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ON DIRECT SUM OF FOUR FUZZY GRAPHS

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ON DIRECT SUM OF FOUR FUZZY GRAPHS

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Abstract: The graph G_1, G_2, G_3 and G_4 which is defined by the direct sum of four fuzzy graphs $G_1 \oplus G_2 \oplus G_3 \oplus G_4$. This is also proved the effective values. The degree of the vertices is $G_1 \oplus G_2 \oplus G_3 \oplus G_4$ is calculated with the establishment of the regular property and the Connectedness of the Direct Sum of four Fuzzy Graphs.

Keywords: Fuzzy graph, Degree of vertices in the direct sum, Regular fuzzy graphs, connected fuzzy graphs and Effective fuzzy graphs.

1. INTRODUCTION

The concept of fuzzy graphs was established by A. Rosenfeld in 1975 [6]. Mordeson .J.N and Peng. S [2] were developed some operations on fuzzy graphs. Further Bhattacharya [1] discussed about the remarks of fuzzy graphs. Also, Dr. K. Radha and Mrs. Arumugam [7] asserted the connectedness and regular properties of direct sum of two fuzzy graphs. Similarly, the direct sum of two fuzzy graphs was extended to three fuzzy graphs in T. Henson and N. Devi [3].

By using numerical example, can be calculated the direct sum of four fuzzy graphs with the degree of nodes. In this whole article V is a fuzzy subset of σ and μ is a symmetric fuzzy relation on σ was represented. In addition, also, with the help of numerical example direct sum four fuzzy graphs of Regularness, Connectedness and Effectiveness of four fuzzy graphs were checked in this paper below.

2. PRELIMINARIES

Let G: (σ, μ) be a fuzzy graph on G^*: (V, E), then the following graphs arrives.

2.1. Definition

The valency of is x defined as $d_G(\mathrm{x}) = \sum_{x \neq y} \mu(xy)$, and if each vertex with same degree K, and if dG (x) = K for every x and y then the graph is said to be a regular fuzzy graph of degree K [4].

2.2. Definition

If every pair of vertices is connected by an edge then the graph is a connected fuzzy graph [4].

3. Direct sum

Let G_1: (σ_1, μ_1), G_2: (σ_2, μ_2), G_3: (σ_3, μ_3), and G_4: (σ_4, μ_4) denote four fuzzy graphs with underlying crisp graphs G_1^*: (V_1, E_2), G_2^*: (V_2, E_2) [7], G_3^*: (V_3, E_3) [3] and G_4^*: (V_4, E_4) respectively.

Let $V = V_1 \cup V_2 \cup V_3 \cup V_4$ and

$E = E_1 \cup E_2 \cup E_3 \cup E_4$

Define: $G_1 \oplus G_2 \oplus G_3 \oplus G_4 = G: (\sigma, \mu)$ by

$$\sigma(x) = \begin{cases} \sigma_1(x_1) \text{ if } x \in V_1 \\ \sigma_2(x_2) \text{ if } x \in V_2 \\ \sigma_3(x_3) \text{ if } x \in V_3 \\ \sigma_4(x_4) \text{ if } x \in V_4 \end{cases}$$

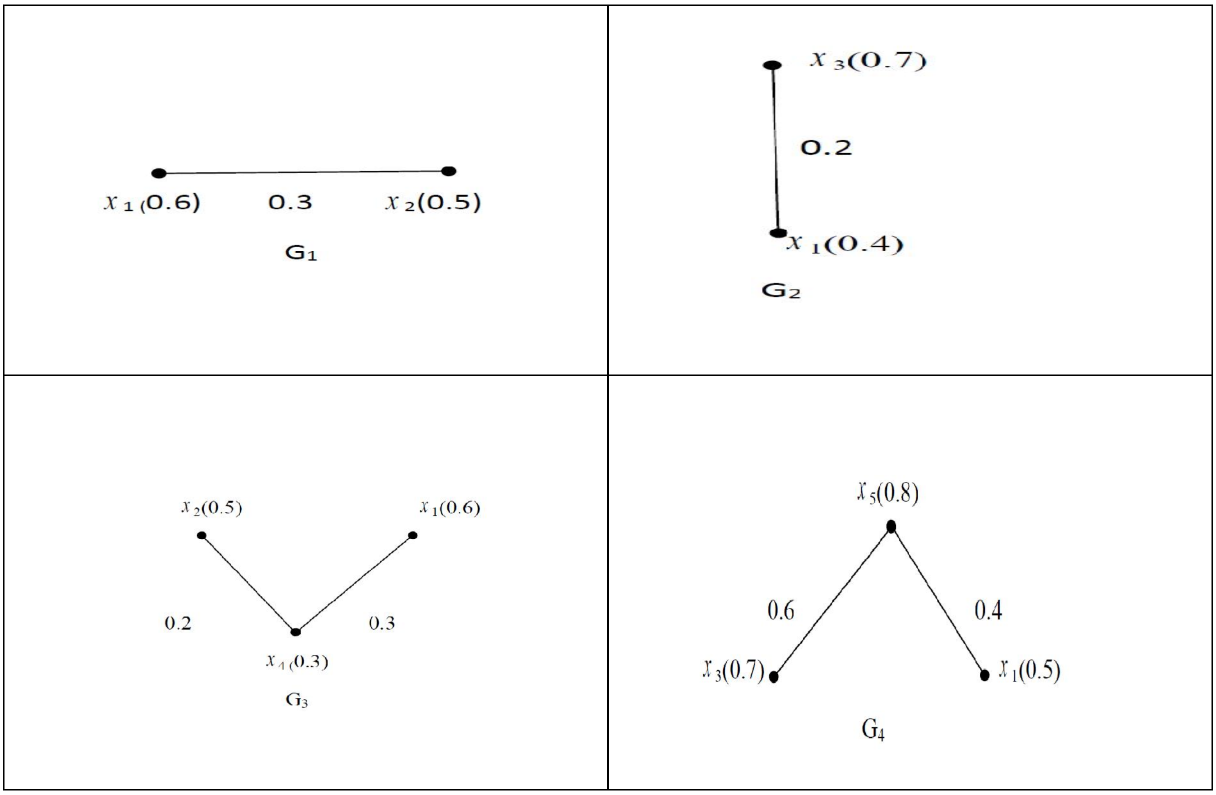
$$\sigma_1(x_1) \cup \sigma_2(x_2) \cup \sigma_3(x_3) \cup \sigma_4(x_4)$$

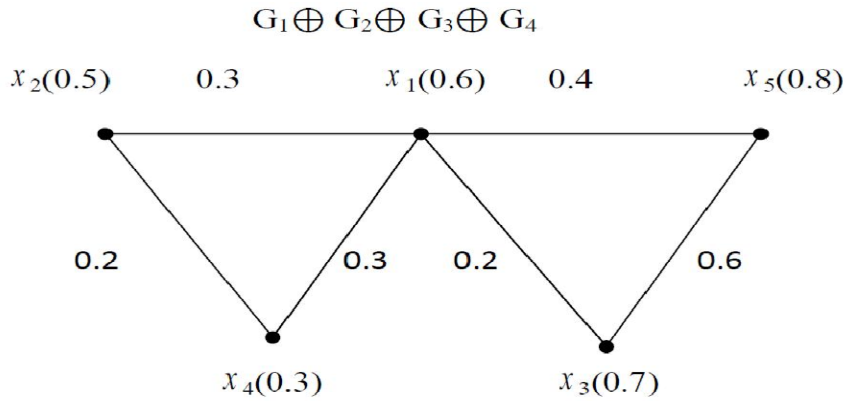
If $x \in V_1 \cup V_2 \cup V_3 \cup V_4$

$$\mu(E) = \begin{cases} \mu_1(E_1) \le \min(\sigma_1) \text{ where } \sigma_1 \in V_1 \\ \mu_2(E_2) \le \min(\sigma_2) \text{ where } \sigma_2 \in V_2 \\ \mu_3(E_3) \le \min(\sigma_3) \text{ where } \sigma_3 \in V_3 \\ \mu_4(E_4) \le \min(\sigma_4) \text{ where } \sigma_4 \in V_4 \end{cases}$$

Therefore $G_1 \oplus G_2 \oplus G_3 \oplus G_4 = G: (\sigma, \mu)$ is the direct sum of four fuzzy graphs [3].

3.1 Example:





Direct sum of four fuzzy graphs

Fig (1)

4. THE DIRECT SUM OF FOUR FUZZY GRAPHS IN THE VALENCY OF NODES

4.1 Theorem: -

Find the valency of nodes in the direct sum of four fuzzy graphs in term of the valency of the node in G_1, G_2, G_3, and G_4 is given by

$$D_{G1\oplus G2\oplus G3\oplus G4}(x) = \begin{cases} D_{G1}(x), \text{ if } x \in V_1 \\ D_{G2}(x), \text{ if } x \in V_2 \\ D_{G3}(x), \text{ if } x \in V_3 \\ D_{G4}(x), \text{ if } x \in V_4 \end{cases}$$

$$D_{G1}(x) + D_{G2}(x) + D_{G3}(x) + D_{G4}(x),$$

$$\text{If } x \in V_1 \cap V_2 \cap V_3 \cap V_4 \text{ and}$$

$$E_1 \cap E_2 \cap E_3 \cap E_4 = \phi$$

Proof:

In $G_1 \oplus G_2 \oplus G_3 \oplus G_4$, for any vertex we have two cases,

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Case (i):-

If $x \in v_1$ or $x \in v_2$ or $x \in v_3$ or $x \in v_4$ then the edge incident at 'x' lies in $E_1 \cap E_2 \cap E_3 \cap E_4$.

$$(\mu_1 \oplus \mu_2 \oplus \mu_3 \oplus \mu_4)(E) = \begin{cases} \mu_1\,(xy) \text{ if } x \in v_1,\ xy \in E_1 \\ \mu_2\,(xy) \text{ if } x \in v_2,\ xy \in E_2 \\ \mu_3\,(xy) \text{ if } x \in v_3,\ xy \in E_3 \\ \mu_4\,(xy) \text{ if } x \in v_4,\ xy \in E_4 \end{cases}$$

Hence,

If $x \in v_1$ then $D_{G1 \oplus G2 \oplus G3 \oplus G4}(x) = \Sigma_{E1}\, \mu\,(E_1) = D_{G1}(x)$

If $x \in v_2$ then $D_{G1 \oplus G2 \oplus G3 \oplus G4}(x) = \Sigma_{E2}\, \mu\,(E_2) = D_{G2}(x)$

If $x \in v_3$ then $D_{G1 \oplus G2 \oplus G3 \oplus G4}(x) = \Sigma_{E3}\, \mu\,(E_3) = D_{G3}(x)$

If $x \in v_4$ then $D_{G1 \oplus G2 \oplus G3 \oplus G4}(x) = \Sigma_{E4}\, \mu\,(E_4) = D_{G4}(x)$

Case (ii): -

If $x \in v_1 \cap v_2 \cap v_3 \cap v_4$ then there is no edge incident to x on $E_1 \cap E_2 \cap E_3 \cap E_4$ but it lies in E_1 or E_2 or E_3 or E_4.

Hence,

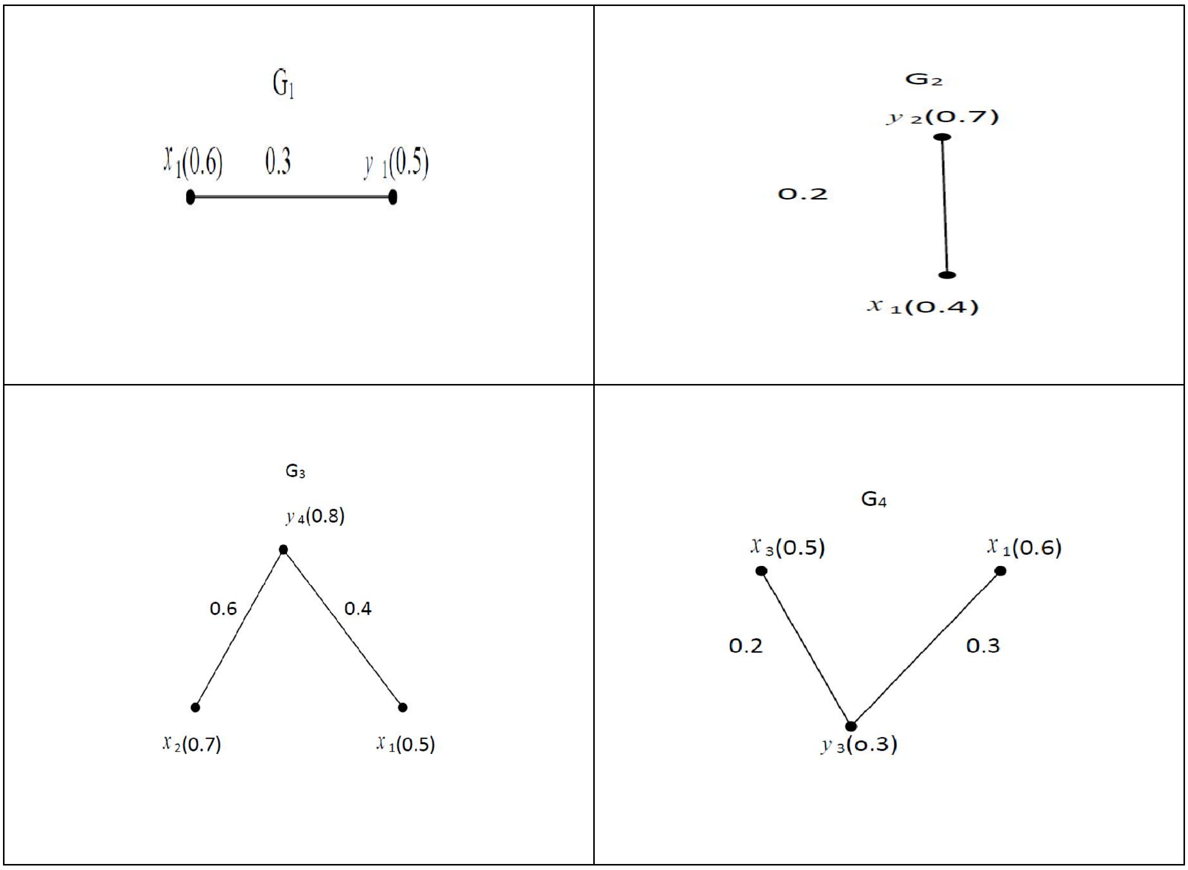
The valency of x in $G_1 \oplus G_2 \oplus G_3 \oplus G_4$ is given by

$$D_{G1 \oplus G2 \oplus G3 \oplus G4}(x) = \Sigma_E\,(\mu_1 \oplus \mu_2 \oplus \mu_3 \oplus \mu_4)\,(E)$$

$$= \Sigma_{E1}\, \mu\,(E_1) + \Sigma_{E2}\, \mu\,(E_2) + \Sigma_{E3}\, \mu\,(E_3) + \Sigma_{E4}\, \mu\,(E_4)$$

$$= D_{G1}(x) + D_{G2}(x) + D_{G3}(x) + D_{G4}(x)$$

4.1. Example: -



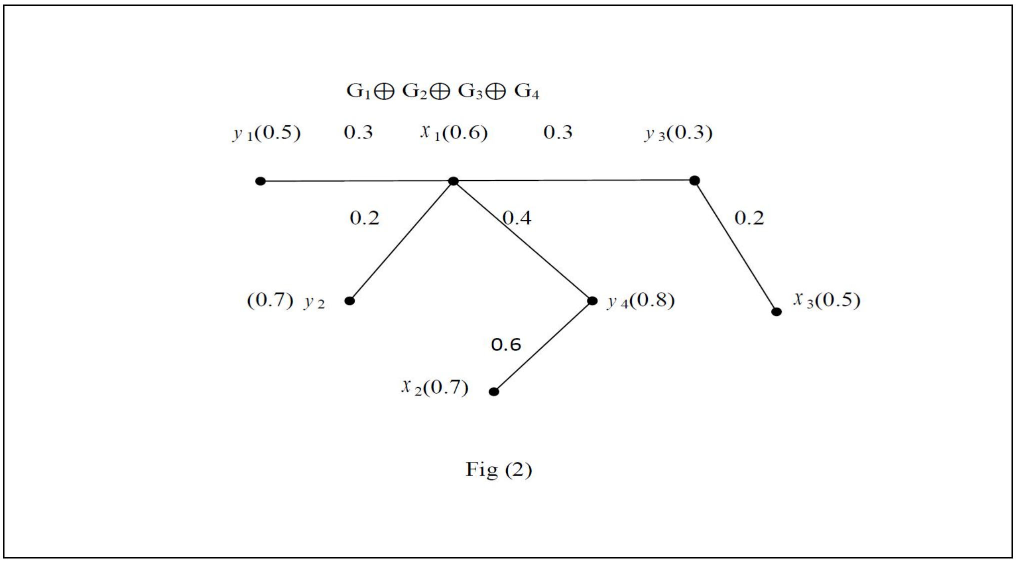


Figure 2 Direct sums of four fuzzy graphs with valency of nodes

The valency of the node in $G_1 \oplus G_2 \oplus G_3 \oplus G_4$ is as follows.

$D_{G1\oplus G2\oplus G3\oplus G4}(x_1) = 0.3 + 0.3 + 0.2 + 0.4 = 1.2$

$D_{G1\oplus G2\oplus G3\oplus G4}(x_2) = 0.6$

$D_{G1\oplus G2\oplus G3\oplus G4}(x_3) = 0.2$

$D_{G1\oplus G2\oplus G3\oplus G4}(y_1) = 0.3$

$D_{G1\oplus G2\oplus G3\oplus G4}(y_2) = 0.2$

$D_{G1\oplus G2\oplus G3\oplus G4}(y_3) = 0.3+0.2 = 0.5$

$D_{G1\oplus G2\oplus G3\oplus G4}(y_4) = 0.6+0.4 = 1$

Now,

G_1, G_2, G_3 and G_4.

$D_{G1\oplus G2\oplus G3\oplus G4}(x_1) = D\,G_1(x_1) + D\,G_2(x_1) + D\,G_3(x_1) + D\,G_4(x_1)$

$= 0.3+0.2+0.3+0.4$

$= 1.2$

$D_{G1\oplus G2\oplus G3\oplus G4}(x_2) = D\,G_4(x_2) = 0.6$

$D_{G1\oplus G2\oplus G3\oplus G4}(x_3) = D\,G_3(x_3) = 0.2$

$D_{G1\oplus G2\oplus G3\oplus G4}(y_1) = D\,G_1(y_1) = 0.3$

$D_{G1\oplus G2\oplus G3\oplus G4}(y_2) = D\,G_2(y_2) = 0.2$

$D_{G1\oplus G2\oplus G3\oplus G4}(y_3) = D\,G_3(y_3) = 0.2 + 0.3 = 0.5$

$D_{G1\oplus G2\oplus G3\oplus G4}(y_4) = D\,G_4(y_4) = 0.6 + 0.4 = 1$

Hence the degree of nodes is verified by the direct sum of four fuzzy graphs.

5. REGULAR FUZZY GRAPHS ON FOUR DIRECT SUMS:

5.1 Theorem: -

If $G_1: (\sigma_1, \mu_1)$, $G_2: (\sigma_2, \mu_2)$, $G_3: (\sigma_3, \mu_3)$ and $G_4: (\sigma_4, \mu_4)$ are regular fuzzy graphs with degrees k_1, k_2, k_3 and k_4 respectively and $V_1 \cap V_2 \cap V_3 \cap V_4 \neq \phi$ then $G_1 \oplus G_2 \oplus G_3 \oplus G_4 : (\sigma, \mu)$ is regular if and only if $k_1 = k_2 = k_3 = k_4$.

Proof:

Let,

$G_1: (\sigma_1, \mu_1)$ be a k_1- regular fuzzy graph

$G_2: (\sigma_2, \mu_2)$ be a k_2- regular fuzzy graph

G_3: (σ_3, μ_3) be a k_3- regular fuzzy graph

G_4: (σ_4, μ_4) be a k_4- regular fuzzy graph

Let us consider $G_1 \oplus G_2 \oplus G_3 \oplus G_4$: ($\sigma$, μ) is regular,

We know that,

$$D_{G1 \oplus G2 \oplus G3 \oplus G4}(x) = \begin{cases} D_{G1}(x), \text{ if } x \in v_1 \\ D_{G2}(x), \text{ if } x \in v_2 \\ D_{G3}(x), \text{ if } x \in v_3 \\ D_{G4}(x), \text{ if } x \in v_4 \\ D_{G1}(x) + D_{G2}(x) + D_{G3}(x) + D_{G4}(x), \end{cases}$$

If $x \in v_1 \cap v_2 \cap v_3 \cap v_4$ and

$E_1 \cap E_2 \cap E_3 \cap E_4 = \phi$

Since $v_1 \cap v_2 \cap v_3 \cap v_4 \neq \phi$

$$D_{G1 \oplus G2 \oplus G3 \oplus G4}(x) = \begin{cases} d_{G1}(x) = k_1, \text{ if } x \in v_1 \\ d_{G2}(x) = k_2, \text{ if } x \in v_2 \\ d_{G3}(x) = k_3, \text{ if } x \in v_3 \\ d_{G4}(x) = k_4, \text{ if } x \in v_4 \end{cases}$$

Since $G_1 \oplus G_2 \oplus G_3 \oplus G_4$: ($\sigma$, μ) is regular.

So, we conclude that $k_1 = k_2 = k_3 = k_4$

Converse part:

Consider $k_1 = k_2 = k_3 = k_4 = k$ (say)

Then k- regular fuzzy graphs be G_1, G_2, G_3, G_4 such that $v_1 \cap v_2 \cap v_3 \cap v_4 \neq \phi$

Then the valency of node in the direct sum is given by

$$D_{G1 \oplus G2 \oplus G3 \oplus G4}(x) = \begin{cases} d_{G1}(x) = k, \text{ if } x \in v_1 \\ d_{G2}(x) = k, \text{ if } x \in v_2 \\ d_{G3}(x) = k, \text{ if } x \in v_3 \\ d_{G4}(x) = k, \text{ if } x \in v_4 \end{cases}$$

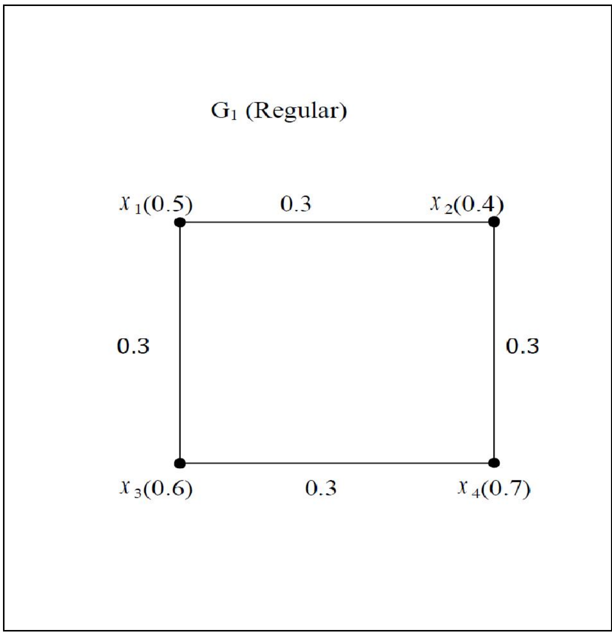
Therefore, the degree of direct sum, of four fuzzy graphs is k.

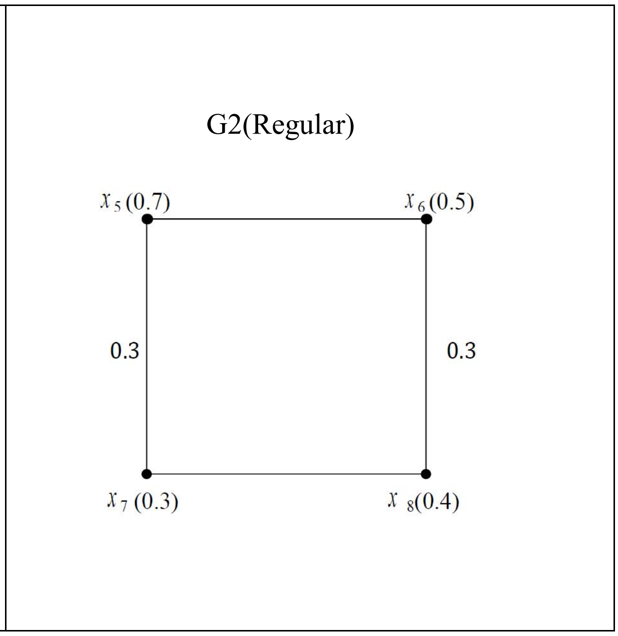
Journal of Physics: Conference Series **1850** (2021) 012055 doi:10.1088/1742-6596/1850/1/012055

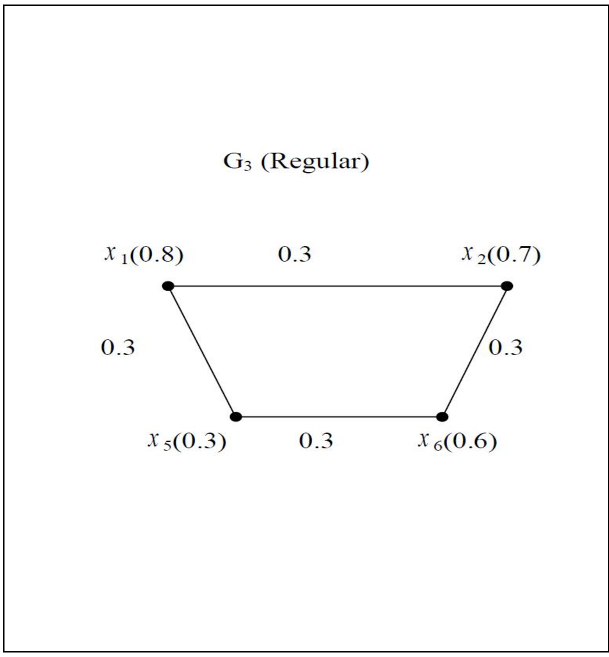
Hence $G_1 \oplus G_2 \oplus G_3 \oplus G_4: (\sigma, \mu)$ is regular.

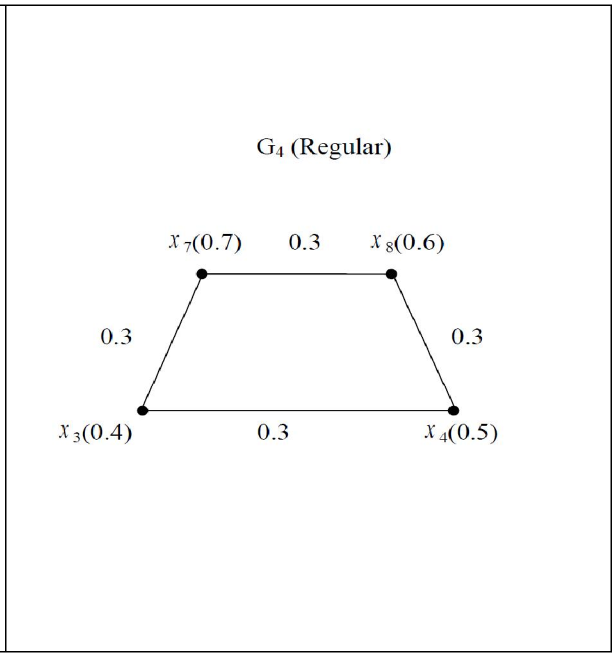
5.1. Example

Consider the four regular fuzzy graphs.









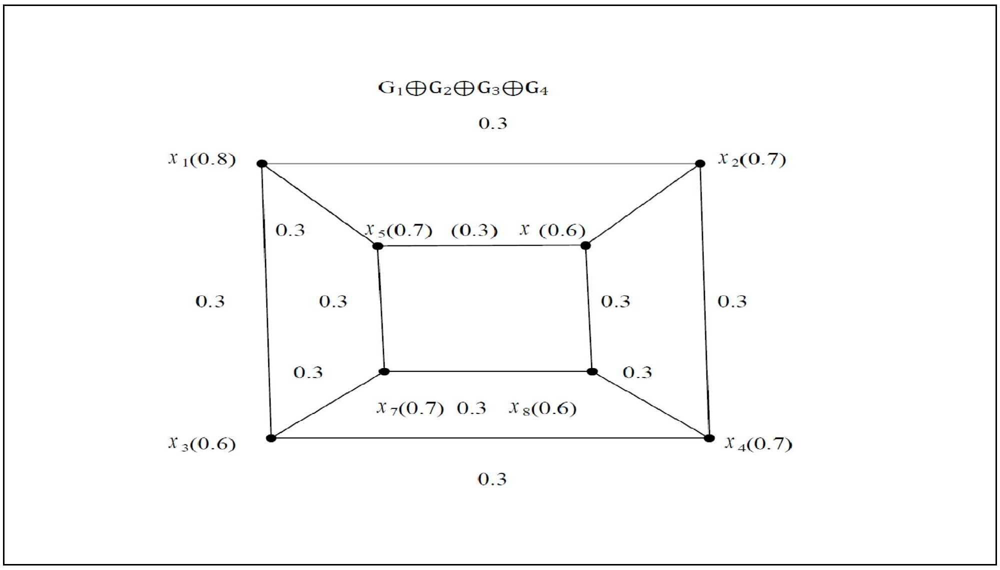


Figure 3 Direct Sums of Regular Fuzzy Graphs

6. DIRECT SUM OF FOUR CONNECTED FUZZY GRAPHS

6.1 Theorem: -

If G_1: (σ_1, μ_1), G_2: (σ_2, μ_2), G_3: (σ_3, μ_3), G_4: (σ_4, μ_4) are four connected fuzzy graphs with underlying Crisp graphs G_1^*: (σ_1, μ_1), G_2^*: (σ_2, μ_2), G_3^*: (σ_3, μ_3), G_4^*: (σ_4, μ_4) respectively such that $E_1 \cap E_2 \cap E_3 \cap E_4 = \phi$, $v_1 \cap v_3 \neq \phi$, $v_2 \cap v_4 \neq \phi$ then their direct sum $G_1 \oplus G_2 \oplus G_3 \oplus G_4$: (σ, μ) is Connected Fuzzy Graphs.

Proof:

Let,

G_1: (σ_1, μ_1) is a connected fuzzy graph, $\mu_1^\infty (E_1) > 0$

G_2: (σ_2, μ_2) is a connected fuzzy graph, $\mu_2^\infty (E_2) > 0$

G_3: (σ_3, μ_3) is a connected fuzzy graph, $\mu_3^\infty (E_3) > 0$

G_4: (σ_4, μ_4) is a connected fuzzy graph, $\mu_4^\infty (E_4) > 0$

Then, $v_1 \cap v_3 \neq \phi$ and $v_2 \cap v_4 \neq \phi$

At least one vertex in $v_1 \cap v_3$, one vertex in $v_2 \cap v_4$, and no edges in $E_1 \cap E_2 \cap E_3 \cap E_4$

Two vertices exist a path $G_1 \oplus G_2 \oplus G_3 \oplus G_4$: (σ, μ), that is μ^∞ $G_1 \oplus G_2 \oplus G_3 \oplus G_4$ $(E) > 0$.

Which implies that $G_1 \oplus G_2 \oplus G_3 \oplus G_4$: (σ, μ) is connected.

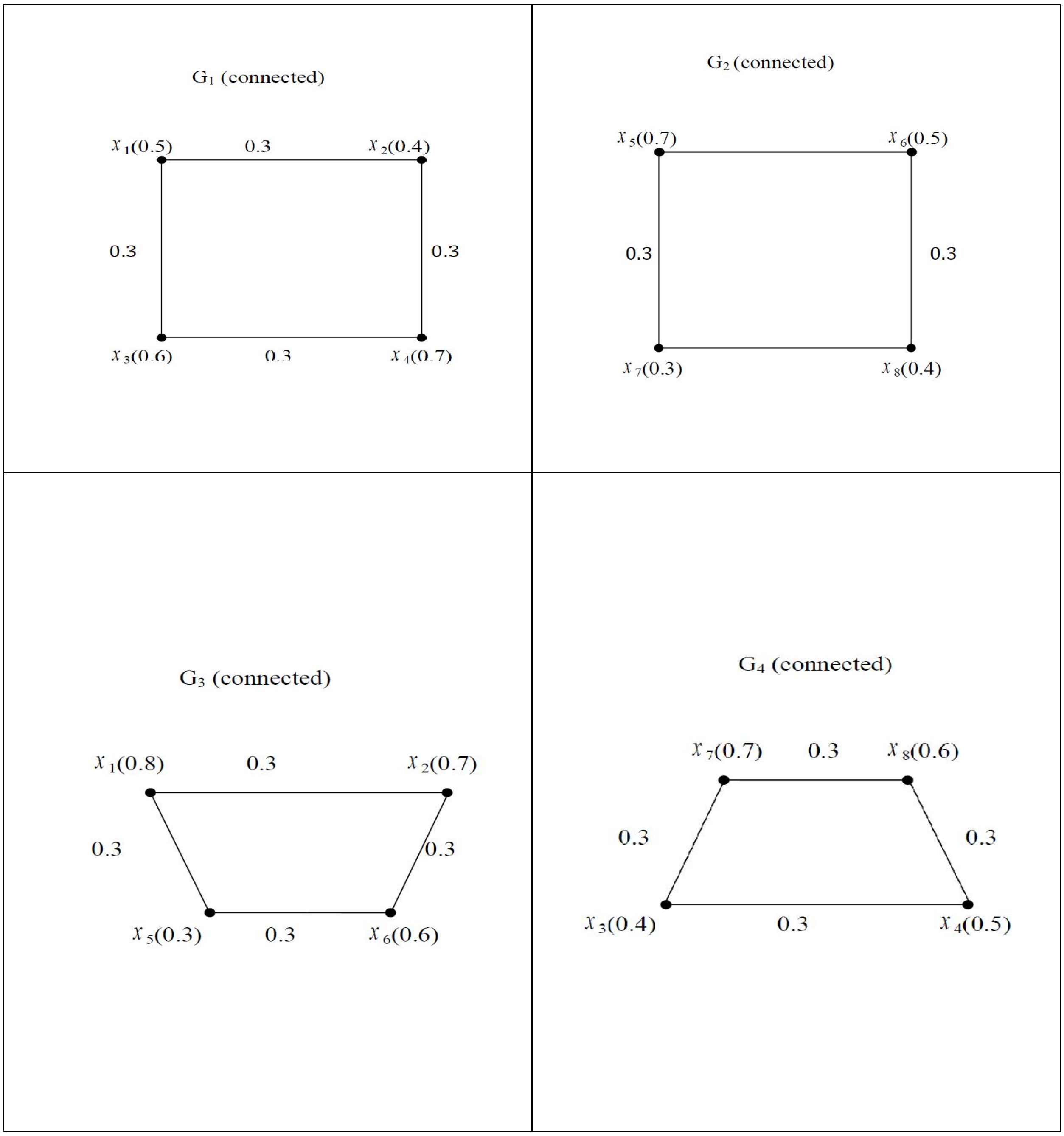
6.1. Example

If G_1: (σ_1, μ_1), G_2: (σ_2, μ_2), G_3: (σ_3, μ_3), and G_4: (σ_4, μ_4) are four connected fuzzy graphs.

With ∩ $(v_1 \cap v_3) = 1$ and ∩ $(v_2 \cap v_4) = 1$. Then $G_1 \oplus G_2 \oplus G_3 \oplus G_4$: (σ, μ) is the connected fuzzy graph.

Solution:-

Consider the direct sum of four connected fuzzy graph.



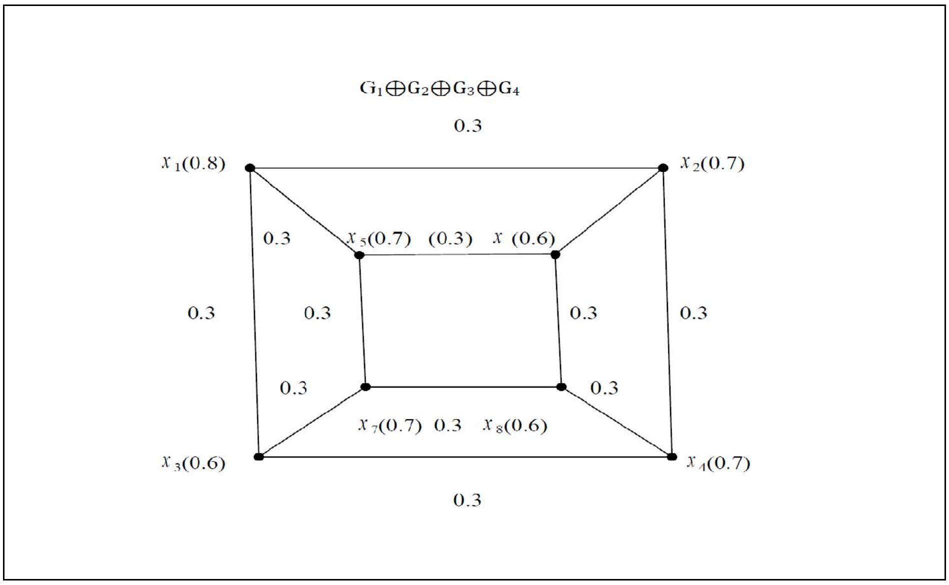


Figure 4 The Direct sums of four Connected Fuzzy Graphs

7. DIRECT SUM OF FOUR FUZZY GRAPHS - EFFECTIVE FUZZY GRAPHS

7.1 Theorem

If G_1, G_2, G_3, and G_4 are four effective fuzzy graphs with $x \in v_1 \cap v_2 \cap v_3 \cap v_4$ and there is no edge common in $E_1 \cap E_2 \cap E_3 \cap E_4$. Such that $\sigma_1(x) \geq \sigma_1(y)$, $\sigma_2(x) \geq \sigma_2(y)$, $\sigma_3(x) \geq \sigma_3(y)$ and $\sigma_4(x) \geq \sigma_4(y)$ then $G_1 \oplus G_2 \oplus G_3 \oplus G_4$ is an Effective Fuzzy Graphs.

Proof: -

Assume that x, y be an edge $G_1 \oplus G_2 \oplus G_3 \oplus G_4$

If x, y $\in v_1 \cup v_2 \cup v_3 \cup v_4$

Then $x, y \in v_1$ or $x, y \in v_2$ or $x, y \in v_3$ or $x, y \in v_3$

Now consider x y $\in v_1$, then x y $\in E_1$

Therefore

$\sigma(x) = \sigma_1(x)$

$\sigma(y) = \sigma_1(y)$ and

$\mu(x, y) = \mu_1(x, y)$

Since G_1 is an Effective Fuzzy Graphs

$\mu_1(x\,y) = \sigma_1(x) \wedge \sigma_1(y)$

$\mu(x\,y) = \sigma(x) \wedge \sigma(y)$

Consider x, y $\in V_2$, then x y $\in E_2$

Therefore,

$\sigma(x) = \sigma_2(x)$

$\sigma(y) = \sigma_2(y)$ and

$\mu(x, y) = \mu_2(x, y)$

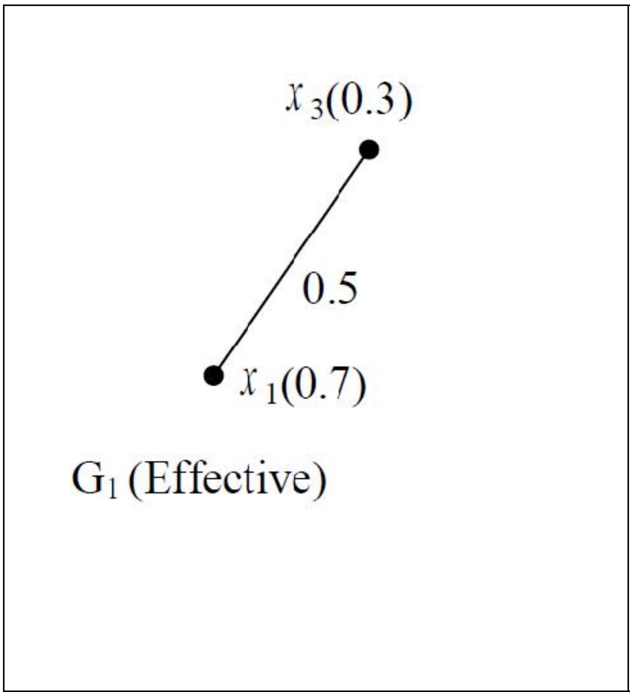
G_2 is an effective fuzzy graph.

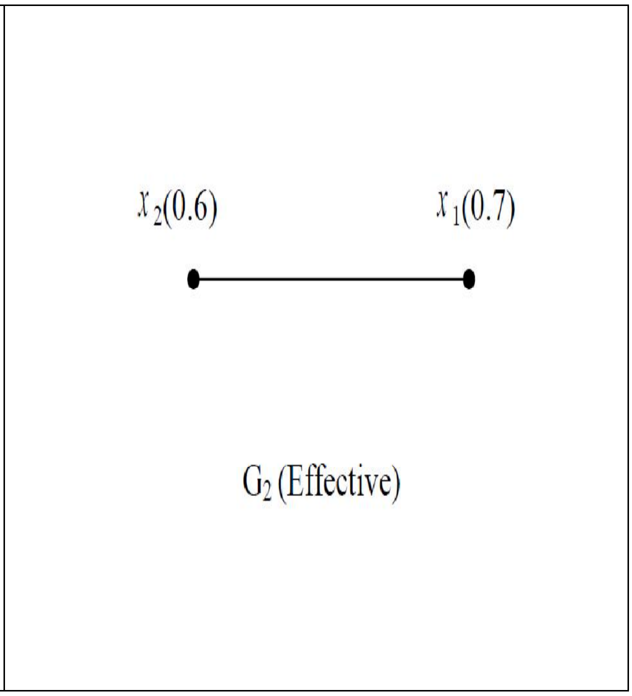
$\mu_2(x\,y) = \sigma_2(x) \wedge \sigma_2(y)$

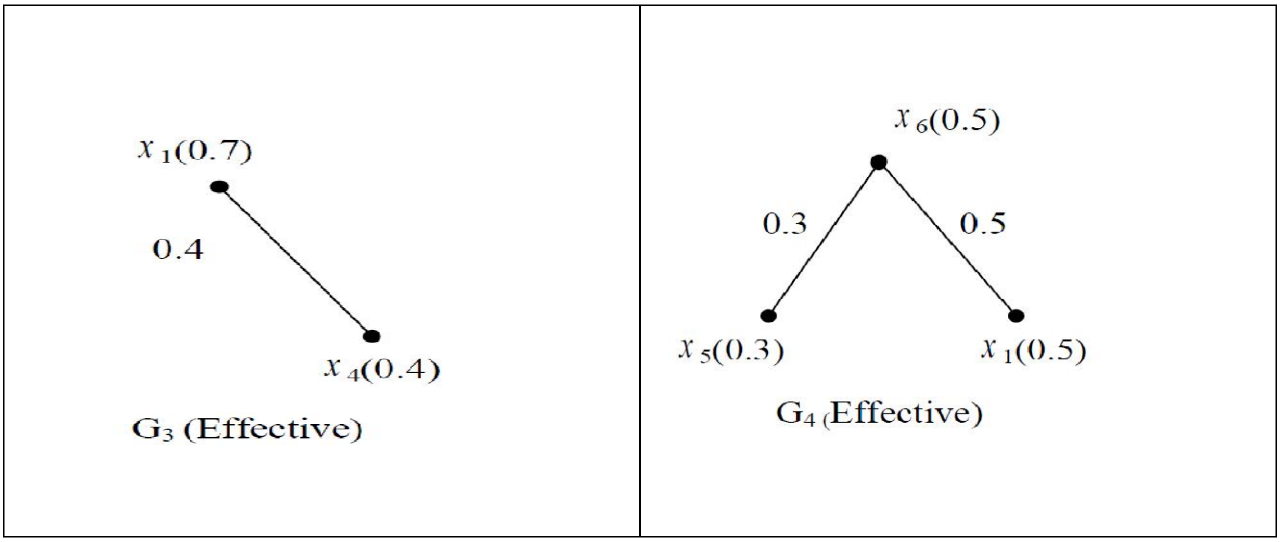
$\mu(x\,y) = \sigma(x) \wedge \sigma(y)$

This proof is similar to other two.

7.1. Example







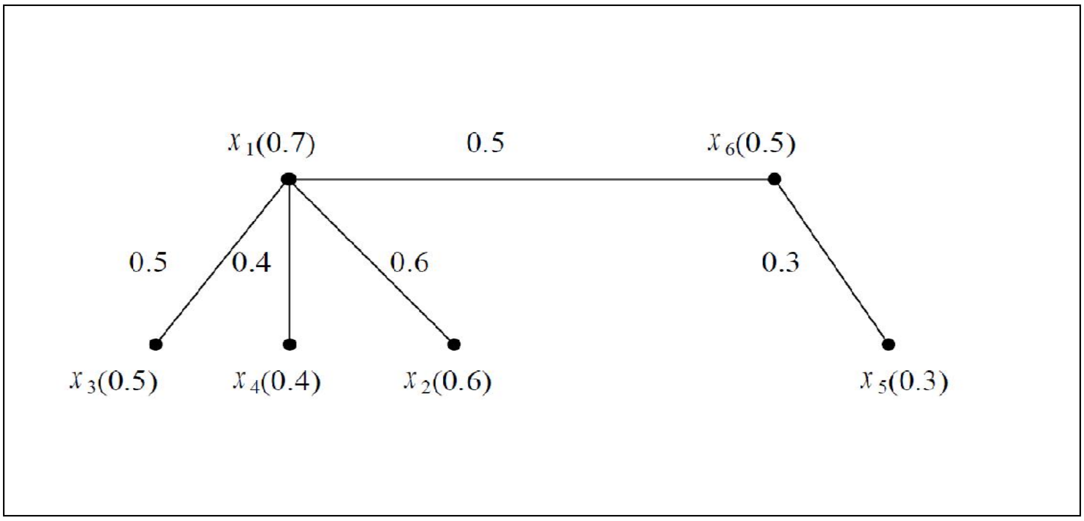


Figure 5 The Direct sums of four Effective Fuzzy Graphs.

8. CONCLUSION

We conclude that, the valency of nodes for the direct sum of four fuzzy graphs is proposed by the formulas and the Regular, Connected and Effective Fuzzy Graphs are verified with the characteristic of the direct sum with an example. In future this work can be taken as next stages like the direct sum of five, six etc., and this direct sum was applicable to the traffic light signals and the roadways.

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