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# Stochastic Decomposition of Geo/Geo/1 Production Inventory System

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**Abstract.** This paper considers an  $(s, S)$  inventory system with production in which demand occurs according to a Bernoulli process and service time follows a geometric distribution. The maximum inventory that can be accommodated in the system is  $S$ . When the items in the on-hand inventory are reduced to a pre-assigned level of  $s$  due to service, production is started. The production time follows a geometric distribution. The production process is stopped as soon as the inventory level reaches the maximum. Customer who arrives during the inventory level is zero is assumed to be lost. Using the stochastic decomposed solution obtained for the steady-state probability vector, we analyzed the inventory cycle. A suitable cost function is defined using the performance measures. Numerical experiments are also incorporated to highlight the minimum value of the cost function against the parameter values.

**Keywords:** Discrete-time production inventory, Bernoulli process, Geometric distribution, Stochastic decomposition, Matrix-Analytic Method.

**Mathematics Subject Classification:** 60K25, 90B05 and 91B70

## 1. Introduction

Queueing systems with inventory have extensively studied by several researchers for the last few decades. A complete review of queueing inventory models was done in the survey by Krishnamoorthy et al. [7]. Positive service time in Queueing inventory was introduced by Sigman and Simchi-Levi [20]. Assuming Poisson arrivals and arbitrary distribution for service time, they developed an explicit expression for the expected delay in terms of underlined system parameters. Berman et al. [3] proposed the processing time of inventory with deterministic service. The notion in that paper resulted in the situation for the analysis of queueing inventory in which demands occur when items in the inventory are stock out. In the production inventory system with positive service time and no restriction for customers to enter in the system when the inventory level zero is analysed by Krishnamoorthy and Narayanan [8]. The authors assumed that the server goes on vacation when either there is no inventory or no customers present. Krishnamoorthy and Jose [6] compared three retrial queueing inventory systems and analysed them using Matrix-Analytic Method. They obtained algorithmic solutions for the model and found out the most profitable model within the given parameter values.

The first reported work on closed-form expression in the queueing inventory system is carried out by Schwarz et al. [18]. The product form solution to this model is obtained by considering the



assumption that customers are not allowed to enter the system when the inventory level is zero. Schwarz and Daduna [17] developed approximation for performance measures in the M/M/1 queueing system in which delivery of inventory is considered as service with the assumption that customers can join in the system even when the inventory level is zero (back-ordering). To get a closed-form expression and avoid the loss of customers, Schwarz et al. [19] considered a queueing inventory model by incorporating the assumption that the demand that comes during stock the out period is re-routed to other service stations. Saffari and Haji [16] analysed a queueing inventory model with the product form solution by assuming high replenishment cost for instantaneous replenishment during the stock out period to prevent the loss of customers. M/M/1 queueing inventory system under  $(s, Q)$  policy is analysed by Saffari et al. [15] in which the demand during the stock out period is assumed to be lost. They obtained explicit expression for system performance measures. To avoid the loss of customers during the stock out period, the concept of local purchase is discussed in [9, 11]. All the mentioned works on product form solutions are studied in a continuous-time setup. The analogue work on discrete-time inventory is not reported so far. Notable work on a discrete-time queue is done by Meisling [13]. The author derived the results in the continuous system as a limiting case discrete system. Dafermos and Neuts [4] illustrated an example of a single server queueing model in the discrete case and discussed the time-dependent character of the queue using a bivariate Markov chain. Lian et al. [12] introduced inventory in a discrete-time inventory system having common life. They derived a closed-form expression for the expected total cost. We use the discrete version of the Matrix-Analytic Method (MAM), explained in Alfa [1, 2], to analyse the stability of the model. For elementary details of MAM, one can refer to Neuts [14].

The investigation of stochastic decomposition of production  $(s, S)$  inventory system in discrete time is not received much attention from researchers. To obtain an explicit expression for the steady-state distribution, Krishnamoorthy and Viswanath [10] restricted the entry of customers according to the inventory level. They obtained an explicit expression for the production cycle and optimized the cost function associated with the model with respect to maximum storage  $S$ . Deepthi [5] considered a similar model in discrete time with another distribution in the steady-state. The author obtained the optimum value of the expected total cost for the maximum inventory level  $S$ . The present paper is a generalization of Krishnamoorthy and Viswanath [10] to a discrete-time setup and the modification of the paper by Deepthi [5]. The objective of this work is to provide a suitable blocking set to obtain the desired form of steady-state probability vector a similar model with a different steady-state distribution.

The rest of the content of this paper is organized as follows. Section 2 provides mathematical modeling and analysis. The stability condition is derived in section 3. Steady-state probability vector and its explicit expression are discussed in section 4. The analysis of production is explained in section 5. The distribution of waiting time is discussed in section 6. Some relevant performance measures and their explicit forms are included in section 7. Finally, section 8 contains numerical experiments.

## 2. Mathematical Modeling and Analysis

This paper looks into a single server production  $(s, S)$  production inventory system in which the arrival of customers follows a Bernoulli process with parameter  $p$ , service time follows a geometric distribution with parameter  $q$ . Each customer receives one inventory after completing the service. When the items in the inventory reduce  $s$  due to demands, production starts. The production time of the individual item in the inventory follows a geometric distribution with parameter  $r$ . The production is stopped when the inventory is reached to the maximum level of  $S$ . We assume that arrival and service completion (if any) occurs at the beginning of a slot and production of the individual item (if any) takes place at the end of a slot.

**Notations**

$N(n)$  : Number of customers in the system at an epoch  $n$ .

$I(n)$  : Inventory level at the epoch  $n$ .

$C(n)$  : The production status, which is  $\begin{cases} 0, & \text{when production is off} \\ 1, & \text{when the production is on} \end{cases}$

$\bar{x} : 1 - x$ , for  $0 \leq x \leq 1$ .

Then  $\{(N(n), I(n)), c(n); n = 0, 1, 2, 3, \dots\}$  is a Quasi Birth Death process with state space

$$\{(i, j); 0 \leq j \leq s\} \cup \{(i, j, k); s + 1 \leq j \leq S - 1, k = 0, 1\} \cup \{(i, S)\}, \text{ for } i \geq 0$$

considering order of the state space as the dictionary order, the transition probability matrix of the above QBD process is given by,

$$P = \begin{bmatrix} B_1 & B_0 & & & \\ A_2 & A_1 & A_0 & & \\ & A_2 & A_1 & A_0 & \\ & & \ddots & \ddots & \ddots \end{bmatrix},$$

where, the blocks  $B_0, B_1, A_0, A_1$  and  $A_2$  square matrix of order  $2S - s$  and are given by

$$B_1 = \begin{bmatrix} \bar{r} & r & & & & & & & & & \\ & \bar{p}\bar{r} & \bar{p}r & & & & & & & & \\ & & \ddots & \ddots & & & & & & & \\ & & & \bar{p}\bar{r} & B_1^0 & & & & & & \\ & & & & B_1^1 & B_1^2 & & & & & \\ & & & & & \ddots & \ddots & & & & \\ & & & & & & B_1^1 & B_1^3 & & & \\ & & & & & & & \bar{p} & & & \end{bmatrix} \text{ with } B_1^0 = \begin{bmatrix} 0 & \bar{p}r \end{bmatrix},$$

$$B_1^1 = \begin{bmatrix} \bar{p} & 0 \\ 0 & \bar{p}\bar{r} \end{bmatrix},$$

$$B_1^2 = \begin{bmatrix} 0 & 0 \\ 0 & \bar{p}r \end{bmatrix}, B_1^3 = \begin{bmatrix} 0 \\ \bar{p}r \end{bmatrix}$$
  

$$B_0 = \begin{bmatrix} 0 & 0 & & & & & & & & & \\ & p\bar{r} & pr & & & & & & & & \\ & & \ddots & \ddots & & & & & & & \\ & & & p\bar{r} & B_0^0 & & & & & & \\ & & & & B_0^1 & B_0^2 & & & & & \\ & & & & & \ddots & \ddots & & & & \\ & & & & & & B_0^1 & B_0^3 & & & \\ & & & & & & & p & & & \end{bmatrix} \text{ with } B_0^0 = \begin{bmatrix} 0 & pr \end{bmatrix},$$

$$B_0^1 = \begin{bmatrix} p & 0 \\ 0 & p\bar{r} \end{bmatrix},$$

$$B_0^2 = \begin{bmatrix} 0 & 0 \\ 0 & pr \end{bmatrix}, B_0^3 = \begin{bmatrix} 0 \\ pr \end{bmatrix}$$
  

$$A_0 = \begin{bmatrix} 0 & 0 & & & & & & & & & \\ & p\bar{q}\bar{r} & p\bar{q}r & & & & & & & & \\ & & \ddots & \ddots & & & & & & & \\ & & & p\bar{q}\bar{r} & A_0^0 & & & & & & \\ & & & & A_0^1 & A_0^2 & & & & & \\ & & & & & \ddots & \ddots & & & & \\ & & & & & & A_0^1 & A_0^3 & & & \\ & & & & & & & p\bar{q} & & & \end{bmatrix} \text{ with } A_0^0 = \begin{bmatrix} 0 & p\bar{q}r \end{bmatrix},$$

$$A_0^1 = \begin{bmatrix} p\bar{q} & 0 \\ 0 & p\bar{q}\bar{r} \end{bmatrix},$$

$$A_0^2 = \begin{bmatrix} 0 & 0 \\ 0 & p\bar{q}r \end{bmatrix}, A_0^3 = \begin{bmatrix} 0 \\ p\bar{q}r \end{bmatrix}$$





*Proof.* From the structure of the transition probability matrix  $P$  and  $\hat{P}$ ,

$$B_1 + \frac{p}{\bar{p}q} A_2 = \hat{P}$$

$$\frac{\bar{p}q}{p} B_0 + A_1 + \frac{p\bar{q}}{\bar{p}q} A_2 = \hat{P}$$

$$\frac{\bar{p}q}{p\bar{q}} A_0 + A_1 + \frac{p\bar{q}}{\bar{p}q} A_2 = \hat{P}$$

Now

$$\pi_0 B_1 + \pi_1 A_2 = \left(\frac{q-p}{q}\right) \hat{\pi} \left(B_1 + \frac{p}{\bar{p}q} A_2\right)$$

$$= \left(\frac{q-p}{q}\right) \hat{\pi} \hat{P}$$

$$= \left(\frac{q-p}{q}\right) \hat{\pi}$$

$$= \pi_0$$

$$\pi_0 B_0 + \pi_1 A_1 + \pi_2 A_2 = \left(\frac{q-p}{q}\right) \frac{p}{\bar{p}q} \hat{\pi} \left(\frac{\bar{p}q}{p} B_0 + A_1 + \frac{p\bar{q}}{\bar{p}q} A_2\right)$$

$$= \left(\frac{q-p}{q}\right) \frac{p}{\bar{p}q} \hat{\pi} \hat{P}$$

$$= \pi_1$$

and for  $i \geq 2$ ,

$$\pi_{i-1} A_0 + \pi_i A_1 + \pi_{i+1} A_2 = \left(\frac{q-p}{q}\right) \frac{p}{\bar{p}q} \left(\frac{p\bar{q}}{\bar{p}q}\right)^{i-1} \hat{\pi} \left(\frac{\bar{p}q}{p\bar{q}} A_0 + A_1 + \frac{p\bar{q}}{\bar{p}q} A_2\right)$$

$$= \left(\frac{q-p}{q}\right) \frac{p}{\bar{p}q} \left(\frac{p\bar{q}}{\bar{p}q}\right)^{i-1} \hat{\pi} \hat{P}$$

$$= \pi_i$$

Using the above equation,  $\mathbf{\Pi}P = \mathbf{\Pi}$ , and on summing, we have  $\mathbf{\Pi}e = 1$  □

## 5. Production Process

The production will start when the items in the inventory reduced to  $s$  and will be stopped as soon as the inventory level reached to  $S$ . During the production process the inventory level varies from 0 to  $S - 1$  for any number of customers. Let  $N(n)$  be the number of customers in the system and  $I(n)$  the number of items in the inventory at an epoch  $n$ . Consider the absorbing discrete time Markov Chain  $(N(n), I(n))$  with state space  $\{(i, j); i \geq 0, 0 \leq j \leq S - 1\} \cup \{\Delta\}$ , where  $\Delta$  represents the absorbing state when the production process is stopped. The transition probability matrix  $P_p$  of the process is given by

$$P_p = \begin{bmatrix} T & t \\ 0 & 1 \end{bmatrix}$$

$$\text{Where } T = \begin{bmatrix} C_1 & C_0 & & \\ D_2 & D_1 & D_0 & \\ & \ddots & \ddots & \ddots \end{bmatrix} \text{ and } t = e - Te$$

$$C_1 = \begin{bmatrix} \bar{r} & r & & & \\ & \bar{p}\bar{r} & \bar{p}r & & \\ & & \ddots & \ddots & \\ & & & \bar{p}\bar{r} & \bar{p}r \\ & & & & \bar{p}\bar{r} \end{bmatrix}, C_0 = \begin{bmatrix} 0 & 0 & & & \\ & \bar{p}\bar{r} & pr & & \\ & & \ddots & \ddots & \\ & & & \bar{p}\bar{r} & pr \\ & & & & \bar{p}\bar{r} \end{bmatrix}, D_2 = \begin{bmatrix} 0 & 0 & & & \\ & \bar{p}q\bar{r} & pqr & & \\ & & \ddots & \ddots & \\ & & & \bar{p}q\bar{r} & pqr \\ & & & & \bar{p}q\bar{r} \end{bmatrix}$$

$$D_1 = \begin{bmatrix} \bar{r} & r & & & \\ p\bar{q}\bar{r} & \bar{p}\bar{q}\bar{r} + pqr & \bar{p}r & & \\ & \ddots & \ddots & \ddots & \\ & & p\bar{q}\bar{r} & \bar{p}\bar{q}\bar{r} + pqr & \bar{p}r \\ & & & p\bar{q}\bar{r} & \bar{p}\bar{q}\bar{r} + pqr \end{bmatrix}, D_0 = \begin{bmatrix} 0 & 0 & & & \\ & \bar{p}q\bar{r} & pqr & & \\ & & \ddots & \ddots & \\ & & & \bar{p}q\bar{r} & pqr \\ & & & & \bar{p}q\bar{r} \end{bmatrix}$$

Let  $\hat{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \dots)$  be the initial probability vector representing the state of the system when production starts

$$\text{Then } \alpha_j = \begin{cases} \frac{q-p}{q} e_{s+1} & \text{for } j = 0 \\ \left(\frac{q-p}{q}\right) \frac{p}{\bar{p}q} \rho^{j-1} e_{s+1} & \text{for } j \geq 1 \end{cases}$$

Where  $e_{s+1}$  is the row vector of dimension  $S$ , whose  $(s + 1)^{th}$  entry is one and the remaining entries are zeros. To calculate the time till the absorption we have to truncate the length of the queue. For this, choose  $\epsilon > 0$  and  $N$  large enough so that

$$\sum_{i=N+1}^{\infty} \rho^{i-1} < \frac{\epsilon}{\left(\frac{q-p}{q}\right) \frac{p}{\bar{p}q}}$$

Using the truncation level  $N$ , we truncate  $T$  to a finite square matrix  $T^*$  of order  $(N + 1)S$  and  $\hat{\alpha}$  to a finite row vector  $\hat{\alpha}^*$  of size  $(N + 1)S$  defined by,

$$\hat{\alpha}^* = (\alpha_1^*, \alpha_2^*, \dots, \alpha_N^*) \text{ with } \alpha_j^* = \frac{\alpha_j}{\left(1 - \frac{p}{q} \rho^N\right)}$$

Hence the length of the production time is a d phase-type distribution in discrete time with parameters  $(\hat{\alpha}^*, T^*)$

Then, the expected length of the production process,  $EPR$ ,

$$EPR = \hat{\alpha}^*(I - T^*)^{-1}e$$

### 6. Waiting Time Distribution

For computing the waiting time distribution of arriving customer in the queue, by considering the position(rank) of the arriving customer, we use discrete phase-type distribution. Suppose that the arriving customer in the queue is of rank  $r_1$ . Consider the absorbing discrete-time Markov chain  $(N'(n), I(n), c(n))$  where  $N'(n)$  denotes the rank (position) of the arriving customer,  $I(n)$  the items in the inventory and  $c(n)$  the production status at the epoch  $n$ , Then the state-space of this Markov process is  $\{1, 2, \dots, r_1\} \times \{0, 1, 2, \dots, s, (s + 1, 0), (s + 1, 1), \dots, (S - 1, 0), (s -$



$$\begin{aligned}
EIL &= \sum_{i=0}^{\infty} \sum_{j=0}^S j\pi_{ij} \\
&= \frac{r}{r-p} (s+1 + s+2 + \dots + S-1) \hat{\pi}_S + S \hat{\pi}_S \\
&\quad + \frac{p}{r-p} k^{s-1} \left(1 + 2 \cdot \frac{1}{k} + \dots + s \cdot \frac{1}{k^{s-1}}\right) \hat{\pi}_S \\
&\quad - \frac{p}{r-p} k^{S-1} \left(1 + 2 \cdot \frac{1}{k} + \dots + (S-1) \cdot \frac{1}{k^{S-2}}\right) \hat{\pi}_S \\
&= \left[ \frac{r}{r-p} \frac{(S+s)(S-s-1)}{2} + \frac{p}{r-p} \frac{(k^{s-1} - k^{S-1} + S)k^2 - (S+s)k + s}{(k-1)^2} + S \right] \hat{\pi}_S
\end{aligned}$$

iii) Expected rate of production,  $EPR$ , is

$$\begin{aligned}
EPR &= r \sum_{j=0}^s \hat{\pi}_i + r \sum_{i=s+1}^S \hat{\pi}_{i,1} \\
&= (1 - (S-s)r) \hat{\pi}_S
\end{aligned}$$

iv) Expected loss of customers,  $ELR$  is given by

$$\begin{aligned}
ELR &= \hat{\pi}_0 p \\
&= \frac{p^2}{r} (1 - k^{S-s}) k^s \hat{\pi}_S
\end{aligned}$$

v) Expected production switching on rate,  $E_{ON}$ , is

$$E_{ON} = q \sum_{i=1}^{\infty} \left( \frac{q-p}{q} \right) \frac{p}{pq} \rho^{i-1} \hat{\pi}_{s+1,0} = p \hat{\pi}_S$$

## 8. Numerical Experiments

### 8.1. Cost Analysis

Based on the above performance measures, we define a suitable cost function. For this, we define individual cost  $c_0, c_1, c_2, c_3$  and  $c_4$  as

- $c_0$  : switching cost for a production to start
- $c_1$  : production / unit inventory / unit time
- $c_2$  : holding of inventory / unit / unit time
- $c_3$  : holding cost of customers / unit / unit time
- $c_4$  : cost due to loss of customers / unit / unit time

Define expected total cost ( $ETC$ ) per unit time as,

$$ETC = c_0 E_{ON} + c_1 EPR + c_2 EIL + c_3 ELR$$

### 8.2. Tabular Illustrations

Through this section, we provide a numerical interpretation of the above system performance measures and optimization of the defined cost function. Since explicit expressions for system performance measures and hence cost function are obtained, we can optimize the cost function using the calculus method. One can also decide the nature of the system characteristics with respect to the underlined parameters. The numerical illustrations are given below.

Table 1 shows the effect of production probability  $r$  with various performance measures and cost function by keeping all other parameters constant. As  $r$  creases, the expected rate of production and expected inventory increase while expected switching rate and expected loss rate of customers decrease since the possibility of stock out period decreases. The optimum value of expected total cost is 14.289 at  $r = 0.48$ .

Similarly, Table 2 illustrates the effect of  $p$  on various measures and expected total cost. As  $p$  increases, the expected loss rate of customers decreases since the possibility of being stock out increases. The expected production rate and production switching rate also decrease with an increase of  $p$  due to the loss of some arrivals during the stock out period. The optimum value of ETC is obtained at  $p = 0.4$  and the corresponding minimum expected total cost is 13.866.

The two dimensional Table 3 is designed to find the optimum value of ETC by varying the re-order point and maximum inventory level. From this table, it is clear that the minimum value of ETC is 15.8231 and is obtained at  $s = 4$  and  $S = 24$ . This indicates that an optimum  $(s, S)$  pair for the model is  $(4, 24)$ .

Table 1: Effect of  $r$  on  $ETC$

$$(c_0, c_1, c_2, c_3, p, q, s, S) = (150, 30, 0.1, 250, 0.4, 0.6, 8, 20)$$

$r$	$\hat{\pi}_S$	EIL	EPR	Eon	ELR	ETC
0.42	0.00575	9.07	0.39104	0.00230	0.00896	15.2220
0.44	0.00847	10.22	0.39530	0.00339	0.00470	14.5640
0.46	0.01131	11.09	0.39759	0.00452	0.00241	14.3180
0.48	0.01410	11.72	0.39877	0.00564	0.00123	<b>14.2890</b>
0.50	0.01677	12.19	0.39938	0.00671	0.00062	14.3620
0.52	0.01928	12.54	0.39968	0.00771	0.00032	14.4800
0.54	0.02163	12.80	0.39984	0.00865	0.00016	14.6130
0.56	0.02382	13.01	0.39992	0.00953	0.00008	14.7480
0.58	0.02587	13.17	0.39996	0.01035	0.00004	14.8780
0.60	0.02778	13.30	0.39998	0.01111	0.00002	15.0010

Table 2: Effect of  $p$  on  $ETC$

$$(c_0, c_1, c_2, c_3, q, r, s, S) = (250, 15, 0.5, 250, 0.6, 0.5, 8, 20)$$

$p$	$\hat{\pi}_S$	EIL1	EPR1	Eon1	ELR1	ETC
0.34	0.02668	13.10	0.33994	0.00907	0.00006	13.9340
0.36	0.02336	12.87	0.35986	0.00841	0.00014	13.9690
0.38	0.02005	12.57	0.37970	0.00762	0.00030	13.9610
0.4	0.01677	12.19	0.39938	0.00671	0.00062	13.9190
0.42	0.01355	11.69	0.41872	0.00569	0.00128	13.8680
0.44	0.01043	11.02	0.43741	0.00459	0.00259	<b>13.8660</b>
0.46	0.00751	10.14	0.45492	0.00346	0.00508	14.0270
0.48	0.00494	9.01	0.47038	0.00237	0.00962	14.5580

Table 3:  $ETC$  vs.  $(S, s)$ 

$$(c_0, c_1, c_2, c_3, p, q, r) = (150, 30, 0.1, 250, 0.45, 0.55, 0.6)$$

$S \setminus s$	1	2	3	4	5	6	7
21	16.5509	16.1164	15.9195	15.8565	15.8724	15.9373	16.0357
22	16.5009	16.0868	15.8990	15.8385	15.8525	15.9121	16.0023
23	16.4603	16.0649	15.8857	15.8278	15.8403	15.8958	15.9793
24	16.4278	16.0497	15.8784	<b>15.8231</b>	15.8348	15.8869	15.9649
25	16.4024	16.0402	15.8764	15.8237	15.8348	15.8841	15.9577
26	16.3832	16.0357	15.8790	15.8287	15.8396	15.8867	15.9565
27	16.3694	16.0357	15.8856	15.8378	15.8485	15.8937	15.9604

### Conclusion

This paper analysed a discrete production queueing inventory system with positive service time and obtained the closed-form solution to the model. The production process and waiting-time distribution of an arriving customer are analyzed using absorbing Markov chain. Explicit expression for other relevant performance measures are obtained. Numerical interpretations are incorporated to highlight the convexity of the cost function. The loss of customers during stock out period can be minimized either by decreasing the service rate or increasing the production rate when the on-hand inventory level is less than a prefixed level. The model can be extended by avoiding the lost sale due to zero inventory by introducing a local purchase, (an instantaneous purchase with some additional cost). One can also minimize the local purchase quantity to optimize profit.

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