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# The kernel in fixed topological spaces

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**Abstract:** The goal of the article is to extend and study the proper fixed spaces. The sets called D- ,and Kernel sets in this space have been studied and introduced . In this paper we include theorems and examples related to these concepts in fixed topological space. This research finds the characteristics and properties of such notions. Also the relation between them are studied and investigated in fixed topological spaces. Since the definition of the mathematics usual topology differs from the definition of fixed topology, we have noticed that some of the concepts that have been studied in both are different in terms of their verification in the proofs and examples and evidence for that the of definitions D-sets,  $\alpha$ ,  $R_o$ ,  $D_o$ ,  $D_1$  – spaces and the kernel set. Also we were obtained that the relationships between these concepts are different from those in previous studies.

**Keywords:** Fixed topological spaces, F-open set , D-set, kernel set and neighborhood

**Mathematics subject classification:** 54A05 , 54C05 , 54D99.

## 1.Introduction:

There is a definition derived from the definition of usual topology called a supra topological and it has been proposed by ( Mashhour et al. ,1983) [1] and it has also been studied in (Meera Devi et al. , 2016) [2]. The fixed topological spaces were introduced, and studied in (Raad Aziz Hussain Al-Abdulla ,2019 ) [3] under the title (proper fixed spaces). (Davis ,1961) [4] had defined  $R_o$ - space as if for every open set of  $G$  and for every  $x \in G$ , therefore  $\overline{\{x\}} \subseteq G$  and this definitions were studied in (Bishwambhar Roy et al. ,2010) [5]. In addition, ( Munkres ,2000) [6] introduced the definition of the term the neighborhood. Also in ( Kalaivani et al. ,2017) [7] ; ( Kalavathi et al. , 2016) [8] ; ( Saravanakumar et al. , 2014) [9] & (Tanay Bakir et al. , 2014) [10] some topological propertied have been observed. Moreover, in (Bishwambhar Roy et al. ,2010) applied the definition of the kernel set, also this definition was studied in (Sindhu , 2019) [11] . The aim of writing this article is to explain these concepts of D- and kernel set in the fixed topological spaces, studying their characteristics and the relation through them. We used the  $(ft-s)$  symbol to denote the fixed topological spaces. Our study deepens the new concept called fixed technology, which differs in nature from the usual topology, as the two concepts are originally independent. In the second chapter of this work, we recalled the



definition of the F-sets, and we discussed all the definitions, theorems and examples related to them that had previously been discussed. As for the third chapter, we identified a new sets under the name D-sets. Then a new spaces were defined using the D-sets under the name  $\alpha$ ,  $D_0$  and  $D_1$  spaces. For fixed topological the relationships between these spaces are found and the possibility of equivalence between them are discusses. In the fourth chapter, we studied a new sets under the name kernel sets and we reached many important theories that belong to this sets related to fixed topological space, among them the behavior of the kernel sets in the presence of space  $D_0$  and also the existence of a sets of intersections of all sets present in the fixed topological space.

**Notation:** We will use the  $(ft-s)$  symbol to denote the fixed topological spaces

## 2. On The F-open Sets:

In this chapter, we recalled the description of the F-open sets and its related theories and examples.

**Definition 2.1 : [3]** Suppose  $X$  is a non-empty set and  $\mathcal{F}t$  is a collection of sets on  $X$  fulfills the following axioms :

- (i) If  $U_\alpha \in \mathcal{F}$  for all  $\alpha \in \Lambda$  such that  $\Lambda$  is any arbitrary set , thus  $\cup \{ U_\alpha : \alpha \in \Lambda \} \neq X$ .
- (ii) If  $U_\alpha \in \mathcal{F}$  for all  $\alpha \in \Lambda$  such that  $\Lambda$  is any arbitrary set , then  $\cap \{ U_\alpha : \alpha \in \Lambda \} \neq \emptyset$ .

Hence  $\mathcal{F}$  is named fixed topology on  $X$  and  $(X, \mathcal{F})$  is named  $(ft-s)$ . The members in  $\mathcal{F}$  are said F - open sets. The set  $B \subseteq X$  is said F - closed set incase  $X - B \in \mathcal{F}$ . It is clear that every F - (open and closed) sets are proper subsets in  $X$ . The empty set and  $X$  are not F - open and not F - closed sets in  $X$ .

**Example 2.2 : [3]** Let  $X = \{m, n, r, s, w\}$  and define  $\mathcal{F} = \{\{m\}, \{m, n\}, \{m, n, s\}\}$  , thus  $\mathcal{F}$  is a fixed topological of  $X$  also the  $\mathcal{F}$  - open sets are :  $\{m\}$  ,  $\{m, n\}$  ,  $\{m, n, s\}$ .

**Definition 2.3 : [3]** Let  $(X, \mathcal{F})$  be  $(ft-s)$  ,  $E$  is subset of  $X$ . A point  $y \in X$  is the limit point of  $E$  in the case of :  $(U \cap E) - \{y\} \neq \emptyset$  for each  $U \in \mathcal{F}$  such as  $y \in U$ .

The set of every limit points of  $E$  is said derived set of  $E$  , we have represented as  $d(E)$ .

**Definition 2.4 : [3]** Let  $(X, \mathcal{F})$  is  $(ft-s)$  ,  $B \subseteq X$ . Then  $B \cup d(B)$  is called the closure of  $B$ . It was denoted as  $\bar{B}$ .

**Example 2.5 : [3]** Suppose  $X = \{1, 2, 3, \dots\} = \mathbb{N}$  and  $\mathcal{F} = \{\{2, 3, 4, \dots\}, \{3, 4, 5, \dots\}, \{4, 5, 6, \dots\}, \dots\}$ . Thus  $\mathcal{F}$  is a fixed topological of  $X$ . If  $B \subseteq X$ , then:

$$\bar{B} = \begin{cases} \{1, \dots, m\} & \text{if } B \text{ finite and } m = \max \{n: n \in B\} \\ X & \text{if } B \text{ infinite} \end{cases}$$

**Theorem 2.6 :[3]** If  $G$  and  $H$  are sets of a  $(ft-s)$   $(X, \mathcal{F})$  while  $G$  is  $F$ -open set, thus  $G \cap \bar{H} = \emptyset$  if and only if  $G \cap H = \emptyset$ .

**Theorem 2.7 :[3]** Assume  $(X, \mathcal{F})$  is  $(ft-s)$ ,  $A \subseteq X$  and  $a \in X$ . Therefore  $a \in \bar{A}$  if and only if  $A \cap U \neq \emptyset$  for each  $U \in \mathcal{F}$  such that  $a \in U$ .

**Theorem 2.8 :[3]** Suppose  $(X, \mathcal{F})$  is  $(ft-s)$ ,  $B \subseteq X$ . Thus  $\bar{B} = X$  if and only if  $B \cap H \neq \emptyset$  for each  $H \in \mathcal{F}$ .

**Theorem 2.9 :[3]** Let  $(X, \mathcal{F})$  be  $(ft-s)$ ,  $B \subseteq X$ . Therefore  $B \subseteq \{x: \overline{\{x\}} \cap B \neq \emptyset\}$  and hence  $\{x: \overline{\{x\}} \cap B \neq \emptyset\} \cap B = B$ .

**Theorem 2.10 :[3]** Assume  $(X, \mathcal{F})$  is  $(ft-s)$ ,  $B$  is  $F$ -open subset of  $X$ . Then  $\{x: \overline{\{x\}} \cap B \neq \emptyset\} = B$ .

**Definition 2.11 : [3]** Assume  $(X, \mathcal{F})$  is a  $(ft-s)$ ,  $C, D \subseteq X$  and  $x \in C$ . Therefore :

- (1)  $C$  is said a neighborhood (nbd) set of  $x$  if there exists  $G$  an  $F$ -open set such as  $x \in G \subseteq C$  and it denoted as  $C \in N(x)$ .
- (2)  $C$  is said a neighborhood (nbd) set of  $D$  if there is  $G$  an  $F$ -open set such as  $D \subseteq G \subseteq C$  and it referred to as  $C \in N(D)$ .
- (3)  $C$  is said a neighborhood (nbd) set in  $X$  if there is  $G$  an  $F$ -open set such as  $G \subseteq C$ . The set of every nbd sets in  $X$  is referred to as  $N(X)$ ,  $N(X) = \{N(G): G \in \mathcal{F}\}$ .

It is evident that in the case of  $G \in \mathcal{F}$ , thus  $G \in N(X)$ .

**Example 2.12 : [3]** Let  $X = \{m, n, r, s, w\}$  and  $\mathcal{F} = \{\{m, n, r\}, \{r, s\}\}$  are a fixed topological of  $X$ . Thus  $N(X) = \{\{m, n, r\}, \{m, n, r, s\}, \{m, n, r, w\}, \{r, s\}, \{m, r, s\}, \{n, r, s\}, \{r, s, w\}, \{m, r, s, w\}, \{n, r, s, w\}, X\}$

### 3. On D-Sets:

In this chapter, we studied the definition of the D-set and the effect of this definition on spaces within the fixed topological space in terms of the different results obtained

**Definition 3.1 :** A non empty subset  $B$  of a  $(ft-s)$   $(X, \mathcal{F})$  is named D-set if there is

$F$ -open sets  $W$  and  $H$  such as  $B = W - H$ . It is evident that if  $B$  is a D-set, then  $B \neq X$ .

**Example 3.2 :** Let  $X = \{q, n, r, s, w\}$  and  $\mathcal{F} = \{\{q, n, r\}, \{q, n, s\}, \{q, r, s, w\}\}$  be a fixed topological of  $X$ . The D-sets in  $X$  are :  $\{r\}, \{n\}, \{s\}, \{s, w\}$  and  $\{r, w\}$ . It is obvious , not every D-set is F-open, and not every F-open set is D-set.

As in Theorem (2.10) if  $A$  is F-open set in  $(ft-s) (X, \mathcal{F})$ , thus  $\{x: \overline{\{x\}} \cap A \neq \emptyset\} = A$ . But if  $A$  is D-set, then it is not necessary that  $\{x: \overline{\{x\}} \cap A \neq \emptyset\} = A$ . This in the following case :

**Example 3.3 :** Let  $X = \{m, n, r\}$  and  $\mathcal{F} = \{\{m\}, \{m, n\}\}$  be fixed topological of  $X$ . Thus  $A = \{n\}$  is D-set in  $X$ . It clear  $\{x: \overline{\{x\}} \cap A \neq \emptyset\} = \{m, n\}$ . Therefore  $\{x: \overline{\{x\}} \cap A \neq \emptyset\} \neq A$ .

**Theorem 3.4 :** Let  $E$  be D-set in  $(ft-s) (X, \mathcal{F})$ . Thus  $\{x: \overline{\{x\}} \cap E \neq \emptyset\} \cap \bar{E} = E$ .

**Proof :** Since  $E \subseteq \bar{E}$  and from Theorem (2.9),  $E \subseteq \{x: \overline{\{x\}} \cap E \neq \emptyset\}$ , thus  $E \subseteq \{x: \overline{\{x\}} \cap E \neq \emptyset\} \cap \bar{E}$ . Let  $x \notin E$  such as  $E = V - H$  while  $V$  and  $H$  are F-open sets. Then  $x \in H$  or  $x \in X - V$ . If  $x \in H$ . Since  $H \cap E = \emptyset$ , then by Theorem (2.6),  $H \cap \bar{E} = \emptyset$ . Hence  $x \notin \bar{E}$ . Thus  $x \notin \{x: \overline{\{x\}} \cap E \neq \emptyset\} \cap \bar{E}$ . If  $x \in X - V$ . Suppose  $x \in \{x: \overline{\{x\}} \cap E \neq \emptyset\}$ . Thus  $\overline{\{x\}} \cap E \neq \emptyset$ . Hence there is  $y \in \overline{\{x\}} \cap E$ . Thus  $y \in \overline{\{x\}}$  and  $y \in E$ . Therefore  $x \in U_y$  for each  $U_y$  F-open set such as  $y \in U_y$  which is a contradiction since  $V$  is F-open set,  $y \in V$  ( since  $y \in E$  and  $E \subseteq V$  ) and  $x \notin V$ . Then  $x \notin \{x: \overline{\{x\}} \cap E \neq \emptyset\}$ . Thus  $x \notin \{x: \overline{\{x\}} \cap E \neq \emptyset\} \cap \bar{E}$ . Therefore  $\{x: \overline{\{x\}} \cap E \neq \emptyset\} \cap \bar{E} \subseteq E$ . Then  $\{x: \overline{\{x\}} \cap E \neq \emptyset\} \cap \bar{E} = E$ .

**Theorem 3.5 :** Suppose that  $A$  is D-set in  $(ft-s) (X, \mathcal{F})$ . Therefore  $A = G \cap \bar{A}$  for some  $G$  an F-open set.

**Proof :** Suppose  $A = G - H$  where  $G$  and  $H$  are F-open sets. Since  $G \cap (X - H) \subseteq G$  and  $A \subseteq \bar{A}$ , then  $A = G \cap (X - H) \subseteq G \cap \bar{A}$ . Since  $H \cap A = \emptyset$ , then by Theorem (2.6),  $H \cap \bar{A} = \emptyset$  and hence  $\bar{A} \subseteq X - H$ . Therefore  $G \cap \bar{A} \subseteq G \cap (X - H) = A$ . Then  $A = G \cap \bar{A}$ .

**Example 3.6 :** Suppose  $X = \{a, b, c, d, e\}$  and  $\mathcal{F} = \{\{a, b, c\}, \{c, d\}, \{c\}\}$  are fixed topological of  $X$ . Then  $\overline{\{c\}} = X$  and there is F-open set  $\{c\}$  such that  $\{c\} = \{c\} \cap \overline{\{c\}}$  but  $\{c\}$  is not D-set.

**Example 3.7 :** Let  $X = \{a, b, c\}$ ,  $\mathcal{F} = \{\{b\}, \{b, c\}\}$  be fixed topological of  $X$ . Suppose  $E = \{b\}$ . Thus  $\bar{E}^c = X^c = \emptyset$ . Therefore  $E \cup \bar{E}^c = \{b\}$ . Hence  $E \cup \bar{E}^c$  is F-open set while  $E$  is not D-set.

**Example 3.8 :** Let  $X = \{a, b, c, d, e\}$  and  $\mathcal{F} = \{\{a, b, c\}, \{c, d\}, \{c\}\}$  be fixed topological of  $X$ . Suppose  $A = \{a, b\}$ . Therefore  $\bar{A}^c = \{a, b, e\}^c = \{c, d\}$  and  $A \cup \bar{A}^c = \{a, b, c, d\}$ . Therefore  $A$  is D-set and  $A \cup \bar{A}^c$  is not F-open set.

**Example 3.9 :** Let  $X = \{a, b, c, d, e, f\}$  and  $\mathcal{F} = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d, e\}\}$  be a fixed topological of  $X$ . The D-sets are :  $\{c\}$ ,  $\{b\}$ ,  $\{d\}$ ,  $\{d, e\}$  and  $\{c, e\}$ . It is quick to show,  $\{b\}$  and  $\{d\}$  are D-sets but  $\{b\} \cup \{d\} = \{b, d\}$  is not D-set. Also  $\{d, e\}$  and  $\{c, e\}$  are D-sets but  $\{d, e\} \cap \{c, e\} = \{e\}$  is not D-set.

**Theorem 3.10 :** In a  $(ft-s)$   $(X, \mathcal{F})$ ,  $A \cap B = \emptyset$  if and only if  $A \cap \bar{B} = \bar{A} \cap B = \emptyset$  for every D-sets  $A, B$  in  $X$ .

**Proof :** Let  $A$  and  $B$  be D-sets in  $X$  such as  $A \cap B = \emptyset$ . Then  $A = U - R$  and  $B = V - S$  where  $U, R, V$  and  $S$  are F-open sets. If  $S - R = \emptyset$ , thus  $(S - R) \cap A = [S - (S \cap R)] \cap A = \emptyset$ . If  $S - R \neq \emptyset$ , then  $S - R$  and  $A$  are D-sets. Therefore by the hypothesis  $(S - R) \cap A = [S - (S \cap R)] \cap A = \emptyset$ . Since  $A \cap R = \emptyset$ , then  $A \cap (S \cap R) = \emptyset$ . Since  $S = (S - (S \cap R)) \cup (S \cap R)$ , then  $S \cap A = [(S - (S \cap R)) \cup (S \cap R)] \cap A = [(S - (S \cap R)) \cap A] \cup [(S \cap R) \cap A] = \emptyset \cup \emptyset = \emptyset$ . Since  $V = B \cup (V \cap S)$ , then  $V \cap A = [B \cup (V \cap S)] \cap A = [(B \cup V) \cap (B \cup S)] \cap A = [V \cap (B \cup S)] \cap A = (V \cap A) \cap (B \cup S) = (V \cap A \cap B) \cup (V \cap A \cap S) = (V \cap \emptyset) \cup (V \cap \emptyset) = \emptyset$ .

Then by Theorem (2.6),  $V \cap \bar{A} = \emptyset$  and hence  $B \cap \bar{A} = \emptyset$ . By the same way we can prove  $A \cap \bar{B} = \emptyset$ .

Conversely, it is clear.

**Theorem 3.11 :** Suppose  $(X, \mathcal{F})$  is  $(ft-s)$ ,  $B \subseteq X$ . If  $B \subseteq U$  for some F-open set  $U$ , thus the below comments are identical :

- 1-  $B \cap A^c = \emptyset$  for each D-set  $A$ .
- 2-  $\bar{B} = X$ .

**Proof :** (1)  $\rightarrow$  (2) Suppose that  $\bar{B} \neq X$ . Then by Theorem (2.18) there exist F-open set  $H$  such as  $H \cap B = \emptyset$ . Since by Definition (2.1),  $H \cap U \neq \emptyset$  and since  $B \subseteq U, B \cap H = \emptyset$ , then  $A = U - H \neq \emptyset$ . Therefore  $A$  is D-set. Since  $B \subseteq U$  and  $B \subseteq H^c$ , then  $B \subseteq U - H = A$ . Hence  $B \cap A^c = \emptyset$  which is a contradiction with the hypothesis. Then  $\bar{B} = X$ .

(2)  $\rightarrow$  (1) Let  $\bar{B} = X$ . Suppose  $B \cap A^c \neq \emptyset$  for some D-set  $A$ . Then  $B \not\subseteq A$ . Suppose  $A = U - G$  for some F-open sets  $U$  and  $G$ . Then  $G \cap A = \emptyset$ . Therefore  $G \cap B = \emptyset$  which is a contradiction with Theorem (2.6).

**Definition 3.12 :** A  $(ft-s)$   $(X, \mathcal{F})$  is called  $\alpha$ -space if for every  $x, y \in X$  such that  $x \neq y$  there are F-open set  $U$  and any non F-open set  $B$  such as  $x \in U, y \in B$  or  $y \in U, x \in B$  and  $U \cap B = \emptyset$ .

**Example 3.13 :** Let  $X = \{a, b, c, d, e\}$  and  $\mathcal{F} = \{\{a, b, c\}, \{c, d\}\}$  be fixed topological of  $X$ . It is obvious  $a \neq b$  but there are no F-open set  $C$  and non F-open set  $E$  such as  $a \in C, b \in E$  or  $b \in C, a \in E$  and  $C \cap E = \emptyset$ . Hence,  $(X, \mathcal{F})$  is not  $\alpha$ -space.

**Example 3.14 :** Let  $X = \{1, 2, 3, 4, \dots\}$  and  $\mathcal{F} = \{\{2, 3, 4, \dots\}, \{3, 4, 5, \dots\}, \{4, 5, 6, \dots\}, \dots\}$  be fixed topological of  $X$ . Suppose  $n, m \in X$  such  $m < n$ . Then  $n \in A = \{n, n+1, n+2, \dots\}, n \neq 1$  and  $m \in B = \{m\}$ .

It is obvious that  $A$  is F-open set,  $B$  non F-open set and  $A \cap B = \emptyset$ . Therefore  $(X, \mathcal{F})$  is an  $\alpha$ -space.

**Example 3.15 :** Let  $X = \{1, 2, 3, \dots\} = \mathbb{N}$  and  $\mathcal{F} = \{A \subseteq \mathbb{N} : 2 \in A, 1 \notin A\}$  be fixed topological on  $X$ . Let  $n, m \in X$  such as  $n \neq m$ . Suppose  $n \neq 2$ . If  $m = 2$ , then  $A = \{m\}$  is F-open set,  $B = \{n\}$  non F-open set, such as  $A \cap B = \emptyset$ . If  $m \neq 2$ , then  $A = \{2, n\}$  is F-open set,  $B = \{m\}$  non F-open set, such as  $A \cap B = \emptyset$ . And in case  $n=2$ , then  $A = \{n\}$  is a F-open set and  $B = \{m\}$  non F-open set, such as  $A \cap B = \emptyset$ . Therefore  $(X, \mathcal{F})$  is  $\alpha$ -space.

**Definition 3.16 :** A  $(ft-s)$   $(X, \mathcal{F})$  is called  $D_o$ -space if for every individual pair of points  $x, y$  in  $X$  there is a D-set or F-open set  $A$  such as  $A$  includes  $x$  not  $y$  or it contains  $y$  not  $x$ .

It is obvious the space  $(X, \mathcal{F})$  in Example (3.2) is a  $D_o$ -space.

**Example 3.17 :**  $X = \{a, b, c, d, e\}$  and  $\mathcal{F} = \{\{a, b, c\}, \{c, d\}, \{c\}\}$  be fixed topological of  $X$ . The D-sets are:  $\{a, b\}, \{d\}$ . Then  $a \neq b$  but there exists no D-set or F-open set  $A$  include one of them but not the other. Hence,  $(X, \mathcal{F})$  is not  $D_o$ -space.

**Theorem 3.18 :** A  $(ft-s)$   $(X, \mathcal{F})$  is  $D_o$ -space if and only if it is  $\alpha$ -space.

**Proof :** Let  $(X, p)$  be  $D_o$  - space and  $x, y \in X$  such as  $x \neq y$ . If there exist F-open  $A$  set such as  $x \in A, y \notin A$ . Let  $B = X - A$ . Then  $B$  is not F-open set such as  $y \in B$  and  $A \cap B = \emptyset$ . If there is a D-set  $A$  such as  $x \in A, y \notin A$ , then  $A = U - V$  where  $U, V$  are F-open sets. Either  $y \in V$ , then  $x \in A$ ,  $A$  not F-open set and  $y \in V$ ,  $V$  F-open set such as  $A \cap V = \emptyset$ . Or  $y \notin V$ . Thus  $y \in (U \cup V)^c = U^c \cap V^c$ . But  $(U^c \cap V^c) \cap U = \emptyset$ . Then by Definition (2.1),  $U^c \cap V^c$  is not F-open set such as  $y \in U^c \cap V^c$  and  $x \in U$ ,  $U$  is F-open set. Therefore  $(X, p)$  is  $\alpha$ -space.

Conversely, Suppose  $x, y \in X$  such that  $x \neq y$ . Hence there are  $U$  as F - open set and  $B$  as non F-open set such as  $x \in U, y \in B$  and  $U \cap B = \emptyset$ . Then  $U$  is F - open set such as  $x \in U, y \notin U$ . Hence  $(X, \mathcal{F})$  is  $D_o$ -space.

**Theorem 3.19 :** A  $(ft-s)$   $(X, \mathcal{F})$  is a  $D_0$ -space if and only if for each  $x, y \in X$  such that  $x \neq y$  there is  $G$  an  $F$ -open set such as  $G$  include one of them not the other.

**Proof :** Let  $(X, \mathcal{F})$  be  $D_0$ -space and  $x, y \in X$  such that  $x \neq y$ . Then there is  $A$   $D$ -set or  $F$ -open set that forms one of them not the other. If  $A$  is  $F$ -open set, so we have the result. If  $A$  is  $D$ -set such that  $x \in A, y \notin A$ . Then  $A = G - K$  where  $G, K$  are  $F$ -open sets. If  $y \in K$ , then  $K$  is  $F$ -open set such as  $y \in K, x \notin K$ . If  $y \notin K$ , thus  $y \in G^c$ . Therefore,  $G$  is  $F$ -open set such as  $x \in G$  and  $y \notin G$ . It is easy to prove the converse part.

**Theorem 3.20 :** Suppose  $(X, \mathcal{F})$  is  $(ft-s)$ . Therefore  $(X, \mathcal{F})$  is  $D_0$ -space if and only if  $\overline{\{x\}} \neq \overline{\{y\}}$  for each  $x, y \in X$  such that  $x \neq y$ .

**Proof :** Let  $(X, \mathcal{F})$  be  $D_0$ -space and  $x, y \in X$  such as  $x \neq y$ . Thus, there exist  $U \in \mathcal{F}$  such as  $x \in U$  and  $y \notin U$ . Therefore  $U \cap \{y\} = \emptyset$ . Hence using Theorem (2.6),  $U \cap \overline{\{y\}} = \emptyset$ . Thus  $x \in U \subseteq \overline{\{y\}}^c$ . Thus  $x \notin \overline{\{y\}}$ . Therefore  $\overline{\{x\}} \neq \overline{\{y\}}$ .

Conversely, let  $x, y \in X$  such as  $x \neq y$ . Therefore  $\overline{\{x\}} \neq \overline{\{y\}}$ . Thus, there is  $z \in X$  such as  $z \in \overline{\{x\}}$  and  $z \notin \overline{\{y\}}$ . Therefore by Theorem (2.7),  $U_z \cap \{x\} \neq \emptyset$  for each  $U_z \in \mathcal{F}$  such that  $z \in U_z$  and there exists  $U \in \mathcal{F}$  such as  $z \in U$  and  $U \cap \{y\} = \emptyset$ . Then,  $y \notin U$  such that  $U \in \mathcal{F}, z \in U$ . Hence there is  $U \in \mathcal{F}$  such as  $x \in U$  and  $y \notin U$ . Therefore by Theorem (3.19),  $(X, \mathcal{F})$  is  $D_0$ -space.

**Definition 3.21 :** A  $(ft-s)$  is called  $D_1$ -space if for every  $x, y \in X$  such as  $x \neq y$  there are  $U$   $D$ -set and  $V$  non  $F$ -open set such as  $x \in U, y \in V$  and  $U \cap V = \emptyset$ .

It is obvious that the  $D_1$ -space is  $D_0$ -space.

**Example 3.22 :** Let  $X = \{a, b, c\}$ ,  $\mathcal{F} = \{\{a\}, \{a, b\}\}$  be a fixed topology on  $X$ . The only  $D$ -set in  $X$  is  $\{b\}$ . Then  $(X, \mathcal{F})$  is  $D_0$ -space, but not  $D_1$ -space, since  $c \neq a$ , that is there does not exist  $D$ -set  $E$  and non  $F$ -open set  $F$  such as  $a \in E, c \in F$  or  $c \in E, a \in F$  and  $E \cap F = \emptyset$ . Therefore not every  $D_0$ -space is  $D_1$ -space.

**Theorem 3.23 :** A  $(ft-s)$   $(X, \mathcal{F})$  is  $D_1$ -space if and only if for each  $x, y \in X$  such as  $x \neq y$  there is  $A$   $D$ -set such as  $A$  include one of them not the other.

**Proof :** It is obvious that if  $(X, \mathcal{F})$  is  $D_1$ -space, then for each  $x, y \in X$  such as  $x \neq y$  there is  $D$ -set  $A$  such as  $A$  contain one of them not the other.



Conversely, let  $x \neq y$ . Thus there is D-set  $A$  such as  $x \in A, y \notin A$ . Let  $A = G - W$  where  $G$  and  $W$  are F-open sets. Then  $y \in W \cup G^c$ . Since  $(W \cup G^c) \cup G = X$ , then by Definition (2.1),  $W \cup G^c$  is not F-open set. Therefore there is A D-set and  $W \cup G^c$  non F - open set such that  $x \in A, y \in W \cup G^c$  and  $(W \cup G^c) \cap A = \emptyset$ . Hence  $(X, p)$  is  $D_1$ -space.

**Remark 3.24 :** If  $(X, \mathcal{F})$  is a (ft-s) such that for each  $x, y \in X$  such that  $x \neq y$  there exists D-sets  $U, V$  such as  $x \in U, y \in V$  and  $U \cap V = \emptyset$ , therefore  $(X, \mathcal{F})$  is  $D_1$ -space.

In general, the opposite of Remark (3.24) is not the case as an example below :

**Example 3.25 :** Let  $X = \{1, 2, 3, \dots\} = \mathbb{N}$  and  $\mathcal{F} = \{\{n, n+1, n+2, \dots\} : n \in \mathbb{N}, n \geq 2\} = \{\{2, 3, 4, \dots\}, \{3, 4, 5, \dots\}, \{4, 5, 6, \dots\}, \dots\}$  be a fixed topological on  $X$ .

The D-sets are :  $\{2\}, \{2, 3\}, \{2, 3, 4\}, \dots, \{3\}, \{3, 4\}, \{3, 4, 5\}, \dots, \{4\}, \{4, 5\}, \{4, 5, 6\}, \dots$

Then the D-sets are :  $\{\{n\}, \{n, n+1\}, \{n, n+1, n+2\}, \dots : n \in \mathbb{N}, n \geq 2\}$ .

It is clear  $(X, \mathcal{F})$  is  $D_1$ -space. But for  $1 \neq 2$ , there is no D-sets  $U, V$  such as  $1 \in U, 2 \in V$  and  $U \cap V = \emptyset$ .

#### 4. On Kernel Sets:

In the fourth chapter, the definition of the kernel set has been included to obtain positive results, using the definitions and concepts that were discussed in the previous chapters.

**Definition 4.1 :** Let  $(X, \mathcal{F})$  be (ft-s).  $x \in X$  and  $A \subseteq X$ . Then we refer to the  $\cap \{U : \forall U \in nbd\{x\}\}$  as the kernel of  $x$  ( or simply  $\ker \{x\}$ ). Also we refer to the  $\cap \{U : \forall U \in nbd\{A\}\}$  as the kernel of  $A$  ( or simply  $\ker \{A\}$ ).

**Theorem 4.2 :** Let  $(X, \mathcal{F})$  be (ft-s) and  $x, y \in X$ . If  $y \in \ker\{x\}$ , thus  $x \in \overline{\{y\}}$ .

**Proof :** Let  $y \in \ker\{x\}$ . Then  $y \in U_x$  for each  $U_x \in nbd\{x\}$ . Therefore  $U_x \cap \{y\} \neq \emptyset$  for each  $U_x \in \mathcal{F}$  such that  $x \in U_x$ . Hence  $x \in \overline{\{y\}}$ .

**Example 4.3 :** Let  $X = \{a, b, c, d, e\}$  and  $\mathcal{F} = \{\{a, b, c\}, \{c, d\}, \{c\}\}$  be fixed topological of  $X$ .

Therefore  $e \in \overline{\{a\}} = \{a, b, e\}$ , but  $a \notin \ker\{e\} = \{e, c\}$ .

**Theorem 4.4 :** Suppose  $(X, \mathcal{F})$  is a (ft-s),  $A \subseteq X$ . Thus  $\ker(A) = A \cup B$  where  $B = \cap \{U : \forall U \in \mathcal{F}\}$ .

**Proof :** Let  $\mathcal{W} = \{W_\alpha : \alpha \in \Lambda\}$  be the collection of all nbds of  $A$  and  $B = \cap \{U_\alpha : \forall U_\alpha \in \mathcal{F}\}$  where  $\mathcal{F} = \{U_\alpha : \alpha \in \Lambda\}$ . Let for each  $\alpha \in \Lambda$ , the collection  $\{W_{\alpha i} : i \in I\}$  such as  $W_\alpha = W_{\alpha i} \cap W_{\alpha j} \cap \dots \quad \forall \alpha \in \Lambda$ . Then  $W_{\alpha i} = A \cup U_\alpha, W_{\alpha j} = (A \cup U_\alpha) \cup P_j, W_{\alpha k} = (A \cup U_\alpha) \cup P_k, \dots$  where  $P_i, P_j, P_k, \dots \in \mathbb{P}(X - (A \cup U_\alpha))$ . Also  $W_{\beta i} = A \cup U_\beta, W_{\beta j} = (A \cup U_\beta) \cup q_j, W_{\beta k} = (A \cup U_\beta) \cup q_k, \dots$  where  $q_i, q_j, q_k, \dots \in \mathbb{P}(X - (A \cup U_\beta)) \dots$

Then  $W_\alpha \cap W_\beta \cap \dots = (W_{\alpha i} \cap W_{\alpha j} \cap \dots) \cap (W_{\beta i} \cap W_{\beta j} \cap \dots) \cap \dots = (A \cup U_\alpha) \cap (A \cup U_\alpha \cup P_j) \cap (A \cup U_\alpha \cup P_k) \cap \dots \cap (A \cup U_\beta) \cap (A \cup U_\beta \cup q_j) \cap (A \cup U_\beta \cup q_k) \cap \dots \cap \dots = (A \cup U_\alpha) \cap (A \cup U_\beta) \cap \dots = A \cup (U_\alpha \cap U_\beta \cap U_\delta \cap \dots) = A \cup B$  where  $B = \cap \{U_\alpha : \forall U_\alpha \in \mathcal{F}\}$ .

**Corollary 4.5 :** Suppose  $(X, \mathcal{F})$  is a  $(ft-s)$  and  $C \subseteq X$ . Thus  $\ker(C) = C$  if and only if  $D \subseteq C$  where  $D = \cap \{U_\alpha : \forall U_\alpha \in \mathcal{F}\}$ .

**Proof :** The proof is by Theorem (4.4) .

**Theorem 4.6 :** Let  $(X, \mathcal{F})$  be  $(ft-s)$ ,  $A \subseteq X$  and  $B = \cap \{U_\alpha : \forall U_\alpha \in \mathcal{F}\}$ . Then

- 1-  $\bar{A} \cap \ker(A) = A \cup B$  if  $A \cap U_\alpha \neq \emptyset$  for each  $U_\alpha \in \mathcal{F}$ .
- 2-  $\bar{A} \cap \ker(A) = A$  if  $A \cap U_\alpha = \emptyset$  for some  $U_\alpha \in \mathcal{F}$ .

**Proof :**

- 1- If  $A \cap U_\alpha \neq \emptyset$  for each  $U_\alpha \in \mathcal{F}$ , then by Theorem (2.8),  $\bar{A} = X$ . Therefore  $\bar{A} \cap \ker(A) = \bar{A} \cap (A \cup B) = X \cap (A \cup B) = A \cup B$ .
- 2- Let  $A \cap U_\alpha = \emptyset$  for some  $U_\alpha \in \mathcal{F}$  and suppose  $x \notin A$  such as  $x \in \ker(A) = A \cup B$ . Then  $x \in B$  and hence  $x \in U_\alpha$  for each  $U_\alpha \in \mathcal{F}$ . Hence  $x \in U$  and  $(A \cap U) - \{x\} = \emptyset$ . Thus  $x \notin d(A)$ . Then  $x \notin \bar{A}$ . Therefore  $x \notin \bar{A} \cap \ker(A)$ . Then  $\bar{A} \cap \ker(A) \subseteq A$ . Since  $A \subseteq \bar{A}$  and  $A \subseteq \ker(A)$ , then  $A \subseteq \bar{A} \cap \ker(A)$ . Therefore  $\bar{A} \cap \ker(A) = A$ .

**Theorem 4.7 :** Suppose  $(X, \mathcal{F})$  is  $(ft-s)$ ,  $A \subseteq X$ . Thus  $\ker(A) \subseteq \{x : \overline{\{x\}} \cap A \neq \emptyset\}$ .

**Proof :** By Theorem (2.9),  $A \subseteq \{x : \overline{\{x\}} \cap A \neq \emptyset\}$ . Let  $y \in B = \cap \{U_\alpha : \forall U_\alpha \in \mathcal{F}\}$ . Then  $y \in U_\alpha$  for each  $U_\alpha \in \mathcal{F}$ . Therefore  $\overline{\{y\}} = X$ . Hence  $\overline{\{y\}} \cap A \neq \emptyset$ . Thus  $B \subseteq \{x : \overline{\{x\}} \cap A \neq \emptyset\}$ . Therefore  $\ker(A) \subseteq \{x : \overline{\{x\}} \cap A \neq \emptyset\}$ .

**Example 4.8 :** Let  $X = \{a, b, c, d, e\}$  and  $\mathcal{F} = \{\{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$  be fixed topological on  $X$ . Let  $A = \{b, c, e\}$ . Hence  $\{x : \overline{\{x\}} \cap A \neq \emptyset\} = X$  and  $\ker(A) = \{a, b, c, e\}$ . Therefore  $\ker(A) \subseteq \{x : \overline{\{x\}} \cap A \neq \emptyset\}$ , but  $\ker(A) \neq \{x : \overline{\{x\}} \cap A \neq \emptyset\}$ .

**Corollary 4.9 :** If  $A$  is D-set in  $(ft-s) (X, \mathcal{F})$ , then  $\{x: \overline{\{x\}} \cap A \neq \emptyset\} \cap \bar{A} = \ker(A) \cap \bar{A}$ .

**Proof :** If  $A$  is D-set, then  $A \cap U = \emptyset$  for some  $U \in \mathcal{F}$ . Therefore the proof follows from Theorems (3.4) and (3.6)(2).

**Theorem 4.10 :** If  $A$  is D-set in  $(ft-s) (X, \mathcal{F})$ , then  $\ker(A) \neq X$ .

**Proof :** Let  $A$  be D-set. Then  $A = U - V$  where  $U$  and  $V$  are F-open sets. Therefore  $A = U - V \subseteq U \neq X$ . Since  $\ker(A) \subseteq U$ , then  $\ker(A) \neq X$ .

**Theorem 4.11 :** Let  $(X, \mathcal{F})$  be a  $(ft-s)$  and  $x, y \in X$  such as  $x \neq y$ . If  $\ker\{x\} = \ker\{y\}$ , hence  $\overline{\{x\}} = \overline{\{y\}}$ .

**Proof :** Let  $\ker\{x\} = \ker\{y\}$ . Then  $\{x\} \cup B = \{y\} \cup B$  such that  $B = \cap \{U_\alpha: \forall U_\alpha \in \mathcal{F}\}$ . Therefore  $x \in B$  and  $y \in B$ . Hence  $\overline{\{x\}} = \overline{\{y\}} = X$ .

**Example 4.12 :** Let  $X = \{a, b, c, d, e\}$  and  $\mathcal{F} = \{\{a, b, c\}, \{c, d\}, \{c\}\}$  be fixed topological on  $X$ . Thus  $\overline{\{a\}} = \overline{\{b\}} = \{a, b, e\}$  but  $\ker\{a\} \neq \ker\{b\}$ .

**Theorem 4.13 :** Suppose  $(X, \mathcal{F})$  is  $(ft-s)$ ,  $E, G \subseteq X$  and  $B = \cap \{U_\alpha: \forall U_\alpha \in \mathcal{F}\}$ . Thus:

- 1-  $\ker(E) \cup \ker(G) = \ker(E \cup G)$
- 2-  $\ker(E) \cap \ker(G) = \ker(E \cap G)$

**Proof :**

- 1-  $\ker(E \cup G) = (E \cup G) \cup B = (E \cup B) \cup (G \cup B) = \ker(E) \cup \ker(G)$
- 2-  $\ker(E \cap G) = (E \cap G) \cup B = (E \cup B) \cap (G \cup B) = \ker(E) \cap \ker(G)$ .

**Theorem 4.14 :** Let  $(X, \mathcal{F})$  be a  $(ft-s)$ . If  $(X, \mathcal{F})$  is  $D_0$ -space, then  $\ker\{x\} \neq \ker\{y\}$  for each  $x, y \in X$  such that  $x \neq y$ .

**Proof :** Let  $(X, \mathcal{F})$  be  $D_0$ -space and  $x, y \in X$  such as  $x \neq y$ . Suppose  $\ker\{x\} = \ker\{y\}$ . Thus  $\{x\} \cup B = \{y\} \cup B$  where  $B = \cap \{U_\alpha: \forall U_\alpha \in \mathcal{F}\}$ . Thus  $x \in B$  and  $y \in B$ . Then there is no  $U \in \mathcal{F}$  such as  $x \in U \wedge y \notin U$  or  $y \in U \wedge x \notin U$ . That is a contradiction with Theorem (3.19). Therefore  $\ker\{x\} \neq \ker\{y\}$ .

**Example 4.15 :** Let  $X = \{a, b, c, d, e\}$  and  $\mathcal{F} = \{\{a, b, c\}, \{c, d\}, \{c\}\}$  be a fixed topological on  $X$ . Hence  $\ker\{x\} \neq \ker\{y\} \forall x, y \in X \ni x \neq y$ , but  $(X, \mathcal{F})$  is not  $D_0$ -space.

The following example show that  $\ker\{x\} = \ker\{y\}$  for some  $x, y \in X$  such that  $x \neq y$  since the space is not  $D_0$ -space.

**Example 4.16 :** Let  $X = \{a, b, c, d, e\}$  and  $\mathcal{F} = \{\{a, b, c\}, \{a, b, d\}\}$  be fixed topological on  $X$ . Then  $(X, \mathcal{F})$  is not  $D_0$ -space, but  $\ker\{a\} = \ker\{b\} = \{a, b\}$ .

## 5.Results:

The most important results that were reached through this research are: The kernel of any set in the space is equal to the union of the set with the set of all intersection of open sets in the fixed topological space sets. If the kernel of two different points are equal, then the closure of the two points are equal too. In the  $D_0$ -space, the kernel of any two different points are not equal. Finally the kernel is distributed over the intersection and the union.

## 6.Conclusion:

In this research, we study the definition of  $(ft-s)$ . This study introduced, examines and studied the  $D$ -set in  $(ft-s)$ . To achieve the aim of the study, some properties and characterizations of these concepts are investigated. The kernel sets, that related to  $F$ -open sets are introduced and studied. We also obtained the fact that the relationships between these concepts are different from those in previous studies, and especially with our main concept, which is kernel set. This work will open a method for other researchers to study the applications of kernel sets.

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