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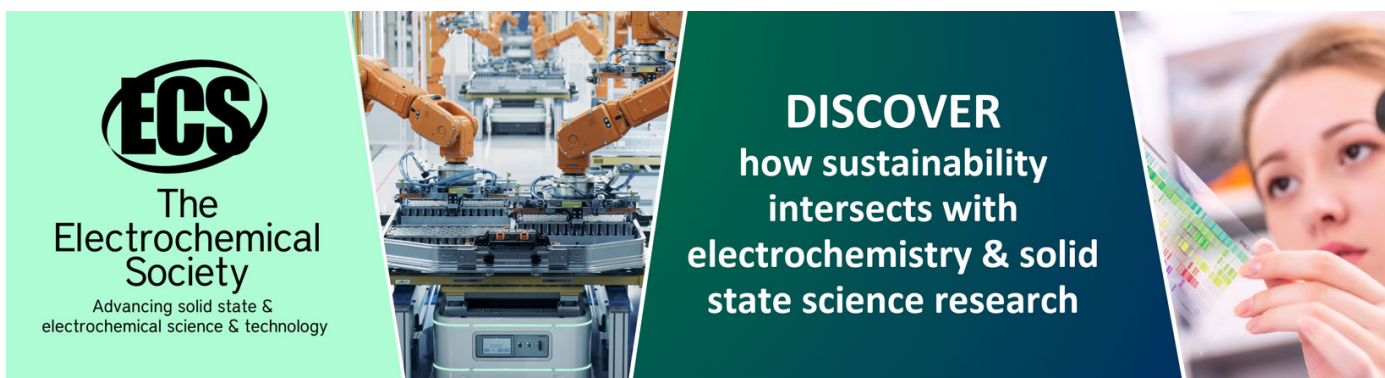
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Effect of time delay on the generation of oscillation in a single degree-of-freedom mass-spring-damper system by nonlinear velocity feedback

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Abstract. The present paper investigates the effect of time delay in a particular type of single degree-of-freedom self-excited oscillator. The self-excited vibration is generated in the system by using linear velocity feedback (to destabilize the static equilibrium of the system) with a nonlinear Rayleigh type feedback (to limit the growth of the instability into a stable limit cycle). The general method of describing function is employed to study the dynamics with the presence of time delay. Also, the analytical results are verified with the simulation result. Without time delay, the control law can generate a stable limit cycle with the proper choice of control parameters. However, the presence of time delay introduces a globally unstable limit cycle in the system with a stable one. Though the amplitude of the stable limit cycle dies down with the increase of time delay and finally vanishes by stabilizing the static equilibrium of the system. The effect of control parameters is also studied.

1. Introduction

Most of the industrial machines like turbine, generator, flywheel etc. produce self-excited vibration during their operation due to dry friction. Cable used in suspension bridge, power transmission line etc. produce self-oscillation due to flow of wind which may fail catastrophically due to fatigue. This vibration is usually detrimental by nature, and several research efforts have been made to control and eliminate this kind of vibration. However, over the last two or three decades, scientists have acknowledged the utilization of artificially generated self-excited vibration in several mechanical and micro-mechanical system like mass sensing [1-6], atomic force microscopy [7-9], pipe crawling microrobot [10], flutter wing mechanism [11], bio-sensor [12] etc. As a result, a number of research works have been carried out to generate the self-excited oscillation in several mechanical and micromechanical system. A few to mention are given below.

Sensors which are based on vibration are preferred for monitoring of system because they provide an instant and fast response of the system continuously. Mass sensing in viscous environment is a challenging research problem. Yabuno et al. [5, 6] have developed Self-excited coupled cantilevers which can be utilized even in highly viscous damping environments for mass sensing. They have used the eigenstate shift approach in almost two identical and weakly coupled cantilevers in-place of eigenfrequency shift approach. Yanagisawa & Yabuno [13] have investigated a method for mass sensing in viscous environments using self-excited oscillation. Recently, Yabuno et al. [1] have developed mass sensor using a virtual cantilever coupled with a real cantilever which is based on the eigenmode shift of coupled cantilevers that provide much higher sensitivity than previously used natural frequency shift method.

Miyaki & Tsukagoshi [14] have synthesized a compact valve for switching the pressure modes of two chambers by self-excited vibration without electricity.

Endo et al. [15] have described ultrasensitive mass detection technique using self-excited coupled microcantilevers which can measure a small mass with very high accuracy.



Kwasniewski et al. [16] have investigated a technique of measuring stress in metal by Self-oscillating Acoustical System, which can be used for determining the state of construction in a faster and accurate way.

Aguilar et al. [17] have worked on a new technique to generate self-excited oscillations to drive two degrees-of-freedom underactuated mechanical systems with the help of a variable structure controller.

The present research investigates the effect of time delay in the nonlinear control utilized to generate self-excited oscillation in single degree-of-freedom mass-spring-damper system. Rayleigh type nonlinear feedback mechanism is utilized for generating self-excited oscillation. The presence of time delay in any active control feedback system is inevitable. Therefore, the effect of time delay is analyzed in the present work. This research primarily focuses on analyzing the behavior of the limit cycle, frequency, amplitude response of the oscillation with the control parameters and time delay.

2. Mathematical Model

The single degree of freedom(1-DOF) spring mass damper system is considered for the present study, as shown in Fig. 1. This system consists of spring, mass and damper in which nonlinear velocity feedback with time delay is acting. Spring and damper have spring constant (k) and damping coefficient (c) respectively. A sensor is attached to mass m which give velocity signal to the controller and finally the feedback control force (F_c) is applied by the actuator on the mass (m) to generate self-oscillation in the system. The main aim to study this system is to analyze the effect of time delay in the feedback on the generation of oscillation.

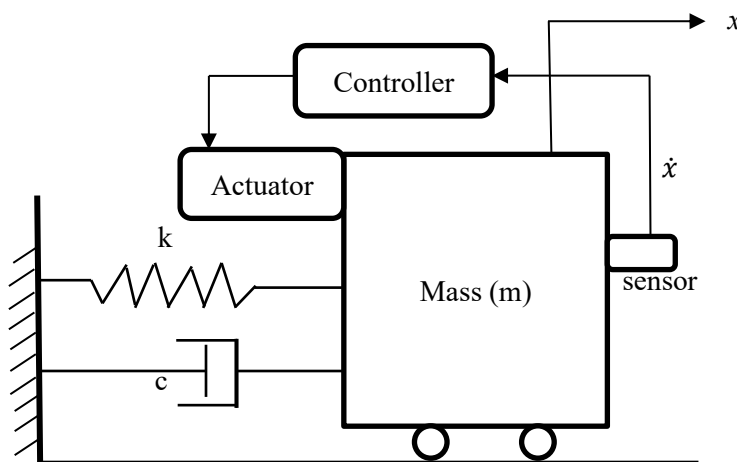


Figure 1. Schematic representation of the mass-spring-damper mechanical system with control

To this end, one can write down the governing equations of motion as follows

$$m\ddot{x} + c\dot{x} + kx = F_c, \quad (1)$$

where x denotes the displacement of the mass m . The prime denotes the differentiation of x with respect to time t . The control force F_c has a linear part (used to destabilize the static equilibrium) and a nonlinear part (to limit the amplitude growth into a limit cycle). The nonlinear part is Rayleigh type, i.e., cubic feedback of filter velocity. Malas and Chatterjee [18] have utilized a similar type of control but without time delay to generate self-excited oscillation in two degrees-of-freedom system. The expression of F_c is given by

$$F_c = \hat{k}_c x'(t - \hat{T}) + \hat{\beta} x'^3(t - \hat{T}), \quad (2)$$

where $\hat{T}$ is the time delay. The term $\hat{k}_c$ and $\hat{\beta}$ are the linear and nonlinear control gains, respectively. Eq. (1) can be replaced by the following non-dimensional form.

$$\ddot{y} + 2\xi\dot{y} + y = f_c, \quad (3)$$

where $y = x/x_0$ is the non-dimensional displacement. $\xi = \frac{c}{2m\omega_n}$ is the damping factor. $\omega_n = \sqrt{k/m}$ is the natural frequency of the system and x_0 is any arbitrary reference length quantity. The nondimensional control is $f_c = k_c\dot{y}(\tau - T) + \beta\dot{y}^3(\tau - T)$. The non-dimensional control parameters are expressed as $k_c = \hat{k}_c/m\omega_n$ and $\beta = \hat{\beta}x_0\omega_n/m$. T is the nondimensional time delay. The 'dot' in the above equation denotes the differentiation with respect to the non-dimensional time $\tau = \omega_n t$.

3. Linear Stability Analysis

In general, the first necessary criteria to generate the self-excited oscillation is to destabilize the static equilibrium of the system. In this section, a linear stability analysis of the static equilibrium has been studied so that one can select the proper value of linear control gain to generate self-oscillation in the system.

To this end, one can linearize the system in the following form

$$\ddot{y} + 2\xi\dot{y} + y = k_c\dot{y}(\tau - T). \quad (4)$$

By taking the Laplace transform of Eq. (4)

$$s^2 + 2\xi s + 1 = k_c s e^{-sT} \quad (5)$$

By substituting $s = j\omega$ in Eq. (5) and separating real and imaginary part, one finally obtains

$$\omega^4 + (4\xi^2 - 2 - k_c^2)\omega^2 + 1 = 0, \quad (6)$$

and

$$\tan(\omega T) = \frac{1 - \omega^2}{2\xi\omega}. \quad (7)$$

Eq. (6) is a fourth order equation of ω which has four roots. Among these four roots, one obtains two roots are real non-negative being critical frequency ($\omega_c = \omega_{c1}, \omega_{c2}$). The corresponding critical value of time delay ($T_c = T_{c1}, T_{c2}$) can be computed from Eq. (7).

$$T_c = \frac{1}{\omega_c} \left[\tan^{-1}\left(\frac{1 - \omega_c^2}{2\xi\omega_c}\right) + 2\pi n \right], \quad (8)$$

where $n=0,1,2,\dots$

For the stability of the equilibrium to switch at the critical values computed above, the eigenvalues must cross the imaginary axis with increasing value of T . Therefore, the sign of the following quantity must also be checked:

$$\begin{aligned} R(\omega_c, T_c) &= \text{sign of} \left[\text{Re} \left(\frac{ds}{d\tau} \right)_{(j\omega_c, T_c)} \right] \\ &= \text{sign of} \left\{ \begin{aligned} &(2\xi + k_c - k_c \cos(\omega_c T_c) + k_c \omega_c T_c \cos(\omega_c T_c)) k_c \omega_c^2 \cos(\omega_c T_c) \\ &+ (2\omega_c + k_c \omega_c T_c \cos(\omega_c T_c) + k_c \sin(\omega_c T_c)) k_c \omega_c^2 \sin(\omega_c T_c) \end{aligned} \right\} \end{aligned} \quad (9)$$

In Eq. (9), $\operatorname{Re}\left(\frac{ds}{d\tau}\right)_{(j\omega_c, T_c)}$ denotes the real value of $R(\omega_c, T_c)$ at critical value of ω_c . Stability and

instability can be examined by the sign of R in the Eq. (9). The equilibrium of the system becomes stable if $R < 0$ and unstable if $R > 0$ for the increasing critical value of T_c .

By utilizing the abovementioned procedure, the unstable region of the static equilibrium for a particular system is shown in Figure 2. To generate a stable limit cycle, one has to choose the linear control gain from the unstable region (white region). However, time delay has an effect on the stability of the static equilibrium. It can be seen that the static equilibrium can be destabilized for positive value of linear control gain with the value of time delay up to one. The stable region of the static equilibrium (shaded region) corresponds to the absence of self-excited oscillation. However, the linear stability analysis predicts an approximate nature about the presence of the limit cycle in the system. The true nature of the oscillation can only be obtained by the nonlinear analysis given in the next section.

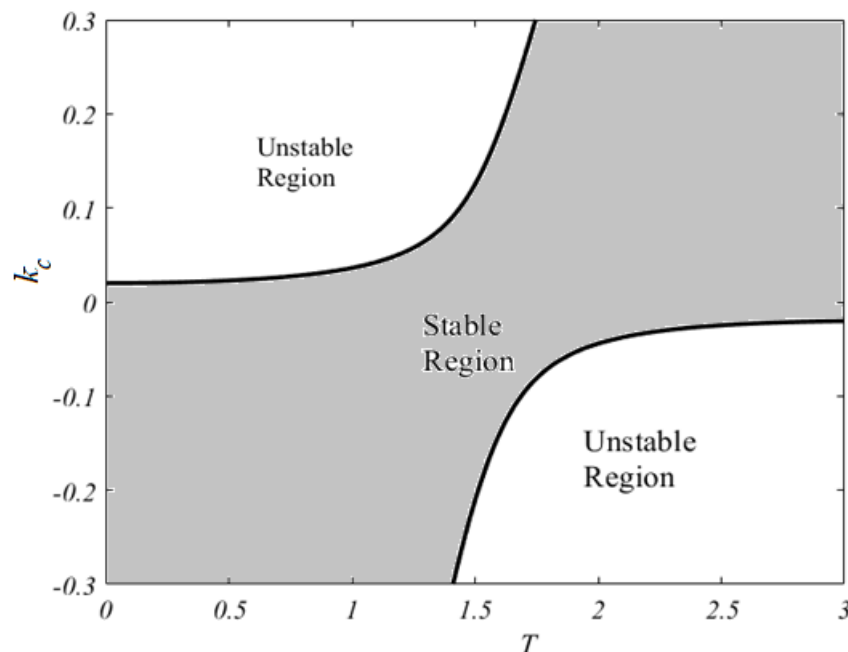


Figure 2. Nature of the static equilibrium of the system with the variation of linear control gain and time delay. $\xi = 0.01$.

4. Nonlinear Analysis

In this section, the describing function method is employed to study the nonlinear dynamics of the system with delayed control. Describing functions method is one of the effective methods for solving nonlinear system, and it can be used for the approximate analysis and find out the prediction of nonlinear behaviour of the nonlinear system.

To this end, one can seek the harmonic solution of the system as

$$y = A \sin(\omega t), \quad (10)$$

where ω and A is the frequency and amplitude of the oscillation, respectively.

According to the general formalism of describing function method, one can represent Eq. (3) in the frequency domain with a linear part $L(s)$ and a describing function of nonlinear part $N(A, \omega)$ as shown in Figure 3. The expression of $L(s)$ and $N(A, \omega)$ is given by

$$L(s) = \frac{1}{s^2 + 2\xi s + 1} \quad (11)$$

and

$$N(A, \omega) = \frac{\omega}{4} (3\beta A^2 \omega^2 + 4k_c) (\sin(\omega T) + j \cos(\omega T)), \quad (12)$$

where s is the standard Laplace variable.

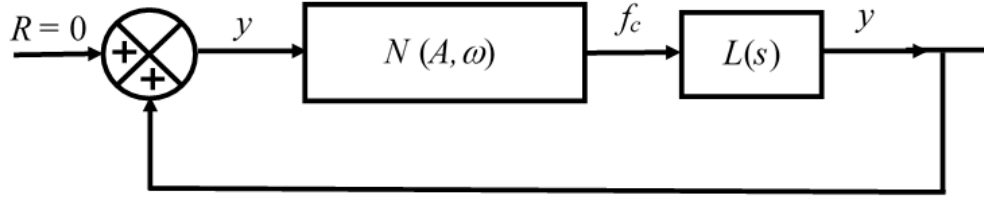


Figure3. Block diagram of the controlled nonlinear system

The characteristic equation of the system can be obtained from the block diagram as

$$1 - NL = 0, \quad (13a)$$

which can be further recast as

$$u + jv = 0, \quad (13b)$$

where $u = 1 - \omega^2 - \frac{1}{4}(3\beta A^2 \omega^3 + 4k_c \omega) \sin(\omega T)$ and $v = 2\xi \omega - \frac{1}{4}(3\beta A^2 \omega^3 + 4k_c \omega) \cos(\omega T)$

Equating real and imaginary part of Eq. (13b), one obtains the following frequency and amplitude equation

$$(1 - \omega^2) \cos(\omega T) - 2\xi \omega \sin(\omega T) = 0, \quad (14)$$

and

$$A = \frac{4}{3\beta \omega^2} \left(\frac{2\xi}{\cos(\omega T)} - k_c \right). \quad (15)$$

The stability of the oscillation can be ascertained by the positive definiteness of the following quantity.

$$S = \frac{\partial u}{\partial A} \frac{\partial v}{\partial \omega} - \frac{\partial u}{\partial \omega} \frac{\partial v}{\partial A}. \quad (16)$$

Without time delay, Eq. (14) to (15) can be rearranged as

$$\omega = 1, \quad (17)$$

$$A = \frac{4}{3\beta} (2\xi - k_c), \quad (18)$$

and

$$S = -3\beta A. \quad (19)$$

It is obvious from Eq. (17) that the system can always be excited at the natural frequency by utilizing the control without time delay. However, the existence condition of the oscillation from Eq. (17) is given by $k_c > 2\xi$ and $\beta < 0$. Further, the stability condition can be obtained from Eq. (19) as $\beta < 0$.

The effect of time delay on the system dynamics is numerically studied in the next section.

5. Results and Discussions

In this section, the analytical result obtained by describing function method is numerically studied and compared with the simulation result. The system damping for the numerical study is chosen as $\xi = 0.01$.

The variation of amplitude with the linear control gain (k_c) is depicted in Figure. 4(a) without time delay, whereas Figure. 4(b) corresponds to the presence of time delay in the control. It can be seen from Figure. 4(b) that the time delay introduces another unstable limit cycle with a stable one and the amplitude of the unstable limit cycle is independent of k_c . Figure 4(c) corresponds to the frequency variation with k_c for the same values of parameters.

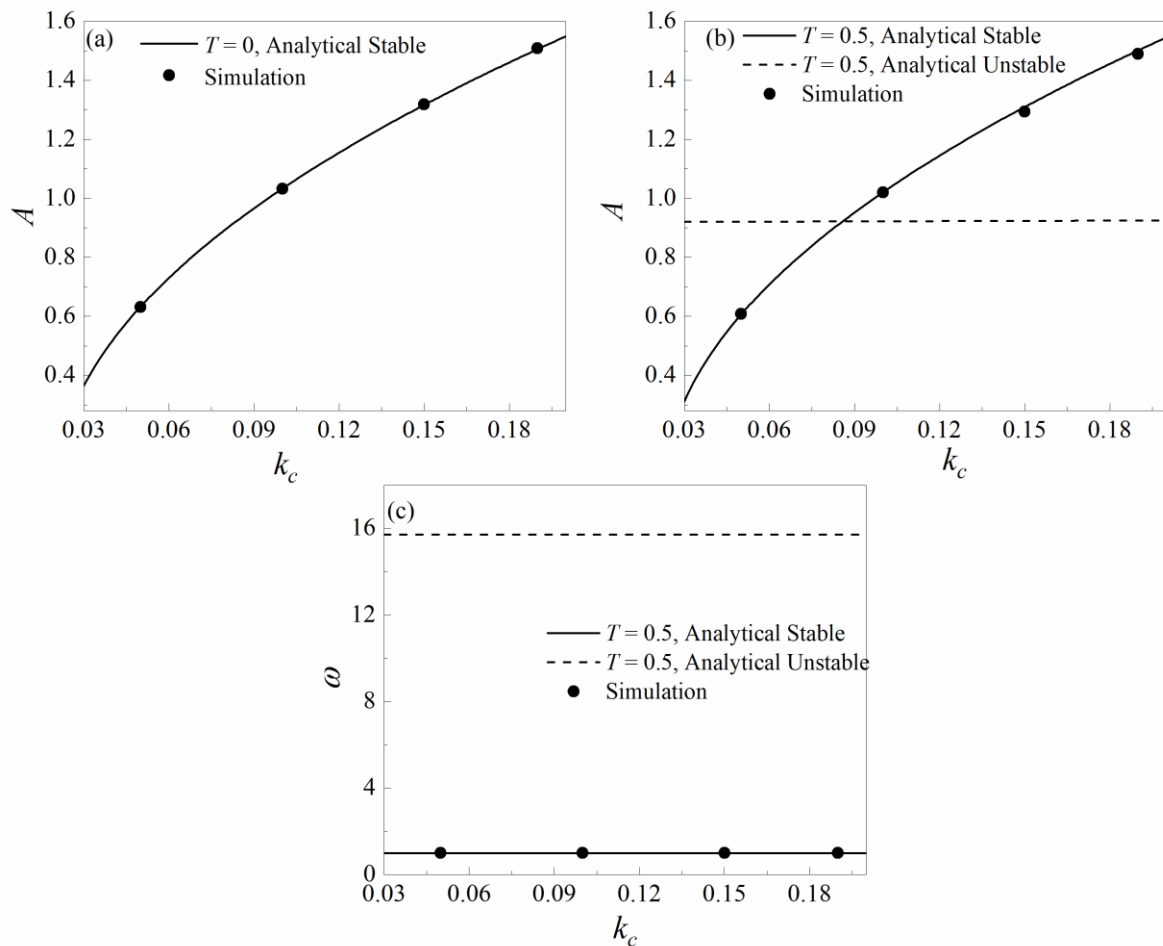


Figure 4. Variation of amplitude and frequency of oscillation with linear control gain k_c .

$$\xi = 0.01; \beta = -0.1$$

The effect of time delay on the oscillation of the system is depicted in Figure. 5. It is evident from Figure 5 that the time delay introduces a globally unstable limit cycle with a stable limit cycle. However, after a particular value of time delay, the stable limit cycle vanishes, and the static equilibrium becomes stable. At the same time, the frequency of unstable limit cycle captures the natural frequency of the system. Figure. 6 shows numerically simulated time history plot of the stable limit cycle for different value of time delay. The result obtained analytically matches very closely with simulation results.

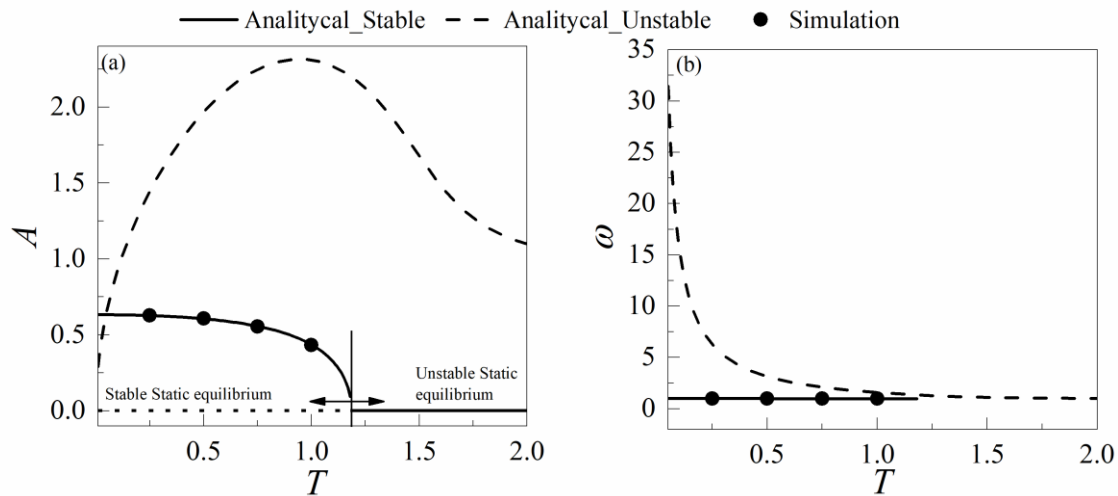


Figure 5. Variation of amplitude and frequency of oscillation with time delay T . $k_c = 0.05$; $\xi = 0.01$; $\beta = -0.1$.

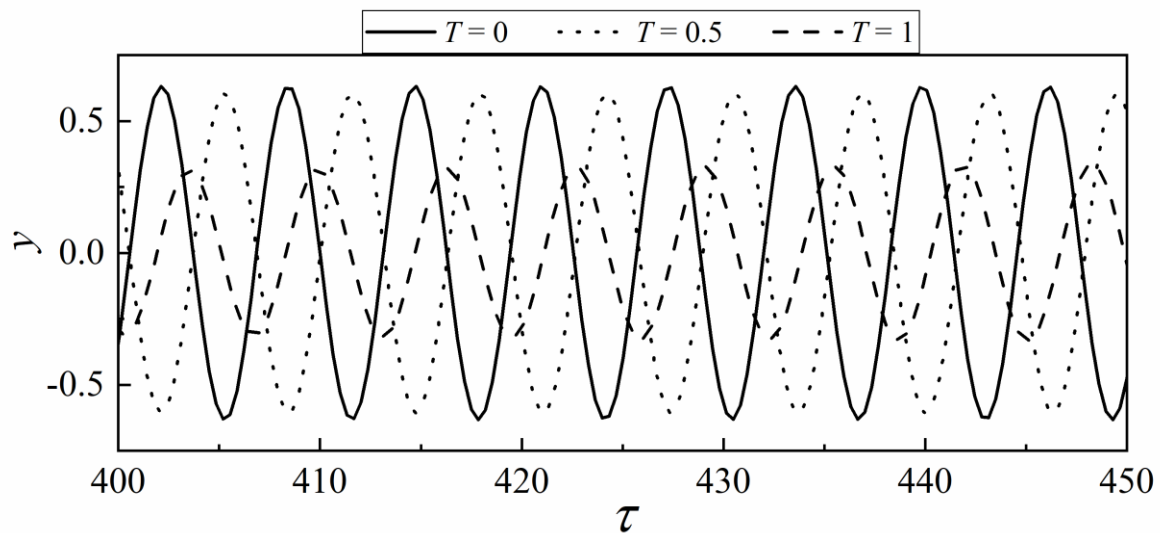


Figure 6. Numerically simulated time history plot of the displacement of the system. $k_c = 0.05$; $\xi = 0.01$; $\beta = -0.1$.

6. Conclusions

The present paper discusses the effect of time delay in a type of single degree-of-freedom self-excited oscillator. The control without time delay exhibits the presence of a stable limit cycle oscillation in the system. However, an unstable limit cycle enters in the dynamics with a stable limit cycle in the presence of time delay. Again, the stable limit cycle is destroyed for high value of time delay. Therefore, it can be concluded that the system can be excited for a small value of time delay with appropriate selection of initial conditions as an unstable limit cycle parallelly presents in the system. Though the presence of time delay in the control fails to produce the desired effect properly (i.e., generating self-excited oscillation) in the system till the present analysis motivates to find out an alternative way to eliminate the undesired unstable limit cycle in the system in further study.

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