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The Transmuted Topp Leone Flexible Weibull (TTLFW) distribution with applications to reliability and lifetime data

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Abstract. The Transmuted Topp Leone Flexible Weibull distribution was developed in this paper using the Transmuted Topp Leone family of distributions and its basic statistical properties were established. Estimation of model parameters was considered using the maximum likelihood estimation (MLE) method and three real life applications were provided. The TTLFW distribution is a promising model as its performance relative to other compounds probability models like the Exponentiated Flexible Weibull, Weibull Flexible Weibull, Kumaraswamy Flexible Weibull, Beta Flexible Weibull, Gamma Flexible Weibull, and Exponentiated Generalized Flexible Weibull distributions is quite credible.

1 Introduction

Probability models have played important roles in many areas of study like finance, biology, engineering, and so on. Many of these probability models have been modelled extended and/or generalized using families of distributions; these families of distributions give room for the introduction of additional shape parameters into the existing probability models. Examples of the existing families of distributions include Kumaraswamy-G family of distribution [1], Gompertz family of distribution [2], Weibull-G family of distribution [3], and a host of others which are mentioned in [4 - 7], and the references therein. However, developing families of distributions have taken a new dimension where two families of distributions are combined to produce a more robust family. For instance, Kumaraswamy Transmuted-G family of distribution [8], exponentiated Weibull-H family of distribution [9], Beta Transmuted {H family of distribution [10], Generalized Transmuted-G family of distribution [11], Transmuted Topp-Leone G family of distribution [12], Marshall-Olkin Topp Leone-G family of distributions [13], and some other ones are notable examples. These families have been used in the literature to extend several existing standard probability models and their potentials have been recorded. The performance of these families is impressive when fitted to real-life events, and this is a major motivation for this present research.

This research aims to explore the work of [12] who combined the Topp Leone family of distributions [14] and Transmuted family of distributions [15]. Particularly, this research considered extending the Flexible Weibull distribution using the Transmuted Topp-Leone G family of distributions. Flexible Weibull (FW) distribution is one of the medications of the well-known Weibull distribution which caters for the disadvantage of not being able to model datasets with a non-monotonic failure rate. The FW distribution exhibits increasing, decreasing, or bathtub shaped failure rate, it has also received

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consideration in clinical studies, life testing experiments, reliability analysis and applied statistics ([16], [17], and [18]). In the next section, the Transmuted Topp-Leone Flexible Weibull distribution is developed and its basic statistical properties are explicitly derived including estimation of model parameters while real life applications are provided in section 3.

2. The Transmuted Topp Leone Flexible Weibull (TTLFW) Distribution

For a random variable y, the Transmuted Topp-Leone family of distribution is defined by the following densities:

$$F(y) = (1+\lambda)\{1 - [1 - G(y)]^2\}^{\alpha} - \lambda\{1 - [1 - G(y)]^2\}^{2\alpha}$$
(1)

and

$$f(y) = 2\alpha g(y) [1 - G(y)] \{1 - [1 - G(y)]^2\}^{\alpha - 1} \{1 + \lambda - 2\lambda \{1 - [1 - G(y)]^2\}^{\alpha}\}$$
(2)

where $y \ge 0$. $\alpha > 0$ and $|\lambda| \le 1$ are shape parameters, F(y) and f(y) are regarded as the cumulative distribution function (c.d.f) and probability density function (p.d.f) respectively.

Also, the densities of the Flexible Weibull distribution are:

$$G(y) = 1 - exp\left(-e^{\beta y - \frac{\theta}{y}}\right) \quad ; y > 0, \beta > 0, \theta > 0 \tag{3}$$

and

$$g(y) = \left(\beta + \frac{\theta}{y^2}\right)e^{\beta y - \frac{\theta}{y}}ex\,p\left(-e^{\beta y - \frac{\theta}{y}}\right) \qquad ; y > 0, \beta > 0, \theta > 0 \tag{4}$$

To derive the c.d.f of the Transmuted Topp-Leone Flexible Weibull distribution, equation (3) is substituted into equation (1) as follows:

$$F(y) = (1+\lambda) \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}} \right) \right]^2 \right\}^{\alpha} - \lambda \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}} \right) \right]^2 \right\}^{2\alpha}$$
(5)

Similarly, the corresponding pdf is obtained by substituting equations (3) and (4) into equation (2) as follows:

$$f(y) = 2\alpha \left(\beta + \frac{\theta}{y^2}\right) e^{\beta y - \frac{\theta}{y}} \left[exp\left(-e^{\beta y - \frac{\theta}{y}}\right) \right]^2 \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}}\right) \right]^2 \right\}^{\alpha - 1} \times \left((1 + \lambda) \left\{ -2\lambda \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}}\right) \right]^2 \right\}^{\alpha} \right\} \right)$$
(6)

for $y \ge 0$. $\alpha > 0$ and $|\lambda| \le 1$, $\beta > 0$, $\theta > 0$.

The plots for the p.d.f and c.d.f of TTLFW distribution are presented in figures 1 and 2 respectively.



Figure 1: PDF of Transmuted Topp Leone Flexible Weibull Distribution

The plot in figure 1 suggests that the shape of the TTLFW distribution is decreasing and unimodal



Figure 2: The c.d.f plot for the TTLFW Distribution

2.1 *Reliability Analysis* Survival Function is obtained as:

$$S(y) = 1 - F(y)$$

Thus, the survival function for TTLFW distribution is:

$$S(y) = 1 - \left((1+\lambda) \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}} \right) \right]^2 \right\}^{\alpha} - \lambda \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}} \right) \right]^2 \right\}^{2\alpha} \right)$$
(7)

The Failure rate is obtained as:

$$h(y) = f(y)/S(y)$$

4

Thus, the failure rate for TTLFW distribution is:

$$h(y) = 2\alpha \left(\beta + \frac{\theta}{y^2}\right) e^{\beta y - \frac{\theta}{y}} \left[exp\left(-e^{\beta y - \frac{\theta}{y}}\right) \right]^2 \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}}\right) \right]^2 \right\}^{\alpha - 1} \\ \times \frac{\left((1+\lambda) \left\{ -2\lambda \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}}\right) \right]^2 \right\}^{\alpha} \right\} \right)}{\left[1 - \left((1+\lambda) \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}}\right) \right]^2 \right\}^{\alpha} - \lambda \left\{ 1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}}\right) \right]^2 \right\}^{2\alpha} \right) \right]}$$
(8)

For $y > 0, \alpha > 0, |\lambda| \le 1, \beta > 0, \theta > 0$

The plot for the failure rate of TTLFW distribution is presented in figure 3.



Figure 3: Plot for the failure rate of the TTLFW Distribution

Figure 3 suggests that the shape for the failure rate of TTLFW distribution is uni-antimodal, unimodal, and increasing.

2.2 Quantile Function

The quantile function of the TTLFW distribution can be obtain by find the inverse equation (5) with some algebra. The quantile function is obtained as:

$$Q(u) = F^{-1}(u)$$

$$Q(u) = \frac{\ln \ln \left[-\ln \ln \left(\rho\right)\right] + \sqrt{\left\{\ln\left[-\ln(\rho)\right]\right\}^2 + u\beta\theta}}{2\beta}$$
(9)

where

$$\rho = \left\{ 1 - \left[\frac{1 + \lambda - \sqrt{(1 + \lambda)^2 - u\lambda u}}{2\lambda} \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}}$$

and $U \sim Uniform(0,1)$

The median can be obtained when 'u' is set at 0.5. Random samples are generated from the TTLFW distribution using:

$$y = \frac{\ln \left| -\ln(\rho) \right| + \sqrt{\left\{ \ln \left| -\ln(\rho) \right] \right\}^2 + u\beta\theta}}{2\beta}$$
(10)

2.3 Estimation of Parameter

Let $y_1, y_2, ..., y_n$ represent random samples from the TTLFW distribution. By the method of maximum likelihood estimation (MLE), the likelihood function of the TTLFW distribution is:

$$f(y_{i} \mid \alpha, \beta, \lambda, \theta) = \prod_{i=1}^{n} [f(y_{1}, y_{2}, ..., y_{n}; \alpha, \beta, \lambda, \theta)]$$

$$f(y_{i} \mid \alpha, \beta, \lambda, \theta) = \prod_{i=1}^{n} 2\alpha \left(\beta + \frac{\theta}{y_{i}^{2}}\right) e^{\beta y_{i} - \frac{\theta}{y}} \left[exp\left(-e^{\beta y_{i} - \frac{\theta}{y_{i}}}\right)\right]^{2}$$

$$\times \left\{1 - \left[exp\left(-e^{\beta y_{i} - \frac{\theta}{y_{i}}}\right)\right]^{2}\right\}^{\alpha - 1} \left\{1 + \lambda - 2\lambda \left\{1 - \left[exp\left(-e^{\beta y - \frac{\theta}{y}}\right)\right]^{2}\right\}^{\alpha}\right\}$$
The log-likelihood function say: $l(\alpha, \beta, \lambda, \theta|y_{i}) = \log \left[f(y_{i}|\alpha, \beta, \lambda, \theta)\right]$ is then given as

The log-likelihood function say; $l(\alpha, \beta, \lambda, \theta | y_i) = \log [f(y_i | \alpha, \beta, \lambda, \theta)]$ is then given as: $l(\alpha, \beta, \lambda, \theta | y_i) = nlog(2) + nlog(\alpha) + \sum_{i=1}^n log \left(\beta + \frac{\theta}{y_i^2}\right) + \beta \sum_{i=1}^n y_i - \theta \sum_{i=1}^n \left(\frac{1}{y_i}\right) - 2e^{\beta \sum_{i=1}^n y_i - \theta \sum_{i=1}^n \left(\frac{1}{y_i}\right)} + (\alpha - 1) \sum_{i=1}^n log \left\{1 - \left[exp \exp\left(-e^{\beta y_i - \frac{\theta}{y_i}}\right)\right]^2\right\} + \sum_{i=1}^n log \left\{1 + \lambda - 2\lambda \left\{1 - \left[exp \exp\left(-e^{\beta y_i - \frac{\theta}{y_i}}\right)\right]^2\right\}^{\alpha}\right\}$ (11)

The partial derivatives of $l(\alpha, \beta, \lambda, \theta | y_i)$ for each of the model parameters are equated to zero; $\frac{dl(\alpha, \beta, \lambda, \theta | y_i)}{d\alpha} = 0, \frac{dl(\alpha, \beta, \lambda, \theta | y_i)}{d\beta} = 0, \frac{dl(\alpha, \beta, \lambda, \theta | y_i)}{d\lambda} = 0, \text{ and } \frac{dl(\alpha, \beta, \lambda, \theta | y_i)}{d\theta} = 0.$

The expressions are solved simultaneously to give the maximum likelihood estimates of the parameters. Since the solutions cannot be obtained in closed forms, numerical methods can be used to obtain the estimates using software like R, MAPLE, and so on. In particular, R software was used in this research to obtain the parameter estimates using iterative methods and with the aid of datasets.

3. Applications

To demonstrate the potentials of the TTLFW distribution, two different datasets are used and models like the Exponentiated Flexible Weibull (EFW), Weibull Flexible Weibull (WFW), Kumaraswamy Flexible Weibull (KuFW), Beta Flexible Weibull (BFW), Gamma Flexible Weibull (GaFW) and Exponentiated Generalized Flexible Weibull (EGFW) distributions are used as the basis for comparison.

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The following selection criteria; Negative Likelihood function (NLL), Bayesian Information Criteria (BIC), Akaike Information Criteria (AIC), Consistent Akaike Information Criteria (CAIC), Hannan and Quinn Information Criteria (HQIC) are used to determine the model with the best fit. The Saphiro Wilk (W) statistic, Kolmogorov Smirnov (KS) statistic, and the Anderson Darling statistic are also obtained. It is preferable to have low values for these selection criteria; therefore, the probability model with the lowest value for these criteria is selected as the distribution with the best fit. R software was used for all the analyses.

First Data: The dataset on time to failure for 40 suits of turbochargers in diesel engines was considered. [19] has previously used the dataset, the observations are:

1.6, 2.0, 2.6, 3.0, 3.5, 3.9, 4.5, 4.6, 4.8, 5.0, 5.1, 5.3, 5.4, 5.6, 5.8, 6.0, 6.0, 6.1, 6.3, 6.5, 6.5, 6.7, 7.0, 7.1, 7.3, 7.3, 7.3, 7.7, 7.7, 7.8, 7.9, 8.0, 8.1, 8.3, 8.4, 8.4, 8.5, 8.7, 8.8, 9.0

The result corresponding to the time to failure dataset is presented in table 1.

Model	Estimates	NLL	AIC	CAIC	BIC	HQIC
TTLFW	$\hat{\alpha} = 0.1517$	78.95826	165.9165	167.0594	172.672	168.3591
	$\hat{\lambda} = 0.577$					
	$\hat{\theta} = 0.6165$					
	$\hat{\beta} = 48.938$					
EFW	$\hat{\alpha} = 14.49$	83.300	172.601	173.268	177.668	174.433
	$\hat{\lambda} = 0.182$					
	$\hat{\theta} = 0.0806$					
WFW	$\hat{\alpha} = 1.6144$	79.790	167.584	168.724	174.336	170.024
	$\hat{\lambda} = 10.67$					
	$\hat{\theta} = 0.361$					
	$\hat{\beta} = 1.425$					
KuFW	$\hat{\alpha} = 17.774$	81.100	170.200	171.343	176.955	172.642
	$\hat{\lambda} = 24.933$					
	$\hat{\theta} = 0.086$					
	$\hat{\beta} = 0.1653$					
BFW	$\hat{\alpha} = 34.432$	82.714	173.425	174.564	180.181	175.686
	$\hat{\lambda} = 6.397$					
	$\hat{\theta} = 0.1019$					
	$\hat{\beta} = 0.0187$					
GaFw	$\hat{\alpha} = 4.7806$	81.339	168.675	169.346	173.746	170.511
	$\hat{\lambda} = 0.2434$					
	$\hat{\theta} = 0.3497$					
EGFW	$\hat{\alpha} = 0.10\overline{22}$	80.706	169.412	170.555	176.162	171.855
	$\hat{\lambda} = 1.9209$					
	$\hat{\theta} = 0.4057$					
	$\hat{\beta} = 0.9977$					

Table 1: Estimates, NLL, AIC, CAIC, BIC, and HQIC for the time-to-failure data

The result for the goodness of fit test is presented in table 2.

Models	W – Statistic	A – Statistic	KS – Statistic	K S p – value
TTLFW	0.0222	0.1880	0.0942	0.8690
EFW	0.0918	0.6668	0.1071	0.7481
WFW	0.0313	0.2363	0.0910	0.8645
KuFW	0.0498	0.3789	0.0979	0.8372
BFW	0.0778	0.5742	0.1050	0.7697
GaFW	0.0536	0.4071	0.0999	0.8192
EGFW	0.0444	0.3385	0.0907	0.8970

Table 2: The W, A, KS Statistics, and p-value using the time-to-failure data

The histogram plot for the data set with the competing models is displayed in figure 4 while the empirical cdf (ecdf) plot is presented in figure 5.



Figure 4: The histogram plot for the time-to-failure dataset



Figure 5: The ecdf plot for the time-to-failure dataset

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Second Data: The dataset on the exceedances of the Wheatson river was also considered. The dataset has been analyzed previously by [20]. The observations are: 1.7, 1.4, 0.6, 9.0, 5.6, 1.5, 2.2, 18.7, 2.2, 1.7, 30.8, 2.5, 14.4, 8.5, 39.0, 7.0, 13.3, 27.4, 1.1, 25.5, 0.3, 20.1, 4.2, 1.0, 0.4, 11.6, 15.0, 0.4, 25.5, 27.1, 20.6, 14.1, 11.0, 2.8, 3.4, 20.2, 5.3, 22.1, 7.3, 14.1, 11.9, 16.8, 0.7, 1.1, 22.9, 9.9, 21.5, 5.3, 1.9, 2.5, 1.7, 10.4, 27.6, 9.7, 13.0, 14.4, 0.1, 10.7, 36.4, 27.5, 12.0, 1.7, 1.1, 30.0, 2.7, 2.5, 9.3, 37.6, 0.6, 3.6, 64.0, 27.0. The result corresponding to the exceedances of the Wheatson river dataset is presented in table 3.

dataset						
Model	Estimates	NLL	AIC	CAIC	BIC	HQIC
TTLFW	$\hat{\alpha} = 4.231$	252.018	512.0	512.6	521.1	515.6
	$\hat{\lambda} = 0.883$					
	$\hat{ heta} = 0.025$					
	$\hat{\beta} = 0.243$					
EFW	$\hat{\alpha} = 2.215$	253.786	513.7	514.1	522.5	516.8
	$\hat{\lambda} = 0.035$					
	$\hat{\theta} = 0.475$					
WFW	$\hat{\alpha} = 1.333$	254.367	516.7	517.3	525.8	520.3
	$\hat{\lambda} = 1.554$					
	$\hat{\theta} = 0.033$					
	$\hat{\beta} = 0.740$					
KuFW	$\hat{\alpha} = 4.252$	253.757	515.1	515.8	524.8	518.8
	$\hat{\lambda} = 2.727$					
	$\hat{\theta} = 0.024$					
	$\hat{\beta} = 0.2.63$					
BFW	$\hat{\alpha} = 2.457$	253.876	515.7	516.4	524.8	519.4
	$\hat{\lambda} = 1.097$					
	$\hat{\theta} = 0.034$					
	$\hat{\beta} = 0.432$					
GaFw	$\hat{\alpha} = 1.596$	254.153	514.3	514.6	521.1	517.0
	$\hat{\lambda} = 0.037$					
	$\hat{\theta} = 0.636$					
EGFW	$\hat{\alpha} = 1.045$	253.877	515.7	516.3	524.8	519.5
	$\hat{\lambda} = 2.370$					
	$\hat{\theta} = 0.035$					
	$\hat{\beta} = 0.446$					

 Table 3: Estimates, NLL, AIC, CAIC, BIC and HQIC for the exceedances of the Wheatson river

 dataset

The result for the goodness of fit test is presented in table 4.

Table 4: The W, A, KS and	p - value using the e	exceedances of the	Wheatson river dataset
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Models	W – Statistic	A – Statistic	K S – Statistic	K Sp – value
TTLFW	0.0845	0.6457	0.0816	0.7229
EFW	0.1259	0.9205	0.0964	0.5140
WFW	0.1516	1.0732	0.1040	0.4172

KuFW	0.1153	0.8482	0.0959	0.5217
BFW	0.1242	0.9098	0.0966	0.5119
GaFW	0.1396	1.0033	0.0989	0.4808
EGFW	0.1245	0.9124	0.0969	0.5084

The histogram plot for the data set with the competing models is displayed in figure 6 while the empirical cdf (ecdf) plot is presented in figure 7.



Figure 6: Histogram of the exceedances of the Wheatson river dataset with the competing models



Figure 7: The ecdf plot of the exceedances of the Wheatson river dataset with the competing models

Clearly, all the two applications provided indicate that the TTLFW distribution is better than the other models based on the values of the criteria used. It has lower values for the NLL, AIC, CAIC, BIC, and HQIC. The histogram and ecdf plots also support all the results presented in tables 1 to 4.

4. Conclusion

The Transmuted Topp-Leone Flexible Weibull (TTLFW) distribution has been successfully derived in this research. It's properties have been established and applications to two real datasets have been provided. The TTLFW distribution exhibits decreasing and unimodal shapes while its failure rate exhibits increasing, unimodal and uni-antimodal shapes. The applications provided reveal that the TTLFW distribution is a good competitor as it performs creditably well than the Exponentiated Flexible Weibull, Weibull Flexible Weibull, Kumaraswamy Flexible Weibull, Beta Flexible Weibull, Gamma Flexible Weibull, and Exponentiated Generalized Flexible Weibull distributions under the considered selection criteria.

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