Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Nano αg_{I}-open set

To cite this article: R B Esmaeel and Sufyan G Saeed 2021 *J. Phys.: Conf. Ser.* **1897** 012031

Nano αg_{I}-open set

R B Esmaeel[1] and Sufyan G Saeed[2]
[1, 2]Department of Mathematics, Ibn Al-Haitham
College of Education University of Baghdad, IRAQ
[1]ranamumosa@yahoo.com, [2]Safsaf201030@gmail.com

ABSTRACT: The main objective of this paper is define the notion $Nano$-αg_{I}-open set by using nano topological space and some properties of this set are studied also, $nano$-αg_{I}-β set and $nano$-αg_{I}-ϑ-$closed$ set are two concepts that are defied by using $Nano$-αg_{I}-open set many examples have been cited two indicate that the reverse of the proposition and remarks is not true. Also an applied example was presented explains how to benefit from $nano$-αg_{I}-$closed$ set.

1- Introduction
An α-open was studied in 1965 by O. Njastad, as a subset Ç is an α-open set where $Ç \subseteq int(cl(int(Ç)))$[1,2]. The notion of ideal was studied by Kuratowski [3,4], that Ị is an ideal on Ӽ, when Ị is a collection of all subsets of Ӽ and Ị have two properties (if Ç, Đ ∈ Ị, then Ç ∪ Đ ∈ Ị) and (if Ç ∈ Ị and Đ ⊆ Ç, then Đ ∈ Ị.

There are many types for the ideal [5-8]

i. $Ị_{\{\emptyset\}}$: the trivial ideal where Ị={∅}.
ii. $Ị_n$: the ideal of all nowhere dense sets
$Ị_n$ ={ Ç ⊆ Ӽ: $int(cl(Ç))$={∅}.
iii. $Ị_f$: the ideal of all finite subsets of Ӽ
$Ị_f$ = { Ç ⊆ Ӽ: Ç is a finite set}.

The collection of all α-open sets denoted by" $\tilde{\iota}_\alpha$" and the collection of all α-closed denoted by" $ɟ_\alpha$".

By using the nano topological space, a new type of near nano open set is presented which is $nano$-αg_{I}-$closed$ set with remarks interpretative table for this type of set. Also concept which is $nano$-αg_{I}-$kernal$ of were given by remarks and example with interpretative table, then other connotation are given; $nano$-αg_{I}-β set, $nano$-αg_{I}-ϑ-$closed$ set with some advantage of those connotation with examples. Finally an introductory example of a specific disease is provided showing how to use the $nano$-αg_{I}-$closed$ set with clarifying in a table.

2- Preliminaries

Definition 2.1:**[9]** For equivalence relation $\mathfrak{R}$ on a set Ӽ ≠ ∅, let Ç ⊆ Ӽ:
i. The lower approximation of Ç via $\mathfrak{R}$ denoted by $\underline{\mathfrak{R}}(Ç)$ where
$\underline{\mathfrak{R}}(Ç) = \bigcup_{ė\in Ç} \{\mathfrak{R}(ė)\,;\mathfrak{R}(ė) \subseteq Ç\}$, and $\mathfrak{R}(ė)$ defined by the equivalence class by ė.
ii. The upper approximation of Ç via $\mathfrak{R}$ denoted by $\overline{\mathfrak{R}}(Ç)$ where
$\overline{\mathfrak{R}}(Ç) = \bigcup_{ė\in Ç} \{\mathfrak{R}(ė)\,;\mathfrak{R}(ė) \cap Ç \neq \emptyset\}$.
iii. The boundary of Ç via $\mathfrak{R}$ denoted by $\mathfrak{R}^b(Ç)$ where
$\mathfrak{R}^b(Ç) = \overline{\mathfrak{R}}(Ç)$- $\underline{\mathfrak{R}}(Ç)$

Definition 2.2:**[11]** For equivalence relation $\mathfrak{R}$ on a set Ҳ $\neq \emptyset$, let Ç $\subseteq$ Ҳ and $\tilde{\imath}_{\mathfrak{R}}$(Ç) = {Ҳ,$\emptyset$, $\underline{\mathfrak{R}}$(Ç), $\overline{\mathfrak{R}}$(Ç), $\mathfrak{R}^b$(Ç)} is topology on Ҳ, then $\tilde{\imath}_{\mathfrak{R}}$(Ç) is called $nano\ topology$ and (Ҳ, $\tilde{\imath}_{\mathfrak{R}}$(Ç)) is called $nano\ topological\ space$. Every element in this prior topology is called $nano$-open set (denoted by; n-open set) and its complement is $nano$-closed set (denoted by; n-closed set). The nano-interior and the nano-closure of Ç denoted by the following symbols n-int(Ç) where n-int(Ç) = $\cup$ { Ơ $\subseteq$ Ҳ; Ơ is an n-open set, where Ơ $\subseteq$ Ç} and n-cl(Ç) = $\cap$ { Ғ $\subseteq$ Ҳ; Ғ is an n-closed set, where Ç $\subseteq$ Ғ}, in respectively.

For any ideal Ị, the space (Ҳ, $\tilde{\imath}_{\mathfrak{R}}$(Ç), Ị) is $nano\ ideal\ topology\ space$.

Example 2.3: Let Ҳ ={$\dot{e}_1$, $\dot{e}_2$, $\dot{e}_3$} and $\mathfrak{R}$ =
ple 2.3: Let Ҳ ={$\dot{e}_1$, $\dot{e}_2$, $\dot{e}_3$} and $\mathfrak{R}$ =
ė1,ė1),(ė2,ė2),(ė3, ė3),(ė1, ė2),(ė2,ė1)}. Then, *P*Ҳ={ Ҳ,$\emptyset$,{$\dot{e}_1$}, {$\dot{e}_2$}, {$\dot{e}_3$},{$\dot{e}_1$, $\dot{e}_2$},{$\dot{e}_2$, $\dot{e}_3$},{$\dot{e}_1$, $\dot{e}_3$}}, $\mathfrak{R}$($\dot{e}_1$)={$\dot{e}_1$, $\dot{e}_2$}= $\mathfrak{R}$($\dot{e}_2$) and $\mathfrak{R}$($\dot{e}_3$)= {$\dot{e}_3$} then Ҳ/ $\mathfrak{R}$={{$\dot{e}_1$, $\dot{e}_2$}, {$\dot{e}_3$}}. Then the following table:

Ç	$\underline{\mathfrak{R}}$(Ç)	$\overline{\mathfrak{R}}$(Ç)	$\mathfrak{R}^b$(Ç)	$\tilde{\imath}_{\mathfrak{R}}$(Ç)
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	{ Ҳ,$\emptyset$}
Ҳ	Ҳ	Ҳ	$\emptyset$	{ Ҳ,$\emptyset$}
{$\dot{e}_1$}	$\emptyset$	{$\dot{e}_1$,$\dot{e}_2$}	{$\dot{e}_1$,$\dot{e}_2$}	{ Ҳ,$\emptyset$,{$\dot{e}_1$,$\dot{e}_2$}}
{$\dot{e}_2$}	$\emptyset$	{$\dot{e}_1$,$\dot{e}_2$}	{$\dot{e}_1$,$\dot{e}_2$}	{ Ҳ,$\emptyset$,{$\dot{e}_1$,$\dot{e}_2$}}
{$\dot{e}_3$}	{$\dot{e}_3$}	{$\dot{e}_3$}	$\emptyset$	{ Ҳ,$\emptyset$,{$\dot{e}_3$}}
{$\dot{e}_1$,$\dot{e}_2$}	{$\dot{e}_1$,$\dot{e}_2$}	{$\dot{e}_1$,$\dot{e}_2$}	$\emptyset$	{ Ҳ,$\emptyset$,{$\dot{e}_1$,$\dot{e}_2$}}
{$\dot{e}_2$,$\dot{e}_3$}	{$\dot{e}_3$}	Ҳ	{$\dot{e}_1$,$\dot{e}_2$}	{Ҳ,$\emptyset$,{$\dot{e}_1$,$\dot{e}_2$},{$\dot{e}_3$}}
{$\dot{e}_1$,$\dot{e}_3$}	{$\dot{e}_3$}	Ҳ	{$\dot{e}_1$,$\dot{e}_2$}	{Ҳ,$\emptyset$,{$\dot{e}_1$,$\dot{e}_2$},{$\dot{e}_3$}}

Table 2.1

Definition 2.4:**[11]** For a space (Ӽ, ῖ$_{\mathfrak{R}}$(Ç)), the set Ḙ ⊆ Ӽ is $nano$-α-open (denoted by n-α-open) whenever Ḙ ⊆ n-int(n-cl(n-int(Ḙ))), where its complement is $nano$-α-closed (denoted by n-α-closed). The family of all $nano$-α-closed symbolize it n-αC(Ӽ) and the family of all $nano$-α-open symbolize it n-αO(Ӽ).

From the table 2.1, the family of all n-α-closed and n-α-open can be determined, according to the given ῖ$_{\mathfrak{R}}$(Ç) in the previous table as the following table:

Ç	ῖ$_{\mathfrak{R}}$(Ç)	n-αO(Ӽ)	n-αC(Ӽ)
∅	{Ӽ,∅}	{Ӽ,∅}	{Ӽ,∅}
Ӽ	{Ӽ,∅}	{Ӽ,∅}	{Ӽ,∅}
{ė$_1$}	{Ӽ,∅,{ė$_1$, ė$_2$}}	{Ӽ,∅,{ė$_1$, ė$_2$}}	{Ӽ,∅,{ė$_3$}}
{ė$_2$}	{Ӽ,∅,{ė$_1$, ė$_2$}}	{Ӽ,∅,{ė$_1$, ė$_2$}}	{Ӽ,∅,{ė$_3$}}
{ė$_3$}	{Ӽ,∅,{ė$_3$}}	{Ӽ,∅,{ė$_3$},{ė$_2$, ė$_3$}, {ė$_1$, ė$_3$}}	{Ӽ,∅,{ė$_1$,ė$_2$}{ ė$_1$},{ė$_2$}}
{ė$_1$, ė$_2$}	{Ӽ,∅,{ė$_1$, ė$_2$}}	{Ӽ,∅,{ė$_1$, ė$_2$}}	{Ӽ,∅,{ė$_3$}}
{ė$_2$, ė$_3$}	{Ӽ,∅,{ė$_1$, ė$_2$},{ė$_3$}}	{Ӽ,∅,{ė$_1$, ė$_2$},{ė$_3$}}	{Ӽ,∅,{ė$_1$, ė$_2$},{ė$_3$}}
{ė$_1$, ė$_3$}	{Ӽ,∅,{ė$_1$, ė$_2$},{ė$_3$}}	{Ӽ,∅,{ė$_1$, ė$_2$},{ė$_3$}}	{Ӽ,∅,{ė$_1$, ė$_2$},{ė$_3$}}

Table 2.2

Definition 2.5:**[10]** In a space (Ӽ, ῖ$_{\mathfrak{R}}$(Ç)), if Ḙ ⊆ Ӽ, then n-Ƙer(Ḙ) = ∩{ Ơ; Ḙ ⊆ Ơ, Ơ ∈ ῖ$_{\mathfrak{R}}$(Ç)} which is shortcut for $nano$-$kernal$ of Ç at Ḙ.

From Table 2.2, if the set Ç ={ė$_1$,ė$_2$} then ῖ$_{\mathfrak{R}}$(Ç) = {Ӽ,∅,{ė$_1$,ė$_2$}}, then according to the given Ḙ ⊆ Ӽ, n-Ƙer(Ḙ) can be determined in the following table:

Ẹ	n-Ꝁer(Ẹ)
$\emptyset$	$\emptyset$
Ꭓ	Ꭓ
$\{\dot{e}_1\}$	$\{\dot{e}_1, \dot{e}_2\}$
$\{\dot{e}_2\}$	$\{\dot{e}_1, \dot{e}_2\}$
$\{\dot{e}_3\}$	Ꭓ
$\{\dot{e}_1, \dot{e}_2\}$	$\{\dot{e}_1, \dot{e}_2\}$
$\{\dot{e}_2, \dot{e}_3\}$	Ꭓ
$\{\dot{e}_1, \dot{e}_3\}$	Ꭓ

Table 2.3

Definition 2.6:**[10]** In a space (Ꭓ,$\tilde{\iota}_{\mathfrak{R}}$(Ç)), if, Ẹ = n-Ꝁer(Ẹ), where Ẹ ⊆ Ꭓ, then Ẹ is said *nano-β* set and briefly n-β set.

From Table 2.3, the set $\emptyset$, Ꭓ and $\{\dot{e}_1,\dot{e}_2\}$ are n-β sets since every one of those sets is equal to it's *nano-kernal*.

Remark 2.7:**[10]** For a space (Ꭓ,$\tilde{\iota}_{\mathfrak{R}}$(Ç)), Ẹ ⊆ Ꭓ, if and only if Ẹ is a *n-open* set, then Ẹ is a n-β set.

Definition 2.8:**[10]** In a space (Ꭓ,$\tilde{\iota}_{\mathfrak{R}}$(Ç)), if Ơ = Ṿ ∩ Ẹ, where Ơ ⊆ Ꭓ, Ṿ is *n-closed* set and Ẹ is n-β set, then Ơ is said *nano-ϑ-closed* set and in shortly n-ϑ-closed set.

From table 2.3 where Ç = $\{\dot{e}_1,\dot{e}_2\}$ then $\tilde{\iota}_{\mathfrak{R}}$(Ç) = {Ꭓ,$\emptyset$,$\{\dot{e}_1,\dot{e}_2\}$} then the family of all *n-ϑ-closed* sets is {Ꭓ,$\emptyset$,$\{\dot{e}_1,\dot{e}_2\}$,$\{\dot{e}_3\}$}.
Example 2.9: Let Ꭓ = $\{\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4\}$ with Ꭓ/$\mathfrak{R}$= {$\{\dot{e}_1\}$,$\{\dot{e}_3\}$$\{\dot{e}_2, \dot{e}_4\}$}} and Ç = $\{\dot{e}_1, \dot{e}_2\}$. Then $\tilde{\iota}_{\mathfrak{R}}$(Ç) = {Ꭓ,$\emptyset$,$\{\dot{e}_1\}$,$\{\dot{e}_2, \dot{e}_4\}$,$\{\dot{e}_1, \dot{e}_2, \dot{e}_4\}$} Then $\{\dot{e}_1\}$ is n-β set and $\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}$ is n-ϑ-closed.

Proposition 2.10:**[10]**

i. Every n-β set is n-ϑ-closed set.
ii. Every n-$open$ set is n-ϑ-closed set.
iii. Every n-$closed$ set is n-ϑ-closed set.

The converse of proposition 2.10, is not true by the example.

Example 2.11: From Table 2.3, Ẹ = {ė₃} where Ç = {ė₁,ė₂}, $\text{ƫ}_{\Re}$(Ç) = {Ẋ,∅,{ė₁,ė₂}} and n-Ꝁer(Ẹ) = Ẋ , then Ẹ is not n-β set, so it is not n-$open$ set, but Ẹ is n-ϑ-closed set since Ẹ = Ẹ ∩ Ẋ. If we take Ẹ = {ė₁,ė₂} with the same set Ç then n-Ꝁer(Ẹ) = { ė₁,ė₂} then Ẹ is a n-β set and Ẹ is a n-ϑ-closed set, but Ẹ is not n-$closed$ set.

Remark 2.12:**[10]** In (Ẋ,$\text{ƫ}_{\Re}$(Ç)),if Ơ ⊆ Ẋ ; Ơ is n-ϑ-closed set, then Ơ = n-Ꝁer(Ơ) ∩ Ṿ where, Ṿ is n-$closed$ set.

3- On Nano $\alpha g_{\text{Ị}}$-closed set

Definition 3.1: In (Ẋ,$\text{ƫ}_{\Re}$(Ç), Ị), Ç is said $nano$-$\alpha g_{\text{Ị}}$-closed set (denoted by, n-$\alpha g_{\text{Ị}}$-closed), if Ç-Ơ ∈ Ị then, cl(Ç)-Ơ ∈ Ị where Ơ ⊆ Ẋ and Ơ is an $nano$-α-open set.

Now, Ç^c is a $nano$-$\alpha g_{\text{Ị}}$-open sets denoted by "n-$\alpha g_{\text{Ị}}$-open". The collection of all $nano$-$\alpha g_{\text{Ị}}$-closed sets, denoted by "n-$\alpha g_{\text{Ị}}C$(Ẋ). The collection of all $\alpha g_{\text{Ị}}$-open sets "n-$\alpha g_{\text{Ị}}O$(Ẋ)".

Example 3.2: From table 2.1 let Ị = {∅,{ė₁},{ė₂},{ė₁,ė₂}} is the ideal, the family of all n-$\alpha g_{\text{Ị}}$-closed sets and it is a complement n-$\alpha g_{\text{Ị}}$-open sets can be determine, according to the given $\text{ƫ}_{\Re}$(Ç) and n-αO(Ẋ) in the table 2.2 as the following table;

Ç	$\tau_{\mathfrak{R}}$(Ç)	n-αO(Ҳ)	n-$\alpha g_I C$(Ҳ)	n-$\alpha g_I O$(Ҳ)
∅	{Ҳ,∅}	{Ҳ,∅}	{Ҳ,∅,{$\dot{e}_3$}, {$\dot{e}_2, \dot{e}_3$}, {$\dot{e}_1, \dot{e}_3$}}	{Ҳ,∅,{$\dot{e}_1$},{$\dot{e}_2$}, {$\dot{e}_1, \dot{e}_2$}}
Ҳ	{Ҳ,∅}	{Ҳ,∅}	{Ҳ,∅,{$\dot{e}_3$}, {$\dot{e}_2, \dot{e}_3$}, {$\dot{e}_1, \dot{e}_3$}}	{Ҳ,∅,{$\dot{e}_1$},{$\dot{e}_2$}, {$\dot{e}_1, \dot{e}_2$}}
{$\dot{e}_1$}	{Ҳ,∅,{$\dot{e}_1, \dot{e}_2$}}	{Ҳ,∅,{$\dot{e}_1, \dot{e}_2$}}	{Ҳ,∅,{$\dot{e}_3$}, {$\dot{e}_2, \dot{e}_3$}, {$\dot{e}_1, \dot{e}_3$}}	{Ҳ,∅,{$\dot{e}_1$},{$\dot{e}_2$}, {$\dot{e}_1, \dot{e}_2$}}
{$\dot{e}_2$}	{Ҳ,∅,{$\dot{e}_1, \dot{e}_2$}}	{Ҳ,∅,{$\dot{e}_1, \dot{e}_2$}}	{Ҳ,∅,{$\dot{e}_3$}, {$\dot{e}_2, \dot{e}_3$}, {$\dot{e}_1, \dot{e}_3$}}	{Ҳ,∅,{$\dot{e}_1$},{$\dot{e}_2$}, {$\dot{e}_1, \dot{e}_2$}}
{$\dot{e}_3$}	{Ҳ,∅, {$\dot{e}_3$}	{Ҳ,∅,{$\dot{e}_3$}, {$\dot{e}_2, \dot{e}_3$}, {$\dot{e}_1, \dot{e}_3$}}	P(Ҳ)	P(Ҳ)
{$\dot{e}_1, \dot{e}_2$}	{Ҳ,∅,{$\dot{e}_1, \dot{e}_2$}}	{Ҳ,∅,{$\dot{e}_1, \dot{e}_2$}}	{Ҳ,∅,{$\dot{e}_3$}, {$\dot{e}_2, \dot{e}_3$}, {$\dot{e}_1, \dot{e}_3$}}	{Ҳ,∅,{$\dot{e}_1$},{$\dot{e}_2$}, {$\dot{e}_1, \dot{e}_2$}}
{$\dot{e}_2, \dot{e}_3$}	{Ҳ,∅, {$\dot{e}_3$}, {$\dot{e}_1, \dot{e}_2$}}	{Ҳ,∅,{$\dot{e}_3$}, {$\dot{e}_1, \dot{e}_2$}}	P(Ҳ)	P(Ҳ)
{$\dot{e}_1, \dot{e}_3$}	{Ҳ,∅, {$\dot{e}_3$}, {$\dot{e}_1, \dot{e}_2$}}	{Ҳ,∅,{$\dot{e}_3$}, {$\dot{e}_1, \dot{e}_2$}}	P(Ҳ)	P(Ҳ)

Table 3.1

Remark 3.3:
i. Every n-$closed$ set in (Ҳ, $\tau_{\mathfrak{R}}$(Ç)) is n-αg_I-$closed$ in (Ҳ, $\tau_{\mathfrak{R}}$(Ç), I).
ii. Every n-$open$ set in (Ҳ, $\tau_{\mathfrak{R}}$(Ç)) is n-αg_I-$open$ in (Ҳ, $\tau_{\mathfrak{R}}$(Ç), I).

Reverse of Remark 3.3 is not true. By example 3.2, if the set Ç = {$\dot{e}_1$,$\dot{e}_2$} then $\tau_{\mathfrak{R}}$(Ç) = { Ҳ,∅,{$\dot{e}_1$,$\dot{e}_2$}} then n-$\alpha g_I C$(Ҳ) = {Ҳ,∅,{$\dot{e}_3$},{$\dot{e}_2$,$\dot{e}_3$},{$\dot{e}_1$,$\dot{e}_3$}} and n-$\alpha g_I O$(Ҳ) = {Ҳ,∅,{$\dot{e}_1$},{$\dot{e}_2$},{$\dot{e}_1$,$\dot{e}_2$}}, the set {$\dot{e}_1$} is n-αg_I-$open$ in (Ҳ, $\tau_{\mathfrak{R}}$(Ç), I) but not n-$open$ set in (Ҳ, $\tau_{\mathfrak{R}}$(Ç)) and the set {$\dot{e}_1$,$\dot{e}_3$} is n-αg_I-$closed$ in (Ҳ, $\tau_{\mathfrak{R}}$(Ç), I) but not n-$closed$ set in (Ҳ, $\tau_{\mathfrak{R}}$(Ç)).

Theorem 3.4: Let Ç and Đ are two n-αg_I-$closed$ sets then Ç ∪ Đ is a n-αg_I-$closed$.
Proof**:** Let Ç and Đ are two n-αg_I-$closed$ sets in (Ҳ, $\tau_{\mathfrak{R}}$(Ç), I) and Ơ ∈ $\tau_{\mathfrak{R}}$(Ç), where (Ç ∪ Đ)- Ơ ∈ I , then Đ-Ơ ∈ I and Ç-Ơ ∈ I, so n-cl(Đ)-Ơ ∈ I and n-cl(Ç)-Ơ ∈ I therefore, (n-cl(Ç)-Ơ) ∪ (n-cl(Đ)-Ơ) ∈ I, *so* n-cl(Ç ∪ Đ)-Ơ ∈ I.Hence Ç ∪ Đ is n-αg_I-closed sets.

Corollary 3.5: Let Ç and Đ are two n-αg_{I}-open sets then Ç ∩ Đ is n-αg_{I}-open.
Proof: Let Ç and Đ are two n-αg_{I}-open set in Ҳ then Ç^c, Đ^c are two n-αg_{I}-closed sets therefore, Ç^c ∪ Đ^c is n-αg_{I}-closed set by theorem 3.4. Hence(Ç ∩ Đ)^c is n-αg_{I}-closed set so Ç ∩ Đ is n-αg_{I}-open set**.**

4- On Nano αg_{I}-kernal of set
*Definition 4.1***:** Let (Ҳ, ῖ$_{\mathfrak{R}}$(Ç), Į) be a nano ideal topological space and Ẹ ⊆ Ҳ, n- αg_{I}-kernal of Ẹ = ∩ { Ơ: Ẹ ⊆ Ơ, Ơ ∈ n-$\alpha g_{\mathrm{I}} O$(Ҳ) which is shortcut for n-αg_{I}- Ꝁer(Ẹ). It is clear that if Ẹ ∈ n-$\alpha g_{\mathrm{I}} O$(Ҳ) then Ẹ = n-αg_{I}- Ꝁer(Ẹ).

Example 4.2: From example 3.2, if the set Ç = {ė₁,ė₂} then ῖ$_{\mathfrak{R}}$(Ç) = { Ҳ,∅,{ė$_1$,ė$_2$}} then $\alpha g_{\mathrm{I}} O$(Ҳ) = {Ҳ,∅,{ė$_1$},{ė$_2$},{ė$_1$,ė$_2$}} according to the given Ẹ ⊆ Ҳ, we can determine n-αg_{I}-Ꝁer(Ẹ) in the following table:

Ẹ	n-Ꝁer(Ẹ)	n-αg_{I}-Ꝁer(Ẹ)
∅	∅	∅
Ҳ	Ҳ	Ҳ
{ė$_1$}	{ė$_1$, ė$_2$}	{ė$_1$}
{ė$_2$}	{ė$_1$, ė$_2$}	{ė$_2$}
{ė$_3$}	Ҳ	Ҳ
{ė$_1$, ė$_2$}	{ė$_1$, ė$_2$}	{ė$_1$, ė$_2$}
{ė$_2$, ė$_3$}	Ҳ	Ҳ
{ė$_1$, ė$_3$}	Ҳ	Ҳ

Table 4.1

Proposition 4.3: In (Ҳ, ῑ$_{\mathfrak{R}}$(Ç), Ị), if Ẹ ⊆ Ҳ, then n-$\alpha g_{Ị}$-Ҟer(Ẹ) ⊆ n-Ҟer(Ẹ).
Proof: Let ė ∉ n-Ҟer(Ẹ), ė ∈ Ҳ then ė ∉∩ {Ơ: Ẹ ⊆ Ơ, Ơ ∈ ῑ$_{\mathfrak{R}}$(Ç)}, implies, ∃ Ơ ∈ ῑ$_{\mathfrak{R}}$(Ç), Ẹ ⊆ Ơ; ė ∉ Ơ. Since every n-$open$ set in (Ҳ, ῑ$_{\mathfrak{R}}$(Ç)) is n-$\alpha g_{Ị}$-$open$ in (Ҳ, ῑ$_{\mathfrak{R}}$(Ç), Ị), then ∃ Ơ ∈ n-$\alpha g_{Ị}O$(Ҳ), Ẹ ⊆ Ơ; ė ∉ Ơ, so ė ∉∩ {Ơ: Ẹ ⊆ Ơ, Ơ ∈ ῑ$_{\mathfrak{R}}$(Ç)}. Thus ė ∉ n-$\alpha g_{Ị}$-Ҟer(Ẹ).

The term n-Ҟer(Ẹ) ⊆ n-$\alpha g_{Ị}$-Ҟer(Ẹ) is not true by table 2.2, if we take the set Ẹ = {ė$_1$}, then n-Ҟer(Ẹ) = {ė$_1$,ė$_2$}, but n-$\alpha g_{Ị}$-Ҟer(Ẹ) = {ė$_1$}, then n-Ҟer(Ẹ) ⊈ n-$\alpha g_{Ị}$-Ҟer(Ẹ).

*Remark 4.4***:** For (Ҳ, ῑ$_{\mathfrak{R}}$(Ç), Ị), if Ҳ is a finite space then Ẹ = n-$\alpha g_{Ị}$-Ҟer(Ẹ) if and only if Ẹ ⊆ Ҳ is a n-$\alpha g_{Ị}$-$open$ set.
Proof: (→) Let Ẹ a n-$\alpha g_{Ị}$-$open$ set. Since Ҳ is a finite space then By Corollary 3.5, n- $\alpha g_{Ị}$-kernal of Ẹ = ∩ { Ơ: Ẹ ⊆ Ơ, Ơ ∈ n-$\alpha g_{Ị}O$(Ҳ)}. It is clear that if Ẹ ∈ n-$\alpha g_{Ị}O$(Ҳ) and Ҳ is a finite space then Ẹ = n-$\alpha g_{Ị}$- Ҟer(Ẹ).
(←) Clear.

Definition 4.5: For any Ẹ ⊆ Ҳ of (Ҳ, ῑ$_{\mathfrak{R}}$(Ç), Ị). If Ẹ = n-$\alpha g_{Ị}$-Ҟer(Ẹ) then Ẹ is said n-$\alpha g_{Ị}$-β set.
From example 4.2, the sets ∅, Ҳ, {ė$_1$}, {ė$_2$} and {ė$_1$,ė$_2$} are n-$\alpha g_{Ị}$-β sets.

*Remark 4.6***:** For (Ҳ, ῑ$_{\mathfrak{R}}$(Ç), Ị), Ẹ ⊆ Ҳ, then Ẹ is n-$\alpha g_{Ị}$-β set if Ẹ is a n-$\alpha g_{Ị}$-$open$ set.

*Definition 4.7***:** For any Ẹ ⊆ Ҳ of (Ҳ, ῑ$_{\mathfrak{R}}$(Ç), Ị), if Ẹ = Ṿ ∩ Ħ, where Ṿ is n-$\alpha g_{Ị}$-$closed$ set and Ħ is n-$\alpha g_{Ị}$-β set, then Ẹ is said n-$\alpha g_{Ị}$-ϑ-$closed$ set.

*Example 4.8***:** From example 4.2, where Ç = {ė$_1$,ė$_2$} then ῑ$_{\mathfrak{R}}$(Ç) = { Ҳ,∅,{ė$_1$,ė$_2$}} then n-$\alpha g_{Ị}C$(Ҳ) = {Ҳ,∅,{ė$_3$},{ė$_2$,ė$_3$},{ė$_1$,ė$_3$}}, then every Ẹ ⊆ Ҳ is n-$\alpha g_{Ị}$-ϑ-$closed$ set since Ẹ = Ṿ ∩ Ħ, such that Ṿ is n-$\alpha g_{Ị}$-$closed$ set and Ħ is n-$\alpha g_{Ị}$-β set.

Theorem 4.9: For any space (Ҳ, ῑ$_{\mathfrak{R}}$(Ç), Ị) then:
i. Every n-$\alpha g_{Ị}$-β set is n-$\alpha g_{Ị}$-ϑ-closed set**.**
ii. Every n-$\alpha g_{Ị}$-$open$ set is n-$\alpha g_{Ị}$-ϑ-closed set**.**
iii. Every n-$\alpha g_{Ị}$-$closed$ set is n-$\alpha g_{Ị}$-ϑ-closed set**.**
Proof: (i)(→) Let Ẹ is n-$\alpha g_{Ị}$-β then Ẹ = n-$\alpha g_{Ị}$-Ҟer(Ẹ) but Ẹ = Ẹ ∩ Ҳ, Ҳ is n-$\alpha g_{Ị}$-$closed$, then Ẹ is n-$\alpha g_{Ị}$-ϑ-closed set.
(ii)(→) Let Ẹ is n-$\alpha g_{Ị}$-$open$ set, so Ẹ = n-$\alpha g_{Ị}$-Ҟer(Ẹ), then Ẹ n-$\alpha g_{Ị}$-β set, so Ẹ is n-$\alpha g_{Ị}$-ϑ-closed set, by (part i) of theorem 4.9.
(iii)(→) Let Ẹ is a n-$\alpha g_{Ị}$-$closed$. Since Ҳ is n-$\alpha g_{Ị}$-β set and Ẹ = Ẹ ∩ Ҳ, then Ẹ is n-$\alpha g_{Ị}$-ϑ-$closed$ set.

The converse of Theorem 4.9, is not true.

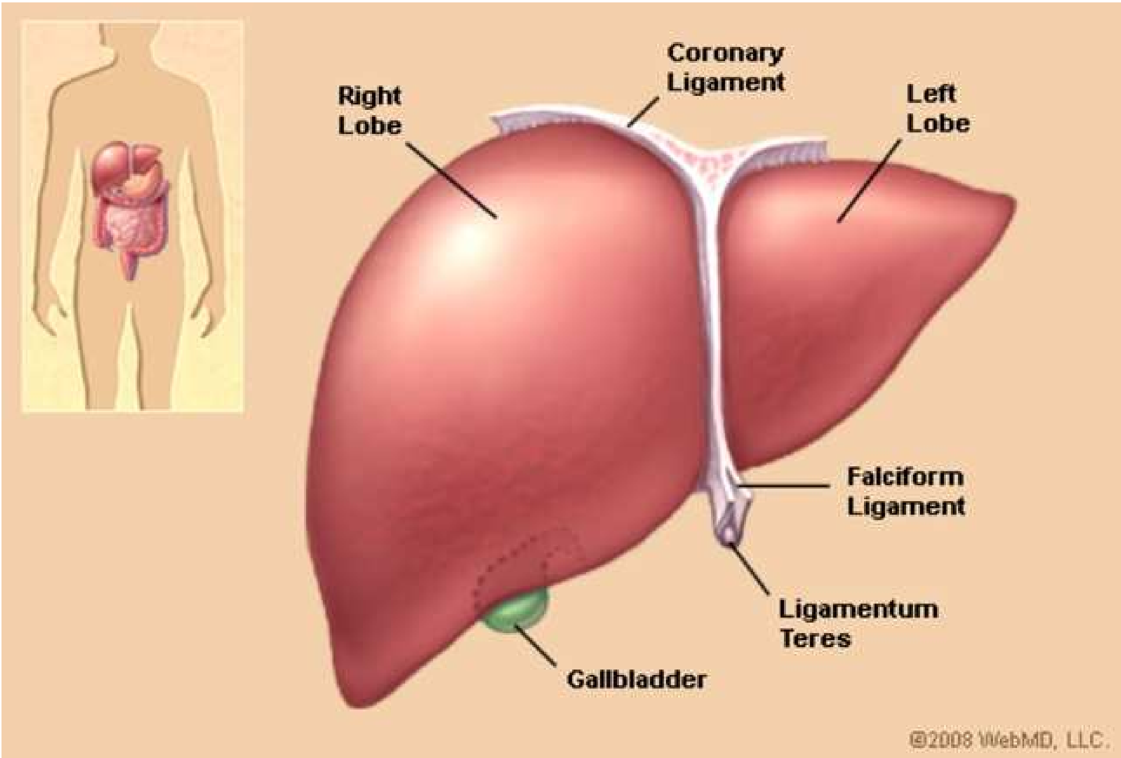
Example 4.10: From example 3.2, if Ẹ = {ė$_2$} where Ç = {ė$_1$, ė$_2$} and ῑ$_{\mathfrak{R}}$(Ç) = {Ҳ,∅,{ė$_1$,ė$_2$}} then n-$\alpha g_{Ị}C$(Ҳ) = {Ҳ,∅,{ė$_3$},{ė$_2$,ė$_3$},{ė$_1$,ė$_3$}}, n-$\alpha g_{Ị}$-Ҟer(Ẹ) = Ҳ. Thus Ẹ is neither n-$\alpha g_{Ị}$-β set nor n-$\alpha g_{Ị}$-$open$ set, but Ẹ is a n-$\alpha g_{Ị}$-ϑ-$closed$ set since Ẹ = Ẹ ∩ Ҳ. In other hand ; if Ẹ ={ė$_1$,ė$_2$} with the same set Ç then n-$\alpha g_{Ị}$-Ҟer(Ẹ) = {ė$_1$,ė$_2$}. Implies Ẹ is n-$\alpha g_{Ị}$-β set, then Ẹ is a n-$\alpha g_{Ị}$-ϑ-$closed$ set but Ẹ is not n-$\alpha g_{Ị}$-$closed$ set.

Proposition 4.11: In $(Ꭓ, \tilde{\tau}_{\Re}(Ç), Ị)$, if $Ꭓ$ is a finite set and $Ơ \subseteq Ꭓ$; $Ơ$ is $n\text{-}\alpha g_{Ị}\text{-}\vartheta\text{-}closed$ set, then $Ơ = n\text{-}\alpha g_{Ị}\text{-Ꝁer}(Ơ) \cap Ħ$, $Ħ$ is $n\text{-}\alpha g_{Ị}\text{-}closed$ set.

Proof: since $Ơ$ is $n\text{-}\alpha g_{Ị}\text{-}\vartheta\text{-}closed$ set, then $Ơ = Ħ \cap Ẹ$ such that $Ẹ$ is $n\text{-}\alpha g_{Ị}\text{-}\beta$ set and $Ħ$ is $n\text{-}\alpha g_{Ị}\text{-}closed$ set. Implies, $Ơ \subseteq n\text{-}\alpha g_{Ị}\text{-Ꝁer}(Ẹ)=Ẹ$ and $Ơ \subseteq n\text{-}\alpha g_{Ị}\text{-Ꝁer}(Ơ)$ which is the smallest $n\text{-}\alpha g_{Ị}\text{-}open$ set containing $Ơ$. Then, $n\text{-}\alpha g_{Ị}\text{-Ꝁer}(Ơ) \subseteq n\text{-}\alpha g_{Ị}\text{-Ꝁer}(Ẹ) = Ẹ$ and $Ơ = Ħ \cap Ẹ$. Therefore, $Ơ = Ħ \cap n\text{-}\alpha g_{Ị}\text{-Ꝁer}(Ơ)$.

5- Some application via $n\text{-}\alpha g_{Ị}\text{-}closed$ sets.

The example that we will deal with in our topic is a viral hepatitis look to the shape 5.1.



Shape 5.1

Example 5.1: Hepatitis A is a viral disease that affects the liver and can cause symptoms that range from mild to severe. The infection is transmitted by eating contaminated food and water, or by direct contact with an infected person.

Almost all people with hepatitis A recover completely with lifelong immunity. However, a very small percentage of people with hepatitis A infection may die from a deadly hepatitis infection. The World Health Organization estimates that in 2016, hepatitis A caused about 7,134 deaths (representing 0.5% of all deaths from viral hepatitis).

The risk of contracting hepatitis A is associated with a lack of safe drinking water and poor sanitation and hygiene (such as infected hands). In countries where the risk of food or water transmission is low, have abnormal sex and who inject drugs. Epidemics can persist and lead to heavy economic losses.

A safe and effective vaccine is available to prevent hepatitis A. Safe water supply, food safety, improved sanitation, hand washing and hepatitis vaccine are among the most effective ways to control the disease. People at high risk, such as those traveling to countries with high levels of infection, men who have same-sex relationships, and drug users can be vaccinated intravenously.

The incubation period for hepatitis A ranges from 14 to 28 days. Symptoms of the infection vary from mild to severe, including fever, malaise, loss of appetite, diarrhea, nausea, abdominal pain, dark urine, diarrhea, and yellowing of the skin and the whites of the eyes. Not all of these symptoms appear on every person with this disease.

Sings and symptoms of the disease are more common in adults than in children. Critical illnesses and death rates are higher among older age groups. Infected children under the age of six do not usually show visible symptoms, and the proportion of infected children is limited to 10%. Infection usually causes more severe symptoms severe attack, and he will soon recover from it.

The following table gives input about 4 patients people $\{ė_1, ė_2, ė_3, ė_4\}$, we will indicate to the symbol **1** if the symptoms are clear to the person and refer the symbol **0** if the symptoms do not appear:

Injured	Yellowing of the skin (Y)	Abdominal pain (A)	Slenderness (S)	The whites of the eyes (E)	Dark urine (D)	Hepatitis
$ė_1$	1	1	1	1	1	1
$ė_2$	1	1	0	1	0	1
$ė_3$	1	1	0	0	0	0
$ė_4$	0	1	0	0	0	0

Table 5.1

In this table, let $Ӿ = \{ė_1, ė_2, ė_3, ė_4\}$ be the set of injured with hepatitis, let $Ị = \{\emptyset, ė_1\}$, $Ç = \{ė_1, ė_3\}$ and $\mathfrak{R}$ be the equivalence on $Ӿ$, where $\mathfrak{R} = \{(ė_i, ė_j);\ ė_i, ė_j \in Ӿ\}$ such that $ė_i, ė_j$ have the same symptoms. Then, $Ӿ/\mathfrak{R}=\{$

$\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3\},\{\dot{e}_4\}\}$, $\tilde{\imath}_{\Re}(Ç) = \{Ӽ,\emptyset,\{\dot{e}_1,\dot{e}_2\}\}$ and n-$\alpha O(Ӽ) = \{Ӽ,\emptyset,$ $\{\dot{e}_1,\dot{e}_3\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$. So n-$\alpha g_! C(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$.

If the whites of the eyes (E) column was cancelled, so $Ӽ/\Re(E) = \{\{\dot{e}_1\},\{\dot{e}_4\},\{\dot{e}_2,\dot{e}_3\}\}$ and $\tilde{\imath}_{\Re(E)}(Ç) = \{Ӽ,\emptyset,\{\dot{e}_1\},\{\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\}\}$. Obviously $\tilde{\imath}_{\Re(E)}(Ç) \neq \tilde{\imath}_{\Re}(Ç)$. Then n-$\alpha O(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_1\},\{\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\}\}$.
And n-$\alpha g_! C(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_4\},\{\dot{e}_1,\dot{e}_4\},\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_3,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$ which is differ from n-$\alpha g_! C(Ӽ)$ with respect to $\tilde{\imath}_{\Re}(Ç)$.

If the abdominal pain (A) column was cancelled, so $Ӽ/\Re(A) = \{\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3\},\{\dot{e}_4\}\} = Ӽ/\Re$ and $\tilde{\imath}_{\Re(A)}(Ç) = \{Ӽ,\emptyset,\{\dot{e}_1,\dot{e}_3\}\} = \tilde{\imath}_{\Re}(Ç)$. Then n-$\alpha O(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_1\},\{\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$ and n-$\alpha g_! C(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$ which is equal from n-$\alpha g_! C(Ӽ)$ with respect to $\tilde{\imath}_{\Re}(Ç)$.

If the slenderness (S) column was cancelled, so $Ӽ/\Re(S) = \{\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3\},\{\dot{e}_4\}\} = Ӽ/\Re$ and $\tilde{\imath}_{\Re(S)}(Ç) = \{Ӽ,\emptyset,\{\dot{e}_1,\dot{e}_3\}\} = \tilde{\imath}_{\Re}(Ç)$. Then n-$\alpha O(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_1\},\{\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$ and n-$\alpha g_! C(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$ which is equal from n-$\alpha g_! C(Ӽ)$ with respect to $\tilde{\imath}_{\Re}(Ç)$.

If the dark urine (D) column was cancelled, so $Ӽ/\Re(D) = \{\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3\},\{\dot{e}_4\}\} = Ӽ/\Re$ and $\tilde{\imath}_{\Re(D)}(Ç) = \{Ӽ,\emptyset,\{\dot{e}_1,\dot{e}_3\}\} = \tilde{\imath}_{\Re}(Ç)$. Then n-$\alpha O(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_1\},\{\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$ and n-$\alpha g_! C(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$ which is equal from n-$\alpha g_! C(Ӽ)$ with respect to $\tilde{\imath}_{\Re}(Ç)$.

If the yellowing of the skin (Y) column was cancelled, so $Ӽ/\Re(Y) = \{\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3,\dot{e}_4\}\}$ and $\tilde{\imath}_{\Re(Y)}(Ç) = \{Ӽ,\emptyset,\{\dot{e}_1\},\{\dot{e}_3,\dot{e}_4\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$. Obviously $\tilde{\imath}_{\Re(Y)}(Ç) \neq \tilde{\imath}_{\Re}(Ç)$. Then n-$\alpha O(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_1\},\{\dot{e}_3,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$ and n-$\alpha g_! C(Ӽ) = \{Ӽ,\emptyset,\{\dot{e}_2\},\{\dot{e}_1,\dot{e}_2\},\{\dot{e}_2,\dot{e}_3\},\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\}\}$ which is differ from n-$\alpha g_! C(Ӽ)$ with respect to $\tilde{\imath}_{\Re}(Ç)$.

From all that, $core(\Re) = \{E,Y\}$. That is mean the yellowing of the skin and the whites of the eyes are the needful and enough to inspire injured develop hepatitis.
The previous $information$ as the next table can be shown:

The collection of equivalent classes	**Nano topology**	$\boldsymbol{n\text{-}\alpha O}(Ꭓ)$	$\boldsymbol{n\text{-}\alpha g_{\mathrm{I}} C}(Ꭓ)$
$Ꭓ/\mathfrak{R}=\{\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3\},\{\dot{e}_4\}\}$	$\tilde{\imath}_{\mathfrak{R}}(Ç)=\{Ꭓ,\emptyset,\{\dot{e}_1,\dot{e}_3\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_1,\dot{e}_3\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$
$Ꭓ/\mathfrak{R}(E)=\{\{\dot{e}_1\},\{\dot{e}_4\},\{\dot{e}_2,\dot{e}_3\}\}$	$\tilde{\imath}\mathfrak{R}(E)Ç=\{\dot{e}1\},\{\dot{e}2,\dot{e}3\},\{\dot{e}1,\dot{e}2,\dot{e}_3\}\}$	$,\{\dot{e}_1\},\{,\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_4\},\{\dot{e}_1,\dot{e}_4\},\{\dot{e}_2,\dot{e}_4\},\{_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$
$Ꭓ/\mathfrak{R}(A)=\{\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3\},\{\dot{e}_4\}\}$	$\tilde{\imath}_{\mathfrak{R}(A)}(Ç)=\{Ꭓ,\emptyset,\{\dot{e}_1,\dot{e}_3\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_1,\dot{e}_3\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$
$Ꭓ/\mathfrak{R}(S)=\{\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3\},\{\dot{e}_4\}\}$	$\tilde{\imath}_{\mathfrak{R}(S)}(Ç)=\{Ꭓ,\emptyset,\{\dot{e}_1,\dot{e}_3\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_1,\dot{e}_3\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$
$Ꭓ/\mathfrak{R}(D)=\{\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3\},\{\dot{e}_4\}\}$	$\tilde{\imath}_{\mathfrak{R}(D)}(Ç)=\{Ꭓ,\emptyset,\{\dot{e}_1,\dot{e}_3\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_1,\dot{e}_3\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_2,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$
$Ꭓ/\mathfrak{R}(Y)=\{\{\dot{e}_1\},\{\dot{e}_2\},\{\dot{e}_3,\dot{e}_4\}\}$	$\tilde{\imath}\mathfrak{R}(Y)Ç=\{Ꭓ,\emptyset,\{\dot{e}1\},\{\dot{e}_3,\dot{e}_4\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_1\},\{\dot{e}_3,\dot{e}_4\},\{_2\},\{\dot{e}_1,\dot{e}_3,\dot{e}_4\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\}\}$	$\{Ꭓ,\emptyset,\{\dot{e}_2\},\{\dot{e}_1,\dot{e}_2\},\{\dot{e}_2,\dot{e}_3\},\{_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_3\},\{\dot{e}_2,\dot{e}_3,\dot{e}_4\},\{\dot{e}_1,\dot{e}_2,\dot{e}_4\}\}$

Table 5.2

References:

[1] O Njastad, 1965 On some classes of nearly open set, *Pacific J. Math*. **15**, pp 961 – 970.
[2] Nadia M Ali, 2004 On New Types of Weakly Open Sets "α-Open and Semi-α-Open Sets", *M.Sc. Thesis, January, Ibn Al-Haithatham Journal for Pure and Applied Science.*
[3] Kuratowski K. 1933 Topology. *NewYork: Acadeic Press* .Vol **I**.
[4] A A Nasef and R B Esamaeel, 2015 Some α- operators vai ideals, *International Electronic journal of Pur and Applied Mathematics* Vol. **9**, No. **3**, pp 149- 159.
[5] A A Nasef, A E Radwan, F A Iprahem and R B Esmaeel, in June-2016 Soft α-compactness via soft ideals Vol. **12**, No. **4**.
[6] M E Abd El-Monsef, A A Nasef, A E Radwan and R B Esmaeel , 2014 On α- open sets with respect to an ideal, Journal of Advances Studies in Topology, 5(**3**), pp 1-9.
[7] R B Esmaeel, 2012 on α-c-compactness, *Ibn Al-Haithatham Journal for Pure and Applied Science*, 22, pp 212-218.
[8] R Englking, 1989 "Outline of general topology" *Amsterdam.*
[9] Z Pawlak, 1982 Rough sets, International journal of computer and Information Sciences. **11**, pp 341-356.
[10] M Parimala and S Jafari, 2018 On some new notions in nano ideai topological space, International Balkan Journal of Mathematics (IBJM), 1(**3**), pp 85-92.
[11] A Steem and J A Seebach, 1970 *Counterexamples in Topology,* Holt, Rinehart and Winster, New York.