) is a very helpful model that can be vastly used in applied statistics reliability analysis, telecommunications engineering Convergence of biology and it is used to analyze age data, health and the existence of several Pilot units [12].

Therefore, in this paper, two models bounded and series were considered to estimate the stress-strength reliability based on two parameters GIRD via different estimation methods. Also, different Bayesian estimation methods and some Shrinkage estimation methods were used. Since, recently with advances in computation and methodology, researchers are using Bayesian methods to solve an increasing variety of complex problems. In many applications, Bayesian methods provide important computational and methodological advantages over classical techniques [13]. Then, Shrinkage estimation methods are used.

The rest of the paper is organized as follows: Section 2. clarifying Generalized Inverse Rayleigh distribution. Section 3. Models Description with Mathematical Formulation. Section 4. Maximum likelihood Estimation. Section 5. Bayesian Estimator. Section 6. Shrinkage Estimation Methods. Section 7. Simulation study. Section 8. demonstrates the effectiveness of the suggested method through numerical results. In the end, in section 9, a conclusion is presented.

2. Generalized Inverse Rayleigh distribution

Generalized Inverted Rayleigh distribution presents a flexible family in the varieties of shapes and is suitable for modeling data with different types of hazard rate function: increasing, decreasing, and upside-down bathtub shape. It is widely used in communication engineering, reliability analysis, and applied statistics. Various applications of this distribution are given [14]. The probability density function (p.d.f.) of the GIRD with scale parameter θ is:

$$f(x) = \frac{2\theta\sigma^2}{x^3} e^{-\frac{\sigma^2\theta}{x^2}} \quad \text{for } x > 0, \theta, \sigma^2 > 0 \quad (1)$$

The cumulative distribution function (c.d.f) is given as follows:

$$F(x) = e^{-\frac{\sigma^2\theta}{x^2}} \quad (2)$$

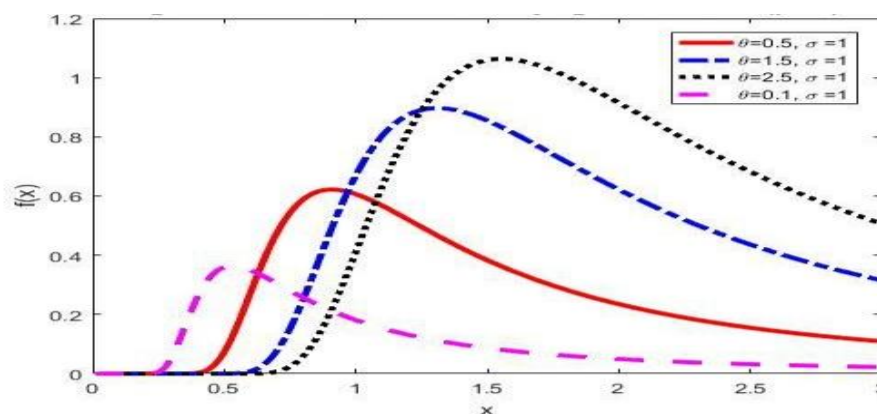


Figure 1: Generalized Inverse Rayleigh Distribution (p.d.f)

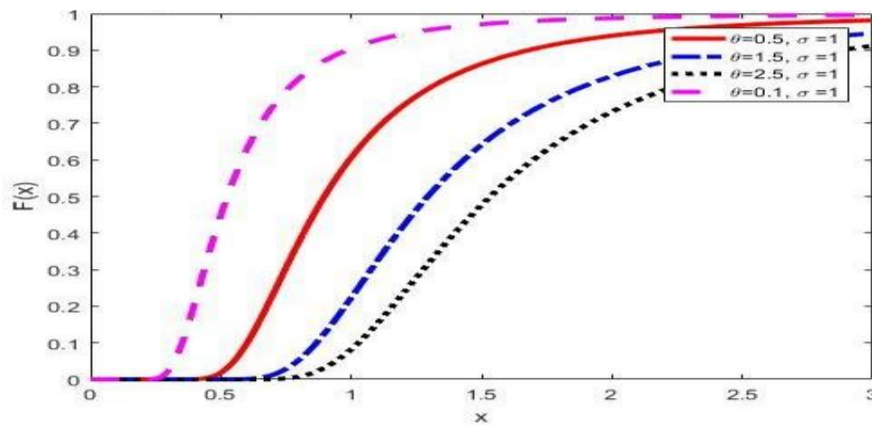


Figure 2: Generalized Inverse Rayleigh Distribution(c.d.f)

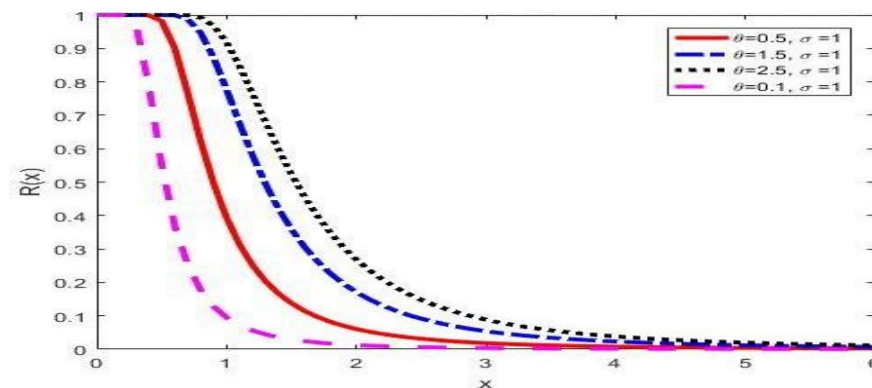


Figure 3: Generalized Inverse Rayleigh Distribution(Reliability)

3. Models Description with Mathematical Formulation

In this section, two models (bounded and series) system reliability were considered as follows:

3.1 Stress-Strength Reliability For The Bounded System

In the S-S model the formula of system reliability (R_b) which defined as $R = P(Y_1 < X < Y_2)$, where X be independent random strength variable such that $X_1 \sim \text{GIRD}(\theta_1, \sigma^2)$ and Y_1, Y_2 are two independent random stress variables such that $Y_1 \sim \text{GIRD}(\theta_2, \sigma^2)$, $Y_2 \sim \text{GIRD}(\theta_3, \sigma^2)$ with known parameter σ , respectively. Therefore, S-S reliability is defined as below:

$$R_b = P(Y_1 < X < Y_2)$$

Therefore,

$$= \int_0^\infty \int_0^x \int_x^\infty f(y_1) dy_1 f(y_2) dy_2 f(x) dx$$

$$= \int_0^\infty F_{y1}(x) \bar{F}_{y2}(x) f(x) dx$$

$$F(x) = e^{-\frac{\sigma^2 \theta_1}{x^2}}$$

$$F_{y1}(x) = e^{-\frac{\sigma^2 \theta_2}{x^2}}$$

$$F_{y2}(x) = e^{-\frac{\sigma^2 \theta_3}{x^2}}$$

$$R_b = \int_0^\infty e^{-\frac{\sigma^2 \theta_2}{x^2}} (1 - e^{-\frac{\sigma^2 \theta_3}{x^2}}) \frac{2\sigma^2 \theta_1}{x^3} e^{-\frac{\sigma^2 \theta_1}{x^2}} dx$$

$$= \int_0^\infty (1 - e^{-\frac{\sigma^2 \theta_3}{x^2}}) \frac{2\sigma^2 \theta_1}{x^3} e^{-\frac{\sigma^2 (\theta_1 + \theta_2)}{x^2}} dx$$

$$= \int_0^\infty \frac{2\sigma^2\theta_1}{x^3} e^{-\frac{\sigma^2(\theta_1+\theta_2)}{x^2}} dx - \int_0^\infty \frac{2\sigma^2\theta_1}{x^3} e^{-\frac{\sigma^2(\theta_1+\theta_2+\theta_3)}{x^2}} dx$$

$$= \frac{\theta_1}{\theta_2+\theta_1} - \frac{\theta_1}{\theta_2+\theta_1+\theta_3}$$

Consequently, by simplifications, we get

$$R_b = \frac{\theta_1\theta_3}{(\theta_2+\theta_1)(\theta_2+\theta_1+\theta_3)}$$

(3)

3.2 Stress-Strength Reliability For Series System

Another important application of random events is the practically important case of a system composed of statistically independent components, arranged logically in series. Additionally, the series systems function properly only when all their components function properly.

In this paper, we used series system reliability (R_s) in S-S model contain three series components, where X_i be independent random strength variable such that $X_1 \sim \text{GIRD}(\theta_1, \sigma^2)$, $X_2 \sim \text{GIRD}(\theta_2, \sigma^2)$, $X_3 \sim \text{GIRD}(\theta_3, \sigma^2)$ and Y is a stress variable such that $Y \sim \text{GIRD}(\theta_4, \sigma^2)$, with known parameter σ which has strengths subject to common stress

$$R_s = P(Y < \min(X_1, X_2, X_3))$$

$$\text{Let } Z = \min(X_1, X_2, X_3),$$

Therefore,

$$R_s = p(y < Z), 0 < y < Z < \infty$$

$$R_s = \int_0^\infty \int_0^z f(y)f(z)dz dy$$

$$F_z(Z) = P(Z < z) = 1 - p(Z \geq z)$$

$$= 1 - [P(x_1 > z)P(x_2 > z)P(x_3 > z)]$$

$$= 1 - \left[\left(1 - e^{-\frac{\sigma^2\theta_1}{z^2}}\right) \left(1 - e^{-\frac{\sigma^2\theta_2}{z^2}}\right) \left(1 - e^{-\frac{\sigma^2\theta_3}{z^2}}\right) \right]$$

$$= 1 - \left[\left(1 - e^{-\frac{\sigma^2\theta_2}{z^2}} - e^{-\frac{\sigma^2\theta_1}{z^2}} + e^{-\frac{\sigma^2(\theta_1+\theta_2)}{z^2}}\right) \left(1 - e^{-\frac{\sigma^2\theta_3}{z^2}}\right) \right]$$

$$= 1 - \left[1 - e^{-\frac{\sigma^2\theta_2}{z^2}} - e^{-\frac{\sigma^2\theta_1}{z^2}} + e^{-\frac{\sigma^2(\theta_1+\theta_2)}{z^2}} - e^{-\frac{\sigma^2\theta_3}{z^2}} + e^{-\frac{\sigma^2(\theta_2+\theta_3)}{z^2}} + e^{-\frac{\sigma^2(\theta_1+\theta_3)}{z^2}} - e^{-\frac{\sigma^2(\theta_1+\theta_2+\theta_3)}{z^2}} \right]$$

$$= e^{-\frac{\sigma^2\theta_2}{z^2}} + e^{-\frac{\sigma^2\theta_1}{z^2}} - e^{-\frac{\sigma^2(\theta_1+\theta_2)}{z^2}} + e^{-\frac{\sigma^2\theta_3}{z^2}} - e^{-\frac{\sigma^2(\theta_2+\theta_3)}{z^2}} - e^{-\frac{\sigma^2(\theta_1+\theta_3)}{z^2}} + e^{-\frac{\sigma^2(\theta_1+\theta_2+\theta_3)}{z^2}}$$

$$\frac{\partial f(z)}{\partial z} = \frac{\sigma^2\theta_2}{z^3} e^{-\frac{\sigma^2\theta_2}{z^2}} + \frac{\sigma^2\theta_1}{z^3} e^{-\frac{\sigma^2\theta_1}{z^2}} - \frac{\sigma^2(\theta_1+\theta_2)}{z^3} e^{-\frac{\sigma^2(\theta_1+\theta_2)}{z^2}} + \frac{\sigma^2\theta_3}{z^3} e^{-\frac{\sigma^2\theta_3}{z^2}}$$

$$- \frac{\sigma^2(\theta_2+\theta_3)}{z^3} e^{-\frac{\sigma^2(\theta_2+\theta_3)}{z^2}} - \frac{\sigma^2(\theta_1+\theta_3)}{z^3} e^{-\frac{\sigma^2(\theta_1+\theta_3)}{z^2}}$$

$$+ \frac{\sigma^2(\theta_1+\theta_2+\theta_3)}{z^3} e^{-\frac{\sigma^2(\theta_1+\theta_2+\theta_3)}{z^2}}$$

$$R_s = \int_0^\infty \frac{\sigma^2\theta_2}{z^3} e^{-\frac{\sigma^2\theta_2}{z^2}} + \frac{\sigma^2\theta_1}{z^3} e^{-\frac{\sigma^2\theta_1}{z^2}} - \frac{\sigma^2(\theta_1+\theta_2)}{z^3} e^{-\frac{\sigma^2(\theta_1+\theta_2)}{z^2}} + \frac{\sigma^2\theta_3}{z^3} e^{-\frac{\sigma^2\theta_3}{z^2}} -$$

$$\frac{\sigma^2(\theta_2+\theta_3)}{z^3} e^{-\frac{\sigma^2(\theta_2+\theta_3)}{z^2}} - \frac{\sigma^2(\theta_1+\theta_3)}{z^3} e^{-\frac{\sigma^2(\theta_1+\theta_3)}{z^2}} + \frac{\sigma^2(\theta_1+\theta_2+\theta_3)}{z^3} e^{-\frac{\sigma^2(\theta_1+\theta_2+\theta_3)}{z^2}} d\theta$$

$$= \int_0^\infty \frac{\sigma^2\theta_2}{z^3} e^{-\frac{\sigma^2\theta_2}{z^2}} + \frac{\sigma^2\theta_1}{z^3} e^{-\frac{\sigma^2\theta_1}{z^2}} - \frac{\sigma^2(\theta_1+\theta_2)}{z^3} e^{-\frac{\sigma^2(\theta_1+\theta_2)}{z^2}} + \frac{\sigma^2\theta_3}{z^3} e^{-\frac{\sigma^2\theta_3}{z^2}}$$

$$- \frac{\sigma^2(\theta_2+\theta_3)}{z^3} e^{-\frac{\sigma^2(\theta_2+\theta_3)}{z^2}} - \frac{\sigma^2(\theta_1+\theta_3)}{z^3} e^{-\frac{\sigma^2(\theta_1+\theta_3)}{z^2}}$$

$$+ \frac{\sigma^2(\theta_1+\theta_2+\theta_3)}{z^3} e^{-\frac{\sigma^2(\theta_1+\theta_2+\theta_3)}{z^2}} d\theta$$

Therefore,

$$= \frac{\theta_2}{\theta_2 + \theta_4} + \frac{\theta_1}{\theta_1 + \theta_4} - \frac{\theta_1 + \theta_2}{\theta_1 + \theta_2 + \theta_4} + \frac{\theta_3}{\theta_3 + \theta_4} - \frac{\theta_2 + \theta_3}{\theta_2 + \theta_3 + \theta_4} - \frac{\theta_1 + \theta_3}{\theta_1 + \theta_3 + \theta_4} + \frac{\theta_1 + \theta_2 + \theta_3}{\theta_1 + \theta_2 + \theta_3 + \theta_4} \quad (4)$$

Maximum Likelihood Estimator (MLE)

The Maximum likelihood method was important and commonly since it contained properties for good estimate [15]. The likelihood function is given as;

$$L(x_1, x_2, \dots, x_n, \theta_i, \sigma^2)$$

$$\prod_{i=1}^n f(x_i)$$

When $x_1, x_2, \dots, x_n$ be a strength random sample of X from GIRD (θ_i, σ^2).

$$\prod_{i=1}^n f(x_i)$$

$$\frac{n}{\prod_{i=1}^n x_i^3} \frac{2\theta_1 \sigma^2}{x_i^3} e^{-\frac{\sigma^2 \theta_1}{x_i^2}}$$

$$\sigma^{2n} \theta_1^n x_i^{-3} e^{-\sum_{i=1}^n \theta_1 \left(\frac{\sigma}{x_i}\right)^2}$$

Take $\ln$ to both sides will be

$$= n \ln 2 + 2n \ln \sigma + n \ln \theta_1 - 3 \ln x_i - \sum_{i=1}^n \theta_1 \left(\frac{\sigma}{x_i}\right)^2$$

The partial derivative of $\ln l$ with respect to θ_1 and equate to zero is given by

$$= \frac{n}{\theta_1} - \sum_{i=1}^n \left(\frac{\sigma}{x_i}\right)^2, \quad \frac{\partial \ln L}{\partial \theta_1} = 0$$

Hence, in the case of the bounded model the MLE estimator for the unknown shape parameter θ_1 will be:

$$MLE_{\theta_1} = \frac{n}{\sum_{i=1}^n \left(\frac{\sigma}{x_i}\right)^2} \quad \text{where } i=1, 2, \dots, n \quad (5)$$

In the same way, let $y_{11}, y_{12}, \dots, y_{1m_1}$ and $y_{21}, y_{22}, \dots, y_{2m_2}$ be stress random variables from GIRD(θ_i, σ^2) respectively, and the MLE for the unknown shape parameter θ_2, θ_3 will be

$$MLE_{\theta_2} = \frac{m_1}{\sum_{j=1}^{m_1} \frac{\sigma^2}{y_{1j}^2}} \quad (6)$$

$$MLE_{\theta_3} = \frac{m_2}{\sum_{t=1}^{m_2} \frac{\sigma^2}{y_{2t}^2}} \quad \text{where } j = 1, 2, \dots, m_1, t = 1, 2, \dots, m_2 \quad (7)$$

And in the case of the Series model

$$MLE_{\theta_1} = \frac{n_1}{\sum_{i=1}^{n_1} \left(\frac{\sigma}{x_{1i}}\right)^2} \quad \text{where } i=1, 2, \dots, n_1 \quad (8)$$

$$MLE_{\theta_2} = \frac{n_2}{\sum_{t=1}^{n_2} \left(\frac{\sigma}{x_{2t}}\right)^2} \quad \text{where } t=1, 2, \dots, n_2 \quad (9)$$

$$MLE_{\theta_3} = \frac{n_3}{\sum_{w=1}^{n_3} \left(\frac{\sigma}{x_{3w}}\right)^2} \quad \text{where } w=1, 2, \dots, n_3 \quad (10)$$

$$MLE_{\theta_4} = \frac{m}{\sum_{j=1}^m \frac{\sigma^2}{y_j^2}} \quad j = 1, 2, \dots, m \quad (11)$$

. Bayesian Estimator

We have studied Bayesian analysis of the parameter is using Jeffrey's Prior Information and Gamma priors under two error loss functions namely; squared error loss function (SELF), and Linear Exponential (LINEX).

Posterior Function of The Parameter Based on Jeffrey's Prior Information

We must find Fisher information since we want to find Jeffrey's Prior Information,

$$\theta \propto \sqrt{I(\theta)}$$

$$\text{Hence, } I(\theta) = \frac{n}{\theta^2} \quad \text{then} \quad g(\theta) = c \sqrt{\frac{n}{\theta^2}}$$

$$g(\theta) = \frac{c}{\theta} \sqrt{n}$$

Then we present the posterior density function is:

$$G_1(\theta | x_1, x_2, \dots, x_n) = \frac{H(x_1, x_2, \dots, x_n; \theta)}{P(x_1, x_2, \dots, x_n; \theta)}$$

When,

$$H(x_1, x_2, \dots, x_n; \theta) = L(x_1, x_2, \dots, x_n; \theta) g(\theta)$$

Then the marginal probability density function of $(x_1, x_2, \dots, x_n)$ is given by

$$P(x_1, x_2, \dots, x_n; \theta) = \int_0^\infty L(x_1, x_2, \dots, x_n; \theta) g(\theta) d\theta$$

Therefore,

$$\text{where } w = \sum_{i=1}^n \left(\frac{\sigma}{x_i}\right)^2 \text{ and let } z = \theta w, \theta = \frac{z}{w}, d\theta = \frac{dz}{w}$$

$$= \frac{c \sqrt{n} 2^n \theta^{n-1} \sigma^{2n} \prod_{i=1}^n x_i^{-3} e^{-\theta w}}{c \sqrt{n} 2^n \sigma^{2n} \prod_{i=1}^n x_i^{-3} \int_0^\infty \left(\frac{z}{w}\right)^{n-1} e^{-z} \frac{dz}{w}}$$

Hence, the posterior density function for θ based on Jeffery's prior information will be

$$G_1(\theta | x_1, x_2, \dots, x_n) = \frac{w^n \theta^{n-1} e^{-z}}{\Gamma(n)}$$

$$G_1(\theta | x_1, x_2, \dots, x_n) = \frac{\sum_{i=1}^n \left(\frac{\sigma}{x_i}\right)^{2n} \theta^{n-1} e^{-\sum_{i=1}^n \theta \left(\frac{\sigma}{x_i}\right)^2}}{\Gamma(n)}$$

We can be identified the posterior density in the equation as a density of Gamma distribution

$$\theta \sim \text{Gamma} \left(n, \frac{1}{w} \right) \text{ with } E(\theta) = \frac{n}{w} \text{ and } \text{var}(\theta) = \frac{n}{w^2}$$

In the case of the bounded model.

$$\theta_1 \sim \text{Gamma} \left(n, \frac{1}{w} \right) \text{ with } E(\theta_1) = \frac{n}{w} \text{ and } \text{var}(\theta_1) = \frac{n}{w^2}$$

$$\theta_2 \sim \text{Gamma} \left(m_1, \frac{1}{w} \right) \text{ with } E(\theta_2) = \frac{m_1}{w} \text{ and } \text{var}(\theta_2) = \frac{m_1}{w^2}$$

$$\theta_3 \sim \text{Gamma} \left(m_2, \frac{1}{w} \right) \text{ with } E(\theta_3) = \frac{m_2}{w} \text{ and } \text{var}(\theta_3) = \frac{m_2}{w^2}$$

In the case of the Series model

$$\theta_1 \sim \text{Gamma} \left(n_1, \frac{1}{w} \right) \text{ with } E(\theta_1) = \frac{n_1}{w} \text{ and } \text{var}(\theta_1) = \frac{n_1}{w^2}$$

$$\theta_2 \sim \text{Gamma} \left(n_2, \frac{1}{w} \right) \text{ with } E(\theta_2) = \frac{n_2}{w} \text{ and } \text{var}(\theta_2) = \frac{n_2}{w^2}$$

$$\theta_3 \sim \text{Gamma} \left(n_3, \frac{1}{w} \right) \text{ with } E(\theta_3) = \frac{n_3}{w} \text{ and } \text{var}(\theta_3) = \frac{n_3}{w^2}$$

$$\theta_4 \sim \text{Gamma} \left(m, \frac{1}{w} \right) \text{ with } E(\theta_4) = \frac{m}{w} \text{ and } \text{var}(\theta_4) = \frac{m}{w^2}$$

5.2 Posterior Function of The Parameter Based on Gamma Prior Information

$$g_2(\theta) = \frac{\beta^\delta \theta^{\delta-1} e^{-\theta\beta}}{\Gamma(\alpha)}; \theta > 0, \beta > 0, \sigma > 0$$

$$G_2(\theta | x_1, x_2, \dots, x_n) = \frac{H(x_1, x_2, \dots, x_n; \theta)}{P(x_1, x_2, \dots, x_n; \theta)}$$

$$= \frac{2^n \theta^n \sigma^{2n} \prod_{i=1}^n x_i^{-3} e^{-\theta w} \frac{\beta^\delta \theta^{\delta-1} e^{-\theta\beta}}{\Gamma(\alpha)}}{2^n \sigma^{2n} \prod_{i=1}^n x_i^{-3} \frac{\beta^\delta}{\Gamma(\alpha)} \int_0^\infty \theta^{n+\delta-1} e^{-\theta(w+\beta)} d\theta}$$

Thus

$$G_2(\theta | x_1, x_2, \dots, x_n) = \frac{p^{n+\delta} \theta^{n+\delta-1} e^{-\theta p}}{\Gamma(n+\delta)}, \text{ where } P = (w + \beta)$$

This can easily be seen

This can be seen easily

$$\theta \sim \text{Gamma} \left(n + \delta, \frac{1}{p} \right) \text{ with } E(\theta) = \frac{n+\delta}{p}, \text{ var}(\theta) = \frac{n+\delta}{p^2}$$

Therefore, in the case of bounded model

$$\begin{aligned}\theta_1 &\sim \text{Gamma}\left(n + \delta, \frac{1}{p}\right) \text{ with } E(\theta_1) = \frac{n+\delta}{p}, \text{ var } (\theta_1) = \frac{n+\delta}{p^2} \\ \theta_2 &\sim \text{Gamma}\left(m_1 + \delta, \frac{1}{p}\right) \text{ with } E(\theta_2) = \frac{m_1+\delta}{p}, \text{ var } (\theta_2) = \frac{m_1+\delta}{p^2} \\ \theta_3 &\sim \text{Gamma}\left(m_2 + \delta, \frac{1}{p}\right) \text{ with } E(\theta_3) = \frac{m_2+\delta}{p}, \text{ var } (\theta_3) = \frac{m_2+\delta}{p^2}\end{aligned}$$

in the case of the Series model

$$\begin{aligned}\theta_1 &\sim \text{Gamma}\left(n_1 + \delta, \frac{1}{p}\right) \text{ with } E(\theta_1) = \frac{n_1+\delta}{p}, \text{ var } (\theta_1) = \frac{n_1+\delta}{p^2} \\ \theta_2 &\sim \text{Gamma}\left(n_2 + \delta, \frac{1}{p}\right) \text{ with } E(\theta_2) = \frac{n_2+\delta}{p}, \text{ var } (\theta_2) = \frac{n_2+\delta}{p^2} \\ \theta_3 &\sim \text{Gamma}\left(n_3 + \delta, \frac{1}{p}\right) \text{ with } E(\theta_3) = \frac{n_3+\delta}{p}, \text{ var } (\theta_3) = \frac{n_3+\delta}{p^2} \\ \theta_4 &\sim \text{Gamma}\left(m + \delta, \frac{1}{p}\right) \text{ with } E(\theta_4) = \frac{m+\delta}{p}, \text{ var } (\theta_4) = \frac{m+\delta}{p^2}\end{aligned}$$

5.3 Bayes Estimator Under Considered Error Loss Functions

5.3.1 Bayes Estimator Under Squared Error Loss Function

By using squared error loss function $l(\hat{\theta}, \theta)$ which is:

$$l(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$$

the mathematical expectation for the loss function (Risk function) is given as follows:

$$R(\hat{\theta}, \theta) = E[l(\hat{\theta}, \theta)] = \int_{\theta} l(\hat{\theta}, \theta) G_1(\theta|x_i) d\theta$$

$\hat{\theta}_S$ by differentiating $R(\hat{\theta}, \theta)$ with respect to $\hat{\theta}$ and equating the derivative to zero to get

$$\hat{\theta}_S = E(\theta|x_i) \quad (12)$$

5.3.1.1 The Jeffrey's prior information case

The Bayes standard estimator for the parameter (θ) is the estimator that makes the risk function as low as possible

$$\hat{\theta}_{1SJ} = E(\theta_1|x_i)$$

To find $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$, we apply equation (5), In the case of bounded

$$\hat{\theta}_{1SJ} = \frac{n}{\sum_{i=1}^n \left(\frac{\sigma}{x_{1i}}\right)^2} \quad (13)$$

$$\hat{\theta}_{2SJ} = \frac{m_1}{\sum_{j=1}^{m_1} \left(\frac{\sigma}{y_{1j}}\right)^2}$$

(14)

$$\hat{\theta}_{3SJ} = \frac{m_2}{\sum_{t=1}^{m_2} \left(\frac{\sigma}{y_{2t}}\right)^2}$$

(15)

Then in the case of series we get $\hat{\theta}_1$, $\hat{\theta}_2$, $\hat{\theta}_3$ and $\hat{\theta}_4$

$$\hat{\theta}_{1SJ} = \frac{n_1}{\sum_{i=1}^{n_1} \left(\frac{\sigma}{x_{1i}}\right)^2}$$

(16)

$$\hat{\theta}_{2SJ} = \frac{n_2}{\sum_{t=1}^{n_2} \left(\frac{\sigma}{x_{2t}}\right)^2}$$

(17)

$$\hat{\theta}_{3SJ} = \frac{n_3}{\sum_{w=1}^{n_3} \left(\frac{\sigma}{x_{3w}}\right)^2} \quad (18)$$

$$\hat{\theta}_{4SJ} = \frac{m}{\sum_{j=1}^m \left(\frac{\sigma}{y_j}\right)^2} \quad (19)$$

5.3.1.2 The Gamma prior information case

The Bayes estimator of Gamma prior information for the parameter (θ) makes the risk function as low as possible

$$\hat{\theta} = E(\theta | x_i)$$

then apply equation (6) For the condition of the bounded we get

$$\hat{\theta}_{1SG} = \frac{n+\delta}{\sum_{i=1}^n \left(\frac{\sigma}{x_i}\right)^2 + \beta}$$

In the same way, find $\hat{\theta}_{2SG}$ and $\hat{\theta}_{3SG}$

$$\hat{\theta}_{2SG} = \frac{m_1+\delta}{\sum_{j=1}^{m_1} \left(\frac{\sigma}{y_{1j}}\right)^2 + \beta} \text{ and } \hat{\theta}_{3SG} = \frac{m_2+\delta}{\sum_{t=1}^{m_2} \left(\frac{\sigma}{y_{2t}}\right)^2 + \beta},$$

And using equation (6) For the condition of the series we get

$$\hat{\theta}_{1SG} = \frac{n_1+\delta}{p} = \frac{n_1+\delta}{(w+\beta)} = \frac{n_1+\delta}{\sum_{i=1}^{n_1} \left(\frac{\sigma}{x_{1i}}\right)^2 + \beta}$$

By the same way find $\hat{\theta}_{2SG}$ and $\hat{\theta}_{3SG}$

$$\hat{\theta}_{2SG} = \frac{n_2+\delta}{\sum_{t=1}^{n_2} \left(\frac{\sigma}{x_{2t}}\right)^2 + \beta}, \hat{\theta}_{3SG} = \frac{n_3+\delta}{\sum_{w=1}^{n_3} \left(\frac{\sigma}{x_{3w}}\right)^2 + \beta} \text{ and } \hat{\theta}_{4SG} = \frac{m+\delta}{\sum_{j=1}^{m_2} \left(\frac{\sigma}{y_j}\right)^2 + \beta}$$

5.3.2 Bayes Estimator Under Linear Exponential Function

The linear error loss function $l(\hat{\theta}, \theta)$ as follows:

$$l(\hat{\theta}, \theta) = e^{\Delta} - \Delta - 1, \text{ where } \Delta = (\hat{\theta} - \theta)$$

The form of the risk function is:

$$\begin{aligned} R(\hat{\theta}, \theta) &= \int_{\theta} \left[e^{(\hat{\theta}-\theta)} - (\hat{\theta} - \theta) - 1 \right] G(\theta | x) d\theta \\ &= e^{\hat{\theta}} \int_{\theta} e^{-\theta} G(\theta | x) d\theta - \hat{\theta} \int_{\theta} G(\theta | x) d\theta + \int_{\theta} \theta G(\theta | x) d\theta - \int_{\theta} G(\theta | x) d\theta \end{aligned}$$

Differentiating $R(\hat{\theta}, \theta)$ with respect to $\hat{\theta}$ and setting the result to zero, we get

$$\frac{\partial R(\hat{\theta}, \theta)}{\partial(\hat{\theta})} = e^{\hat{\theta}} \int_{\theta} e^{-\theta} G(\theta | x) d\theta - \int_{\theta} G(\theta | x) d\theta = 0$$

Which implies that

$$e^{\hat{\theta}} \int_{\theta} e^{-\theta} G(\theta | x) d\theta = 1$$

Hence,

$$e^{\hat{\theta}} = \frac{1}{\int_{\theta} e^{-\theta} G(\theta | x) d\theta}$$

Thus, the Bayesian estimator of θ establish on LINEX loss function is

$$\hat{\theta}_L = -\ln \int_{\theta} e^{-\theta} G(\theta | x) d\theta$$

5.3.2.1 The Case of Jeffrey s Prior Information Case

$$\hat{\theta}_{LJ} = -\ln \int_0^{\infty} e^{-\theta} G_1(\theta | x) d\theta$$

$$\begin{aligned}
&= -\ln \int_0^\infty e^{-\theta} \frac{w^n \theta^{n-1} e^{-\theta w}}{\Gamma(n)} \\
&= \frac{-\ln w^n}{\Gamma(n)} \int_0^\infty \theta^{n-1} e^{-\theta(1+w)} d\theta \\
&= \frac{-\ln w^n}{p^n \Gamma(n)} \Gamma(n), \quad \text{where } p = (1+w), \text{ for the condition of the bounded} \\
\hat{\theta}_{LJ} &= -\ln \left(\frac{w}{p} \right)^n = -\ln \left(\frac{\sum_{i=1}^n \left(\frac{\sigma}{X_i} \right)^2}{(1+w)} \right)^n
\end{aligned}$$

Thus, for the condition of the bounded

$$\hat{\theta}_{1LJ} = -\ln \left(\frac{\sum_{i=1}^n \left(\frac{\sigma}{X_i} \right)^2}{(1+w)} \right)^n \quad (20)$$

By the same way find $\hat{\theta}_{2LJ}$ and $\hat{\theta}_{3LJ}$

$$\hat{\theta}_{2LJ} = -\ln \left(\frac{\sum_{j=1}^{m1} \left(\frac{\sigma}{Y_{1j}} \right)^2}{(1+w)} \right)^{m1} \quad (21)$$

$$\hat{\theta}_{3LJ} = -\ln \left(\frac{\sum_{t=1}^{m2} \left(\frac{\sigma}{Y_{2t}} \right)^2}{(1+w)} \right)^{m2} \quad (22)$$

And For the condition of the series

$$\hat{\theta}_{1LJ} = -\ln \left(\frac{\sum_{i=1}^{n1} \left(\frac{\sigma}{X_{1i}} \right)^2}{(1+w)} \right)^{n1} \quad (23)$$

$$\hat{\theta}_{2LJ} = -\ln \left(\frac{\sum_{t=1}^{n2} \left(\frac{\sigma}{X_{2t}} \right)^2}{(1+w)} \right)^{n2} \quad (24)$$

$$\hat{\theta}_{3LJ} = -\ln \left(\frac{\sum_{w=1}^{n3} \left(\frac{\sigma}{X_{3w}} \right)^2}{(1+w)} \right)^{n3} \quad (25)$$

$$\hat{\theta}_{4LJ} = -\ln \left(\frac{\sum_{j=1}^m \left(\frac{\sigma}{Y_j} \right)^2}{(1+w)} \right)^m \quad (26)$$

5.3.2.2 The Gamma Prior Information Case

$$\begin{aligned}
\hat{\theta}_{LENG} &= -\ln \int_0^\infty e^{-\theta} G_2(\theta|x) d\theta \\
&= -\ln \int_0^\infty e^{-\theta} \frac{p^{n+\delta} \theta^{n+\delta-1} e^{-\theta p}}{\Gamma(n+\delta)} d\theta \\
&= -\ln \int_0^\infty \frac{p^{n+\delta} \theta^{n+\delta-1} e^{-\theta(1+p)}}{\Gamma(n+\delta)} d\theta \\
&= -\ln \frac{p^{n+\delta}}{(1+p)^{n+\delta}} \int_0^\infty \frac{(1+p)^{n+\delta} \theta^{n+\delta-1} e^{-\theta(1+p)}}{\Gamma(n+\delta)} d\theta
\end{aligned}$$

And after, completing the solution steps to find the product of the integral to get

$$\hat{\theta}_{LG} = -\ln\left(\frac{p}{(1+p)}\right)^{n+\delta} = -\ln\left(\frac{(w+\beta)}{(1+(w+\beta))}\right)^{n+\delta}$$

Therefore, for the condition of the bounded

$$\hat{\theta}_{1LG} = -\ln\left(\frac{(w+\beta)}{(1+(w+\beta))}\right)^{n+\delta} \quad (27)$$

By the same way find $\hat{\theta}_{2LG}$ and $\hat{\theta}_{3LG}$

$$\hat{\theta}_{2LG} = -\ln\left(\frac{(w+\beta)}{(1+(w+\beta))}\right)^{m1+\delta} \quad (28)$$

$$\hat{\theta}_{3LG} = -\ln\left(\frac{(w+\beta)}{(1+(w+\beta))}\right)^{m2+\delta} \quad (29)$$

, And for the condition of the series

$$\hat{\theta}_{1LG} = -\ln\left(\frac{(w+\beta)}{(1+(w+\beta))}\right)^{n1+\delta} \quad (30)$$

$$\hat{\theta}_{2LG} = -\ln\left(\frac{(w+\beta)}{(1+(w+\beta))}\right)^{n2+\delta} \quad (31)$$

$$\hat{\theta}_{3LG} = -\ln\left(\frac{(w+\beta)}{(1+(w+\beta))}\right)^{n3+\delta} \quad (32)$$

$$\hat{\theta}_{4LG} = -\ln\left(\frac{(w+\beta)}{(1+(w+\beta))}\right)^{m+\delta} \quad (33)$$

6. Shrinkage Estimation

Shrinkage is where extreme values in a sample are “shrunk” towards a central value like the sample mean. Shrinking data can result in:

- Better, more stable, estimates for true population parameters.
- Reduced sampling and non-sampling errors.
- Smoothed spatial fluctuations.

Thompson gave the shape of the shrinkage estimator as follows:

$$\hat{\theta}_{sh} = \varphi(\hat{\theta})\hat{\theta}_{MLE} + (1 - \varphi(\hat{\theta}))\hat{\theta}_{Len} \quad , \text{ such that } 0 \leq \varphi(\hat{\theta}) \leq 1 \quad (34)$$

Where the unbiased estimator $\hat{\theta}_b$ was applied as usual MLE estimator of θ and θ_0 is a very closed value of θ as prior information (initial estimate) and $\varphi(\hat{\theta})$ denote the shrinkage weight factor as we mentioned above such that $0 \leq \varphi(\hat{\theta}) \leq 1$, which may be a function of $\hat{\theta}_{ub}$; a function of sample size or may be constant or may be found through minimizing the mean square error of $\hat{\theta}_{sh}$ (ad hoc basis). See [16,17,18].

6.1 Constant Shrinkage Weight Factor (Sh1)

Two models bounded and Series used the constant shrinkage in this subsection.

6.1.1 Constant Shrinkage Weight Factor For Bounded Model

The constant shrinkage weight factor (Sh1) for the bounded system will be assumed as $\varphi(\hat{\theta}_i) = e^{-0.01}$, $i=1,2,3$

6.1.1.1 Constant Shrinkage Weight Factor In Square Jeffry

$$\hat{\theta}_{1_{sh1}} = \varphi(\hat{\theta}_1)\hat{\theta}_{1_{MLE}} + (1 - \varphi(\hat{\theta}_1))\hat{\theta}_{1_{LenJ}} \quad (35)$$

$$\hat{\theta}_{2_{sh1}} = \varphi(\hat{\theta}_2)\hat{\theta}_{2_{MLE}} + (1 - \varphi(\hat{\theta}_2))\hat{\theta}_{2_{LenJ}} \quad (36)$$

$$\hat{\theta}_{3_{sh1}} = \varphi(\hat{\theta}_3)\hat{\theta}_{3_{MLE}} + (1 - \varphi(\hat{\theta}_3))\hat{\theta}_{3_{LenJ}} \quad (37)$$

where $\hat{\theta}_{i_{LenJ}}$ are prior information of $\hat{\theta}_i$ in Bayes estimator under Linear Exponential Function, ($i=1,2,3$) and then apply to the following shrinkage capabilities

Then we substitute (35), (36), and (37) in equation (3) we infer the estimation of S-S reliability (R) which consist of the abounded component using shrinkage estimation as below:

$$\hat{R}_{sh1} = \frac{\hat{\theta}_{1_{sh1}} \hat{\theta}_{3_{sh1}}}{(\hat{\theta}_{1_{sh1}} + \hat{\theta}_{2_{sh1}})(\hat{\theta}_{1_{sh1}} + \hat{\theta}_{2_{sh1}} + \hat{\theta}_{3_{sh1}})} \quad (38)$$

6.1.1.2 Constant Shrinkage Weight Factor In Square Gamma

$$\hat{\theta}_{1_{sh1}} = \varphi(\hat{\theta}_1)\hat{\theta}_{1_{MLE}} + (1 - \varphi(\hat{\theta}_1))\hat{\theta}_{1_{LenG}} \quad (39)$$

$$\hat{\theta}_{2_{sh1}} = \varphi(\hat{\theta}_2)\hat{\theta}_{2_{MLE}} + (1 - \varphi(\hat{\theta}_2))\hat{\theta}_{2_{LenG}} \quad (40)$$

$$\hat{\theta}_{3_{sh1}} = \varphi(\hat{\theta}_3)\hat{\theta}_{3_{MLE}} + (1 - \varphi(\hat{\theta}_3))\hat{\theta}_{3_{LenG}} \quad (41)$$

where $\hat{\theta}_{i_{LenG}}$ are prior information of $\hat{\theta}_i$ in Bayes estimator under Linear Exponential Function, ($i=1,2,3$) and then apply to the following shrinkage capabilities

Then we substitute (39), (40), and (41) in equation (3) we infer the estimation of S-S reliability (R) which consist of the abounded component using shrinkage estimation as below

$$\hat{R}_{sh1G} = \frac{\hat{\theta}_{1_{sh1}} \hat{\theta}_{3_{sh1}}}{(\hat{\theta}_{1_{sh1}} + \hat{\theta}_{2_{sh1}})(\hat{\theta}_{1_{sh1}} + \hat{\theta}_{2_{sh1}} + \hat{\theta}_{3_{sh1}})} \quad (42)$$

6.1.2 Constant Shrinkage Weight Factor For Series Model

$$\varphi(\hat{\theta}_i) = e^{-0.01}, \quad i=1,2,3,4$$

6.1.2.1 Constant Shrinkage Weight Factor In Square Jeffry

$$\hat{\theta}_{1_{sh1}} = \varphi(\hat{\theta}_1)\hat{\theta}_{1_{MLE}} + (1 - \varphi(\hat{\theta}_1))\hat{\theta}_{1_{LenJ}} \quad (43)$$

$$\hat{\theta}_{2_{sh1}} = \varphi(\hat{\theta}_2)\hat{\theta}_{2_{MLE}} + (1 - \varphi(\hat{\theta}_2))\hat{\theta}_{2_{LenJ}} \quad (44)$$

$$\hat{\theta}_{3_{sh1}} = \varphi(\hat{\theta}_3)\hat{\theta}_{3_{MLE}} + (1 - \varphi(\hat{\theta}_3))\hat{\theta}_{3_{LenJ}} \quad (45)$$

$$\hat{\theta}_{4sh1} = \varphi(\hat{\theta}_4)\hat{\theta}_{4MLE} + (1 - \varphi(\hat{\theta}_4))\hat{\theta}_{4LenJ}$$

(46)

Then we substitute (43), (44), (45) and (46) in equation (4) we infer the estimation of S-S reliability (R) which consist of a series component using shrinkage estimation as below:

$$\hat{R}_{sh1J} = \frac{\hat{\theta}_{2sh1}}{\hat{\theta}_{2sh1} + \hat{\theta}_{4sh1}} + \frac{\hat{\theta}_{1sh1}}{\hat{\theta}_{1sh1} + \hat{\theta}_{4sh1}} - \frac{\hat{\theta}_{1sh1} + \hat{\theta}_{2sh1}}{\hat{\theta}_{1sh1} + \hat{\theta}_{2sh1} + \hat{\theta}_{4sh1}} + \frac{\hat{\theta}_{3sh1}}{\hat{\theta}_{3sh1} + \hat{\theta}_{4sh1}} - \frac{\hat{\theta}_{2sh1} + \hat{\theta}_{3sh1}}{\hat{\theta}_{2sh1} + \hat{\theta}_{3sh1} + \hat{\theta}_{4sh1}} -$$

$$\frac{\hat{\theta}_{1sh1} + \hat{\theta}_{3sh1}}{\hat{\theta}_{1sh1} + \hat{\theta}_{3sh1} + \hat{\theta}_{4sh1}} + \frac{\hat{\theta}_{1sh1} + \hat{\theta}_{2sh1} + \hat{\theta}_{3sh1}}{\hat{\theta}_{1sh1} + \hat{\theta}_{2sh1} + \hat{\theta}_{3sh1} + \hat{\theta}_{4sh1}}$$

(47)

6.1.2.2 Constant Shrinkage Weight Factor In Square Gamma

$$\hat{\theta}_{1sh1} = \varphi(\hat{\theta}_1)\hat{\theta}_{1MLE} + (1 - \varphi(\hat{\theta}_1))\hat{\theta}_{1LenG}$$

(48)

$$\hat{\theta}_{2sh1} = \varphi(\hat{\theta}_2)\hat{\theta}_{2MLE} + (1 - \varphi(\hat{\theta}_2))\hat{\theta}_{2LenG}$$

(49)

$$\hat{\theta}_{3sh1} = \varphi(\hat{\theta}_3)\hat{\theta}_{3MLE} + (1 - \varphi(\hat{\theta}_3))\hat{\theta}_{3LenG}$$

(50)

$$\hat{\theta}_{4sh1} = \varphi(\hat{\theta}_4)\hat{\theta}_{4MLE} + (1 - \varphi(\hat{\theta}_4))\hat{\theta}_{4LenG}$$

(51)

Then we substitute (48), (49), (50) and (51) in equation (4) we infer the estimation of S-S reliability (R) which consist of a series component using shrinkage estimation as below

$$\hat{R}_{sh1G} = \frac{\hat{\theta}_{2sh1}}{\hat{\theta}_{2sh1} + \hat{\theta}_{4sh1}} + \frac{\hat{\theta}_{1sh1}}{\hat{\theta}_{1sh1} + \hat{\theta}_{4sh1}} - \frac{\hat{\theta}_{1sh1} + \hat{\theta}_{2sh1}}{\hat{\theta}_{1sh1} + \hat{\theta}_{2sh1} + \hat{\theta}_{4sh1}} + \frac{\hat{\theta}_{3sh1}}{\hat{\theta}_{3sh1} + \hat{\theta}_{4sh1}} - \frac{\hat{\theta}_{2sh1} + \hat{\theta}_{3sh1}}{\hat{\theta}_{2sh1} + \hat{\theta}_{3sh1} + \hat{\theta}_{4sh1}} -$$

$$\frac{\hat{\theta}_{1sh1} + \hat{\theta}_{3sh1}}{\hat{\theta}_{1sh1} + \hat{\theta}_{3sh1} + \hat{\theta}_{4sh1}} + \frac{\hat{\theta}_{1sh1} + \hat{\theta}_{2sh1} + \hat{\theta}_{3sh1}}{\hat{\theta}_{1sh1} + \hat{\theta}_{2sh1} + \hat{\theta}_{3sh1} + \hat{\theta}_{4sh1}}$$

(52)

6.2 Shrinkage Weight Function (Sh2)

6.2.1 Shrinkage Weight Function For Bounded Model

We suggested the shrinkage weight factor as a function of n , m_1 and m_2 , respectively in equation () as follows:

$$\gamma(\hat{\theta}_i) = e^{-\hat{\theta}_i s_j}, \quad \text{where } i = 1, 2, 3$$

6.2.1.1 Shrinkage Weight Function With Square Jeffry

The shrinkage estimator should be:

$$\hat{\theta}_{1sh2} = \gamma(\hat{\theta}_1)\hat{\theta}_{1MLE} + (1 - \gamma(\hat{\theta}_1))\hat{\theta}_{1LenJ}$$

(53)

$$\hat{\theta}_{2sh2} = \gamma(\hat{\theta}_2)\hat{\theta}_{1MLE} + (1 - \gamma(\hat{\theta}_2))\hat{\theta}_{2LenJ}$$

(54)

$$\hat{\theta}_{3sh2} = \gamma(\hat{\theta}_3)\hat{\theta}_{3MLE} + (1 - \gamma(\hat{\theta}_3))\hat{\theta}_{3LenJ}$$

(55)

Upon replacement in equation (53), (54) and (55) in equation (3), the estimate of the squared shrinkage of the reliability S-S for bounded component becomes:

$$\hat{R}_{sh2} = \frac{\hat{\theta}_{1sh2} \hat{\theta}_{3sh2}}{(\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2})(\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2} + \hat{\theta}_{3sh2})} \quad (56)$$

6.2.1.2 Shrinkage Weight Function With Square Gamma

$$\hat{\theta}_{1sh2} = \gamma(\hat{\theta}_1)\hat{\theta}_{1MLE} + (1 - \gamma(\hat{\theta}_1))\hat{\theta}_{1Leng} \quad (57)$$

$$\hat{\theta}_{2sh2} = \gamma(\hat{\theta}_2)\hat{\theta}_{1MLE} + (1 - \gamma(\hat{\theta}_2))\hat{\theta}_{2Leng} \quad (58)$$

$$\hat{\theta}_{3sh2} = \gamma(\hat{\theta}_3)\hat{\theta}_{3MLE} + (1 - \gamma(\hat{\theta}_3))\hat{\theta}_{3Leng} \quad (59)$$

Upon replacement in equation (57), (58) and (59) in equation (3), the estimate of the squared shrinkage of the reliability S-S for bounded component becomes:

$$\hat{R}_{sh2} = \frac{\hat{\theta}_{1sh2} \hat{\theta}_{3sh2}}{(\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2})(\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2} + \hat{\theta}_{3sh2})} \quad (60)$$

6.2.2 Shrinkage Weight Function For Series Model

$$\gamma(\hat{\theta}_i) = e^{-\hat{\theta}_i s J} \quad , \text{ where } i = 1, 2, 3, 4$$

6.2.2.1 Shrinkage Weight Function With Square Jeffry

$$\hat{\theta}_{1sh2} = \gamma(\hat{\theta}_1)\hat{\theta}_{1MLE} + (1 - \gamma(\hat{\theta}_1))\hat{\theta}_{1LenJ} \quad (61)$$

$$\hat{\theta}_{2sh2} = \gamma(\hat{\theta}_2)\hat{\theta}_{1MLE} + (1 - \gamma(\hat{\theta}_2))\hat{\theta}_{2LenJ} \quad (62)$$

$$\hat{\theta}_{3sh2} = \gamma(\hat{\theta}_3)\hat{\theta}_{3MLE} + (1 - \gamma(\hat{\theta}_3))\hat{\theta}_{3LenJ} \quad (63)$$

$$\hat{\theta}_{4sh2} = \gamma(\hat{\theta}_4)\hat{\theta}_{4MLE} + (1 - \gamma(\hat{\theta}_4))\hat{\theta}_{4LenJ} \quad (64)$$

Then replacement in equation (61), (62), (63) and (64) in equation (4), the estimate of the squared shrinkage of the reliability S-S for series component becomes:

$$\hat{R}_{sh2G} = \frac{\hat{\theta}_{2sh2}}{\hat{\theta}_{2sh2} + \hat{\theta}_{4sh2}} + \frac{\hat{\theta}_{1sh2}}{\hat{\theta}_{1sh2} + \hat{\theta}_{4sh2}} - \frac{\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2}}{\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2} + \hat{\theta}_{4sh2}} + \frac{\hat{\theta}_{3sh2}}{\hat{\theta}_{3sh2} + \hat{\theta}_{4sh2}} - \frac{\hat{\theta}_{2sh2} + \hat{\theta}_{3sh2}}{\hat{\theta}_{2sh2} + \hat{\theta}_{3sh2} + \hat{\theta}_{4sh2}} - \frac{\hat{\theta}_{1sh2} + \hat{\theta}_{3sh2}}{\hat{\theta}_{1sh2} + \hat{\theta}_{3sh2} + \hat{\theta}_{4sh2}} + \frac{\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2} + \hat{\theta}_{3sh2}}{\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2} + \hat{\theta}_{3sh2} + \hat{\theta}_{4sh2}} \quad (65)$$

6.2.1.2 Shrinkage Weight Function With Square Gamma

$$\hat{\theta}_{1sh2} = \gamma(\hat{\theta}_1)\hat{\theta}_{1MLE} + (1 - \gamma(\hat{\theta}_1))\hat{\theta}_{1Leng} \quad (66)$$

$$\hat{\theta}_{2sh2} = \gamma(\hat{\theta}_2)\hat{\theta}_{1MLE} + (1 - \gamma(\hat{\theta}_2))\hat{\theta}_{2Leng} \quad (67)$$

$$\hat{\theta}_{3sh2} = \gamma(\hat{\theta}_3)\hat{\theta}_{3MLE} + (1 - \gamma(\hat{\theta}_3))\hat{\theta}_{3Leng} \quad (68)$$

$$\hat{\theta}_{4sh2} = \gamma(\hat{\theta}_4)\hat{\theta}_{4MLE} + (1 - \gamma(\hat{\theta}_4))\hat{\theta}_{4Leng} \quad (69)$$

Then replacement in equation (66), (67), (68) and (69) in equation (4), the estimate of the squared shrinkage of the reliability S-S for series component becomes:

$$\hat{R}_{sh2G} = \frac{\hat{\theta}_{2sh2}}{\hat{\theta}_{2sh2} + \hat{\theta}_{4sh2}} + \frac{\hat{\theta}_{1sh2}}{\hat{\theta}_{1sh2} + \hat{\theta}_{4sh2}} - \frac{\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2}}{\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2} + \hat{\theta}_{4sh2}} + \frac{\hat{\theta}_{3sh2}}{\hat{\theta}_{3sh2} + \hat{\theta}_{4sh2}} - \frac{\hat{\theta}_{2sh2} + \hat{\theta}_{3sh2}}{\hat{\theta}_{2sh2} + \hat{\theta}_{3sh2} + \hat{\theta}_{4sh2}} - \frac{\hat{\theta}_{1sh2} + \hat{\theta}_{3sh2}}{\hat{\theta}_{1sh2} + \hat{\theta}_{3sh2} + \hat{\theta}_{4sh2}} + \frac{\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2} + \hat{\theta}_{3sh2}}{\hat{\theta}_{1sh2} + \hat{\theta}_{2sh2} + \hat{\theta}_{3sh2} + \hat{\theta}_{4sh2}} \quad (70)$$

6.3 Squared Shrinkage Weight Factor

This subsection relates to the shrinkage amount based on the squared shrinkage weighting function defined as follows:

6.3.1 Squared Shrinkage Weight Factor For Bounded Model

$$\psi(\hat{\theta}_1) = \frac{[\hat{\theta}_{1MLE} - E(\hat{\theta}_1)]^2}{var(\hat{\theta}_1)}, \quad \psi(\hat{\theta}_2) = \frac{[\hat{\theta}_{2MLE} - E(\hat{\theta}_2)]^2}{var(\hat{\theta}_2)}, \quad \psi(\hat{\theta}_3) = \frac{[\hat{\theta}_{3MLE} - E(\hat{\theta}_3)]^2}{var(\hat{\theta}_3)}$$

Where

$$E(\hat{\theta}_1) = \frac{n(\theta_1)}{n-1}, \quad E(\hat{\theta}_2) = \frac{m_1(\theta_2)}{m_1-1}, \quad E(\hat{\theta}_3) = \frac{m_2(\theta_3)}{m_2-1}$$

$$var(\hat{\theta}_1) = \frac{[n(\theta_1)]^2}{(n_k-1)^2(n_k-2)}, \quad var(\hat{\theta}_2) = \frac{[m_1(\theta_2)]^2}{(m_1-1)^2(m_1-2)} \quad \text{and} \quad var(\hat{\theta}_3) = \frac{[m_2(\theta_3)]^2}{(m_2-1)^2(m_2-2)}$$

6.3.1.1 Squared Shrinkage Weight Factor With Square Jeffry

$$\hat{\theta}_{1sh3} = \psi(\hat{\theta}_1)\hat{\theta}_{1MLE} + (1 - \psi(\hat{\theta}_1))\hat{\theta}_{1Leng} \quad (71)$$

$$\hat{\theta}_{2sh3} = \psi(\hat{\theta}_2)\hat{\theta}_{2MLE} + (1 - \psi(\hat{\theta}_2))\hat{\theta}_{2Leng} \quad (72)$$

$$\hat{\theta}_{3sh3} = \psi(\hat{\theta}_3)\hat{\theta}_{3MLE} + (1 - \psi(\hat{\theta}_3))\hat{\theta}_{3Leng} \quad (73)$$

The corresponding S-S reliability estimation was used using the equation for the bounded component as follows;

$$\hat{R}_{sh3} = \frac{\hat{\theta}_{1sh3} \hat{\theta}_{3sh3}}{(\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3})(\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3} + \hat{\theta}_{3sh3})} \quad (74)$$

6.3.1.2 Squared Shrinkage Weight Factor With Square Gamma

$$\hat{\theta}_{1sh3} = \psi(\hat{\theta}_1)\hat{\theta}_{1MLE} + (1 - \psi(\hat{\theta}_1))\hat{\theta}_{1Leng} \quad (75)$$

$$\hat{\theta}_{2sh3} = \psi(\hat{\theta}_2)\hat{\theta}_{2MLE} + (1 - \psi(\hat{\theta}_2))\hat{\theta}_{2Leng} \quad (76)$$

$$\hat{\theta}_{3sh3} = \psi(\hat{\theta}_3)\hat{\theta}_{3MLE} + (1 - \psi(\hat{\theta}_3))\hat{\theta}_{3Leng} \quad (77)$$

The corresponding S-S reliability estimation was used using the equation for the bounded component as follows;

$$\hat{R}_{sh3} = \frac{\hat{\theta}_{1sh3} \hat{\theta}_{3sh3}}{(\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3})(\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3} + \hat{\theta}_{3sh3})} \quad (78)$$

6.3.2 Squared Shrinkage Weight Factor For Series Model

$$\psi(\hat{\theta}_1) = \frac{[\hat{\theta}_{1MLE} - E(\hat{\theta}_1)]^2}{var(\hat{\theta}_1)}, \psi(\hat{\theta}_2) = \frac{[\hat{\theta}_{2MLE} - E(\hat{\theta}_2)]^2}{var(\hat{\theta}_2)}, \psi(\hat{\theta}_3) = \frac{[\hat{\theta}_{3MLE} - E(\hat{\theta}_3)]^2}{var(\hat{\theta}_3)},$$

$$\psi(\hat{\theta}_4) = \frac{[\hat{\theta}_{4MLE} - E(\hat{\theta}_4)]^2}{var(\hat{\theta}_4)}$$

Where

$$E(\hat{\theta}_1) = \frac{n_1(\theta_1)}{n_1-1}, E(\hat{\theta}_2) = \frac{n_2(\theta_2)}{n_2-1}, E(\hat{\theta}_3) = \frac{n_3(\theta_3)}{n_3-1} \text{ and } E(\hat{\theta}_4) = \frac{m(\theta_4)}{m-1}$$

$$var(\hat{\theta}_1) = \frac{[n_1(\theta_1)]^2}{(n_1-1)^2(n_1-2)}, var(\hat{\theta}_2) = \frac{[n_2(\theta_2)]^2}{(n_2-1)^2(n_2-2)}, var(\hat{\theta}_3) = \frac{[n_3(\theta_3)]^2}{(n_3-1)^2(n_3-2)} \text{ and } var(\hat{\theta}_4) = \frac{[m(\theta_4)]^2}{(m-1)^2(m-2)}$$

6.3.2.1 Squared Shrinkage Weight Factor With Square Jeffry

$$\hat{\theta}_{1sh3} = \psi(\hat{\theta}_1)\hat{\theta}_{1MLE} + (1 - \psi(\hat{\theta}_1))\hat{\theta}_{1LenJ} \quad (79)$$

$$\hat{\theta}_{2sh3} = \psi(\hat{\theta}_2)\hat{\theta}_{2MLE} + (1 - \psi(\hat{\theta}_2))\hat{\theta}_{2LenJ} \quad (80)$$

$$\hat{\theta}_{3sh3} = \psi(\hat{\theta}_3)\hat{\theta}_{3MLE} + (1 - \psi(\hat{\theta}_3))\hat{\theta}_{3LenJ} \quad (81)$$

$$\hat{\theta}_{4sh3} = \psi(\hat{\theta}_4)\hat{\theta}_{4MLE} + (1 - \psi(\hat{\theta}_4))\hat{\theta}_{4LenJ} \quad (82)$$

The corresponding S-S reliability estimation was used using the equation for the series component as follows;

$$\hat{R}_{sh3J} = \frac{\hat{\theta}_{2sh3}}{\hat{\theta}_{2sh3} + \hat{\theta}_{4sh3}} + \frac{\hat{\theta}_{1sh3}}{\hat{\theta}_{1sh3} + \hat{\theta}_{4sh3}} - \frac{\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3}}{\hat{\theta}_{1sh3} + \hat{\theta}_{2sh2} + \hat{\theta}_{4sh3}} + \frac{\hat{\theta}_{3sh3}}{\hat{\theta}_{3sh3} + \hat{\theta}_{4sh3}} - \frac{\hat{\theta}_{2sh3} + \hat{\theta}_{3sh3}}{\hat{\theta}_{2sh3} + \hat{\theta}_{3sh3} + \hat{\theta}_{4sh3}} -$$

$$\frac{\hat{\theta}_{1sh3} + \hat{\theta}_{3sh3}}{\hat{\theta}_{1sh3} + \hat{\theta}_{3sh3} + \hat{\theta}_{4sh3}} + \frac{\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3} + \hat{\theta}_{3sh3}}{\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3} + \hat{\theta}_{3sh3} + \hat{\theta}_{4sh3}}$$

(83)

6.3.2.2 Squared Shrinkage Weight Factor With Square Gamm

$$\hat{\theta}_{1sh3} = \psi(\hat{\theta}_1)\hat{\theta}_{1MLE} + (1 - \psi(\hat{\theta}_1))\hat{\theta}_{1LenG} \quad (84)$$

$$\hat{\theta}_{2sh3} = \psi(\hat{\theta}_2)\hat{\theta}_{2MLE} + (1 - \psi(\hat{\theta}_2))\hat{\theta}_{2LenG} \quad (85)$$

$$\hat{\theta}_{3sh3} = \psi(\hat{\theta}_3)\hat{\theta}_{3MLE} + (1 - \psi(\hat{\theta}_3))\hat{\theta}_{3LenG} \quad (86)$$

$$\hat{\theta}_{4sh3} = \psi(\hat{\theta}_4)\hat{\theta}_{4MLE} + (1 - \psi(\hat{\theta}_4))\hat{\theta}_{4LenG} \quad (87)$$

Then the corresponding S-S reliability estimation was used using the equation for the series component as follows:

$$\hat{R}_{sh3G} = \frac{\hat{\theta}_{2sh3}}{\hat{\theta}_{2sh3} + \hat{\theta}_{4sh3}} + \frac{\hat{\theta}_{1sh3}}{\hat{\theta}_{1sh3} + \hat{\theta}_{4sh3}} - \frac{\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3}}{\hat{\theta}_{1sh3} + \hat{\theta}_{2sh2} + \hat{\theta}_{4sh3}} + \frac{\hat{\theta}_{3sh3}}{\hat{\theta}_{3sh3} + \hat{\theta}_{4sh3}} - \frac{\hat{\theta}_{2sh3} + \hat{\theta}_{3sh3}}{\hat{\theta}_{2sh3} + \hat{\theta}_{3sh3} + \hat{\theta}_{4sh3}} -$$

$$\frac{\hat{\theta}_{1sh3} + \hat{\theta}_{3sh3}}{\hat{\theta}_{1sh3} + \hat{\theta}_{3sh3} + \hat{\theta}_{4sh3}} + \frac{\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3} + \hat{\theta}_{3sh3}}{\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3} + \hat{\theta}_{3sh3} + \hat{\theta}_{4sh3}}$$

(88)

7. Simulation

In order to check the performance of the estimate, the Monte Carlo simulation was used to investigate the performance comparison of different reliability estimates. Estimated methods were performed in S-S models using a variety of samples (30, 60, and 90). The next steps for Monte Carlo simulations show the statistical results for each sample based on the criteria for mean squared errors for bounded and series models as following steps:

- **For Bounded Model**

Step1: Generate random samples from the uniform distribution defined on the interval (0,1) as $u_1, u_2, \dots, u_n$, $v_1, v_2, \dots, v_{m_1}$ and $w_1, w_2, \dots, w_{m_2}$ respectively

Step 2: Convert the mentioned random samples into GIRD random samples of $X \sim GIRD(\theta_1, \sigma^2)$, and by using the cumulative distribution function in the following way

$$F(x_i) = e^{\frac{-\alpha^2 \theta}{x_i^2}}, \quad U_i = e^{\frac{-\alpha^2 \theta}{x_i^2}}$$

$$x_i = [\alpha^2 \theta_i / -\ln(U_i)]^{\frac{1}{2}}$$

by the same method for $y_1 \sim GIRD(\theta_2, \sigma^2)$, $y_2 \sim GIRD(\theta_3, \sigma^2)$, we get

$$y_j = [\alpha^2 \theta_2 / -\ln(V_j)]^{\frac{1}{2}}$$

$$y_t = [\alpha^2 \theta_3 / -\ln(w_j)]^{\frac{1}{2}}$$

Step3: calling the R_b for the bounded model from equation (3)

Step4: Apply Shrinkage estimators of reliability for Square Jeffry and Gamma by using equations (38), (42), (56), (60), (74), and (78)

Step5: Based on $L=1000$ trials, Calculate the MSE as follows

$$MSE = \frac{1}{L} \sum_{i=1}^L (\hat{R}_{bi} - R_b)^2$$

- **For Series Models**

Step 1: initialize random samples from the uniform distribution defined on the interval (0,1) as $u_{t1}, u_{t2}, \dots, u_{tn_t}$; $t = 1, 2, 3$, and $w_1, w_2, \dots, w_m$, respectively.

Step 2: Using the cumulative distribution function to convert the uniform random samples to random samples of $X_t \sim GIRD(\theta_t, \sigma^2)$, for $t = 1, 2, 3$ as follow:

$$F(x_{it}) = e^{\frac{-\alpha^2 \theta}{x_{it}^2}}, \quad U_{it} = e^{\frac{-\alpha^2 \theta}{x_{it}^2}}$$

$$x_{it} = [\alpha^2 \theta_t / -\ln(U_{it})]^{\frac{1}{2}}$$

by the same method for $y \sim GIRD(\theta_4, \sigma^2)$, we get

$$y_j = [\alpha^2 \theta_4 / -\ln(w_j)]^{\frac{1}{2}}$$

Step3: calling the R_s for the series model from equation (4)

Step4: Compute Shrinkage estimators of reliability using equations (47), (52), (65), (70), (83), and (88) for Square Jeffry and Gamma

Step5: Based on $L=1000$ trials, Calculate the MSE as follows

$$MSE = \frac{1}{L} \sum_{i=1}^L (\hat{R}_{si} - R_s)^2$$

8. Numerical Results

In this section, simulation results were introduced based on three parameters ($\theta_1, \theta_2, \theta_3$) in bounded model and four parameters ($\theta_1, \theta_2, \theta_3, \theta_4$) in series model, and three samples problems size 30, 60, 90 that have been implemented 1000 duplicated. In addition, Tables (1 to 8) explained the results of the proposed estimation methods. The simulation results used to determine the best outcome of the proposed estimation methods (MLE, MSE, Sh1, Sh2, and Sh3) for two kinds of system reliability (R_b , and R_s) in S-S models construct on one parameter Generalized Inverted Rayleigh distribution. The following explanation of the proposed methods for each model is independently discussed.

In the case of estimation, system reliability ($R_b = P(Y_1 < X < Y_2)$) of the first stress - strength model, the simulation results of the proposed estimation methods are illustrated in tables (1,2,3,4) Tables 2 and 4 offer the simulation results for MSE of all the proposed estimation methods. Based on the simulation results, the shrinkage estimator using Shrinkage weight function with Square Jeffry as shown in these tables was the best one and had less MSE for the ($R_b = P(Y_1 < X < Y_2)$) of the Generalized Inverted Rayleigh distribution. While Constant Shrinkage Weight Factor in Square Jeffry had the second rank and followed by Shrinkage weight function with Square Gamma, and MLE, respectively. The following tables (1-4) will be presenting the simulation results.

On the other hand, in the case of estimation for the series system reliability ($R_s = P(Y < \min(X_1, X_2, X_3))$) of the second stress-strength model, the simulation results of the proposed estimation methods are illustrated in tables (5,6,7,8), we have been presented simulation results for MSE through tables 6 and 8. Based on the simulation results, the shrinkage estimator using Shrinkage weight function with Square Gamma as shown in these tables was the best one and had less MSE for the ($R_s = P(Y < \min(X_1, X_2, X_3))$) of the Generalized Inverted Rayleigh distribution. While Constant Shrinkage Weight Factor in Square Jeffry had the second rank and followed by Shrinkage weight function with Square Gamma, and MLE, respectively. The following tables (5-8) will be presenting the simulation results.

Table 1- Estimation value of R , when $\theta_1 = 1$, $\theta_2 = 2.5$ and $\theta_3 = 1.5$

(n, m_1, m_2)	Sh1		Sh2		Sh3	
	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}	Sh_{2LnG}	Sh_{3LnJ}	Sh_{3LnG}
(30,30,60)	0.094725	0.100860	0.094752	0.100723	0.092313	0.093713
(60,30,30)	0.095399	0.100931	0.095543	0.101080	0.093836	0.096050
(90,30,60)	0.095149	0.102073	0.095068	0.101660	0.092542	0.094832
(60,30,60)	0.095655	0.102367	0.095601	0.102021	0.093182	0.095171
(30,30,90)	0.094405	0.100998	0.094369	0.100700	0.091485	0.092775
(60,30,90)	0.094629	0.101773	0.094511	0.101263	0.091993	0.094228
(90,90,90)	0.093103	0.094919	0.093181	0.095039	0.092499	0.092766
(60,60,90)	0.092548	0.095626	0.092600	0.095648	0.091692	0.092616

Table 2- MSE value of $R = 0.085714285714286$ when $\theta_1 = 1$, $\theta_2 = 2.5$ and $\theta_3 = 1.5$

(n, m_1, m_2)	Sh1		Sh2		Sh3	
	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}	Sh_{2LnG}	Sh_{3LnJ}	Sh_{3LnG}
(30,30,30)	0.000548	0.000565	0.000547	0.000565	0.000692	0.001024
(60,30,30)	0.005294	0.005414	0.005266	0.005338	0.006727	0.009338
(90,30,60)	0.004669	0.005470	0.004616	0.005244	0.005083	0.006571
(60,30,60)	0.004152	0.004630	0.004113	0.004463	0.005038	0.006785
(30,30,90)	0.004655	0.005299	0.004618	0.005139	0.005294	0.006860
(90,90,90)	0.001739	0.001741	0.001738	0.001740	0.001956	0.002525
(60,30,90)	0.004064	0.004913	0.004015	0.004680	0.004680	0.006250
(60,60,90)	0.002657	0.002779	0.002652	0.002759	0.002938	0.003672

Table 3- Estimation value of R , when $\theta_1 = 2.3$, $\theta_2 = 3.2$ and $\theta_3 = 1.6$

(n, m_1, m_2)	Sh1		Sh2		Sh3	
	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}	Sh_{2LnG}	Sh_{3LnJ}	Sh_{3LnG}
(30,30,30)	0.097694	0.103549	0.097951	0.104162	0.095667	0.097460
(60,30,30)	0.096427	0.103168	0.096643	0.103671	0.094402	0.097279
(90,30,60)	0.097523	0.106304	0.097537	0.106246	0.094264	0.097035
(60,30,60)	0.098196	0.106687	0.098223	0.106666	0.095301	0.098280
(60,60,60)	0.095209	0.098405	0.095342	0.098724	0.094351	0.095359
(30,30,90)	0.097622	0.105913	0.097630	0.105842	0.094281	0.096039
(60,30,90)	0.097642	0.106775	0.097606	0.106571	0.094149	0.096753
(90,60,90)	0.095726	0.099785	0.095781	0.099884	0.094470	0.095744

Table 4- MSE value of $R = 0.094238156209987$ when $\theta_1 = 2.3$, $\theta_2 = 3.2$ and $\theta_3 = 1.6$

(n, m_1, m_2)	Sh1		Sh2		Sh3	
	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}	Sh_{2LnG}	Sh_{3LnJ}	Sh_{3LnG}
(30,30,30)	0.000488	0.000496	0.000487	0.000502	0.000644	0.001000
(60,30,30)	0.000504	0.000510	0.000503	0.000514	0.000749	0.001239
(90,30,60)	0.004086	0.004956	0.004059	0.00486	0.005207	0.007747
(60,30,60)	0.003992	0.004927	0.003966	0.004844	0.004990	0.007795
(60,60,60)	0.002254	0.002255	0.002252	0.002270	0.002723	0.003955
(30,30,90)	0.003648	0.004497	0.003617	0.004390	0.004385	0.006317
(60,30,90)	0.003732	0.004755	0.003699	0.004617	0.004644	0.006698
(90,60,90)	0.002036	0.002197	0.002032	0.002191	0.002358	0.003274

Table 5- Estimation value of R , when $\theta_1 = 1$, $\theta_2 = 2.5$, $\theta_3 = 1.5$ and $\theta_4 = 1.7$

(n_1, n_2, n_3, m)	Sh1		Sh2		Sh3	
	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}
(60,30,60,60)	0.211514	0.211511	0.211127	0.211514	0.211511	0.211127
(30,30,60,60)	0.213102	0.213090	0.212085	0.213102	0.213090	0.212085
(90,30,60,90)	0.212015	0.211997	0.210915	0.212015	0.211997	0.210915
(30,30,30,30)	0.211854	0.211878	0.212771	0.211854	0.211878	0.212771
(60,90,90,30)	0.209807	0.209880	0.213562	0.209807	0.209880	0.213562
(90,90,30,60)	0.211470	0.211478	0.211646	0.211470	0.211478	0.211646
(60,60,30,90)	0.211721	0.211702	0.210500	0.211721	0.211702	0.210500
(30,60,90,90)	0.211993	0.211985	0.211222	0.211993	0.211985	0.211222

Table 6- MSE value of $R=0.210557718044953$ when $\theta_1 = 1$, $\theta_2 = 2.5$, $\theta_3 = 1.5$ and $\theta_4 = 1.7$

(n_1, n_2, n_3, m)	Sh1		Sh2		Sh3	
	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}	Sh_{2LnG}	Sh_{3LnJ}	Sh_{3LnG}
(60,30,60,60)	0.001007	0.001006	0.000970	0.000908	0.001174	0.001595
(90,90,90,90)	0.005424	0.005422	0.005346	0.005294	0.006169	0.008502
(30,30,60,60)	0.001049	0.001048	0.000995	0.000883	0.001236	0.001674
(90,30,60,90)	0.000725	0.000724	0.000695	0.000631	0.000879	0.001258
(30,30,90,90)	0.000801	0.000800	0.000755	0.000661	0.000925	0.001226
(60,30,60,90)	0.000720	0.000719	0.000687	0.000619	0.000868	0.001236
(60,60,30,90)	0.0007119	0.0007112	0.000678	0.000604	0.000791	0.001026
(30,60,90,90)	0.000707	0.000706	0.000678	0.000611	0.000803	0.001074

Table 7- Estimation value of R , when $\theta_1 = 2.6$, $\theta_2 = 1.4$, $\theta_3 = 2.3$ and $\theta_4 = 3.1$

(n_1, n_2, n_3, m)	Sh1		Sh2		Sh3	
	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}	Sh_{2LnG}	Sh_{3LnJ}	Sh_{3LnG}
(60,30,60,60)	0.144992	0.145023	0.146134	0.150122	0.145339	0.146951
(90,90,90,90)	0.144106	0.144138	0.145393	0.148938	0.144283	0.145204
(30,30,60,60)	0.146552	0.146563	0.146634	0.148442	0.146453	0.146543
(90,30,60,90)	0.143930	0.143937	0.143896	0.145270	0.144158	0.144769
(30,30,90,90)	0.145961	0.145948	0.144859	0.144016	0.145665	0.144847
(60,30,60,90)	0.144952	0.144951	0.144553	0.145154	0.145205	0.145700
(60,60,30,90)	0.145189	0.145183	0.144545	0.144332	0.145052	0.144666
(30,60,90,90)	0.145010	0.145011	0.144663	0.144928	0.145039	0.144951

(n_1, n_2, n_3, m)	Sh1		Sh2		Sh3	
	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}	Sh_{1LnJ}	Sh_{1LnG}	Sh_{2LnJ}
(60,30,60,60)	0.001395	0.001394	0.001354	0.001251	0.001497	0.001748
(30,30,60,60)	0.001583	0.001582	0.001521	0.001387	0.001750	0.002104
(90,30,60,90)	0.000998	0.000997	0.000970	0.000911	0.001062	0.001235
(30,30,30,30)	0.002518	0.002515	0.002398	0.002116	0.002917	0.003588
(60,90,90,30)	0.002234	0.002232	0.002165	0.002030	0.002600	0.003238
(90,90,30,60)	0.001372	0.001371	0.001333	0.001235	0.001532	0.001861
(60,60,30,90)	0.001110	0.001110	0.001074	0.000998	0.001244	0.001516
(30,60,90,90)	0.001148	0.001147	0.001113	0.001040	0.001261	0.001500

Table 8- MSE value of $R = 0.143392225122712$ when $\theta_1 = 2.6$, $\theta_2 = 1.4$, $\theta_3 = 2.3$ and $\theta_4 = 3.1$

9. Conclusion

Reliability of the supported system of the stress strength model was presented when the general parameter pressure and force followed the General Inverse Rayleigh Distribution using two different loss functions, such as the squared error loss function, linear exponential loss function, Jeoffrey presets information, pre-gamma, and the method of shrinkage estimation (constant shrinkage, squared shrinkage, and Shrinkage weight function). Model ($R_b = P(Y_1 < X < Y_2)$) and model ($R_s = P(Y < \min(X_1, X_2, X_3))$) were used to evaluate and verify the performance of methods using different samples (30, 60, and 90). We an attempt has been made to estimate ($R_b = P(Y_1 < X < Y_2)$) and ($R_s = P(Y < \min(X_1, X_2, X_3))$) for stress-strength followed the general inverse Rayleigh distribution.

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