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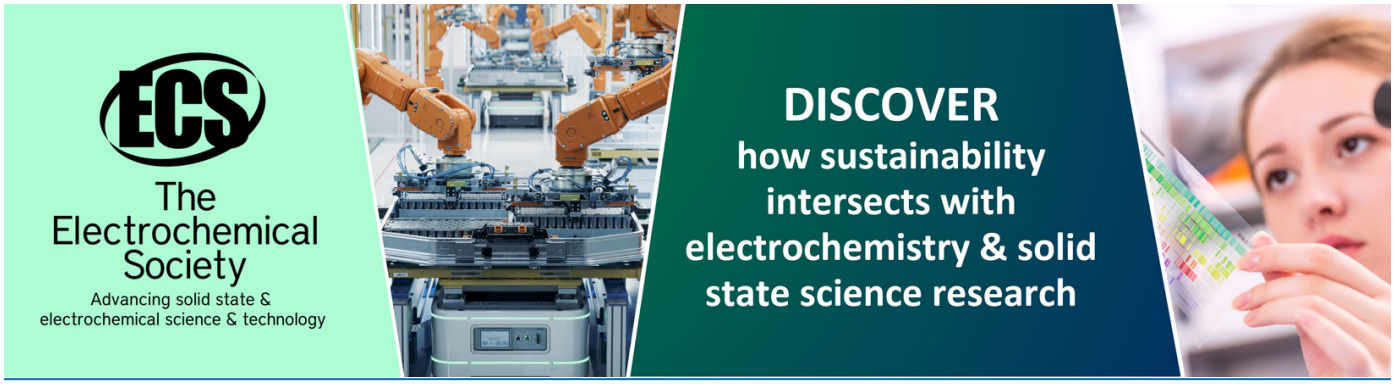

Metacognitive skills of students with high mathematical abilities in solving contextual problems

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Abstract: This study aims to explore the metacognitive skills of students with high mathematical abilities in the mathematics education study program at the Faculty of Teacher Training and Education in the University of Tadulako in solving contextual problems. The process of student metacognition is determined by providing a number of contextual problems that must be solved, then explored in stages by thinking aloud and unstructured interviews. It aims to explore students' metacognitive skills in solving contextual problems. The results of data analysis show that students with high mathematical abilities in solving contextual problems involve their metacognitive skills.

1. Introduction

Changes in the world of work and society are very rapid, resulting in education that is not enough to just transfer knowledge, but students need to be equipped with the ability to solve new problems, design solutions, collaborate, and communicate in everyday life [1]. However, not a few students are only able to solve problems, if the problem is in accordance with the examples given. However, when the context of the problem is changed, sometimes they are no longer able to solve it [2].

Students metacognition skills in solving contextual problems are the exploration of students' metacognition processes that involve awareness and regulation of their thinking in terms of planning, monitoring, evaluating student thinking processes and results when solving contextual problems with the aim of gaining more knowledge [3-10]

The involvement of metacognition in various activities is very good, for example, the involvement of metacognition can improve students' ability in problem solving [11-12], can help students understand reading [3], increase student learning outcomes [3], or increase the ability of students to become successful learners [4]. There is a principle that efforts to involve metacognition in various learning activities are expected to provide benefits to improve the quality of learning carried out.

The simple definition of problem solving is the process of accepting a problem as a challenge to solve it. In line with the above understanding, Cooney argues that problem solving is the process of accepting a problem and trying to solve it [14]. Problem solving is defined as a process of finding a way out of a difficulty or obstacle, the achievement of goals that cannot be immediately understood [15].

One of the methods that can be used to study and teach mathematics is problem solving. Some of the advantages for students who have problem-solving skills, among others, are developing critical thinking skills and strong math skills. The use of problem solving can improve cognitive abilities, and creativity. Problem solving is part of the mathematics application process and can motivate students to learn mathematics [16].

Contextual problems are defined as problems where the problem situation is a real experience for students. Experience in calculating contextual problems obtained from learning mathematics, it is hoped

that students can use it to solve problems in their lives [17]; namely, providing experience to students solving problems, can make students more careful in deciding something in life.

Problem solving, especially in mathematics is an attempt to find a solution to the problem at hand by using one's initial knowledge. This research is concerned with solving mathematical problems. Mathematical problem solving are the steps a person takes to solve a problem. Mathematical steps in this study are to use Polya's steps which include: understand the problem to be resolved, make a plan for resolution, carry out what has been planned, re-examine what has been done [15]. When solving problems, each individual has unique characteristics that are not shared by other individuals. Therefore it can be said that each individual is different from one another. Based on the description above, it is necessary to explore the process of student metacognition in solving contextual problems.

2. Method

This research is a qualitative research and is carried out in the Mathematics Education Study Program of the Teaching and Education Faculty, Tadulako University. The process of student metacognition is determined by providing a number of contextual problems that must be solved, then explored in stages by thinking aloud and unstructured interviews. It aims to explore students' metacognitive skills in solving contextual problems. Furthermore, the data obtained were analyzed in three stages, namely: (1) data condensation, (2) data presentation, and (3) drawing conclusions. In detail, the data analysis in this study was carried out in the following steps: (1) examines all data obtained from interviews, observations, field notes, and the results of contextual problem solving tasks, (2) reducing data by making an abstraction. (3) compiling into units which are further categorized by coding, (4) checking the validity of the data, by means of time triangulation, (5) analyzing the student's metacognition process in solving contextual problems, (6) data interpretation and drawing conclusions [18]. The detailed map of the research road can be seen in Figure 1.

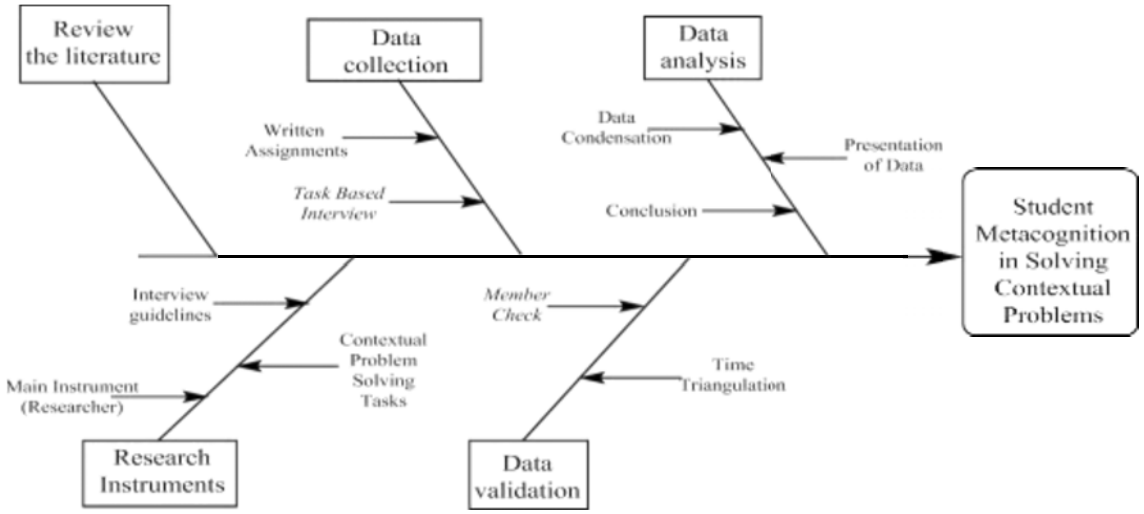

Figure 1. State of the Art

3. Results and Discussion

3.1. The stage of understanding the problem

The contextual problem given to the subject is: Anton intends to cross a river which is 1 kilometer wide and head to a point as far as 4 kilometers downstream. He can swim at 4 kilometers / hour and run 10

kilometers / hour. Assume that he starts swimming and that he swims toward a point x kilometers downstream from his starting point A. How long does it take to reach his destination point D.

Based on the results of the validation, S data shows the consistency of information when understanding the problem when conducting task-based interviews. From the interview excerpt, it can be seen that S response in an effort to understand the problem is to first read the problem / question out loud and with the aim of being able to understand the problem given. To get this information S immediately read the whole question carefully. S reads the questions with the aim that in the process of solving questions directed and in accordance with the aims and objectives of the questions (A(003), A(004), A(005)). This is revealed in the following interview excerpt: A(003)P : Your plan?

A(003)S : I have to read carefully the problem carefully, Sir
A(004)P : What for?
A(004)S : So that the problem solving process is more focused and in accordance with the aims and objectives in the question Sir

Thus S realized that it is necessary to plan what to do to understand the problem.
In an effort to find out the purpose and purpose of the questions and obtain information about what is known and what is asked, S reads the problem carefully, writes, and visualizes the problem by describing the problem to be solved, as in the following picture:

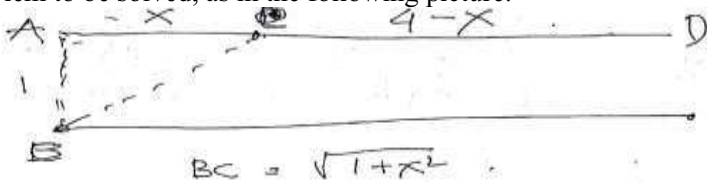

Figure 2. The subject states the problem in another form

This shows that S is aware in planning to find out the intent and purpose of the given problem.
Based on data from interviews A(009) and A(010). After S plans what to do to understand the problem, first read the questions as a whole carefully with the aim of being able to understand the problem given and get the information, S immediately reads out loud. S believes that in a planned way she will understand the problem well after looking at it again.

A(009)P : How to?
A(009)S : Read carefully Sir
A(010)P : Why did you do that?
A(010)S : I read in order to make it easier for me to understand the questions given and solve the questions according to my knowledge

This shows that at this stage S has no difficulty understanding the problem. Therefore S is aware in monitoring the steps taken in understanding the problem [21]
To see whether the effort was made in planning to be able to understand the problem and know the purpose and purpose of the problem and to obtain information about what was known and what was asked, S checked or reflected on the information obtained by reviewing the strategies that had been used and whether the strategy was leading on the desired result or not, as revealed in the following interviews A (038) and A (039):

A(038)P : Do you understand the problem? A(038)S : (S pays attention)
Yes, Sir
A(039)P : Can you write about what is known and asked in the question? A(039)S : (S rechecked the question)

Yes, God willing, I can write it down, sir

Thus S is aware in evaluating information about what is known and what is being asked [11-13].

3.2. The stage of drawing up a plan

To achieve cognitive goals, S plans to plan a solution, which is the stage of choosing a problem- solving strategy. After seeing the questions, S was sure he could plan a solution using certain strategies because the material had been studied in the calculus course. S can estimate the time it will take to develop a problem-solving approach or strategy, which is about 10 minutes, as in interviews A(044) and A(045), as follows:

A(044)P : Before solving the problem, what do you do ?
A(044)S : Draw up completion steps, Sir
A(045)P : You mean?
A(045)S : Develop a strategy or steps for completion, Sir

So it can be said that S is aware in planning what will be done in order to prepare a plan for completion [6, 20, 21].

After planning what will be done in preparing the completion plan and estimating the time that will be used to develop the problem solving approach or strategy. S monitors the cognitive strategies used to ensure they are on the right track after developing problem-solving steps. In this case, S stated that what was known so far was sufficient to achieve the goal and believed that the steps that had been prepared could be used to solve problem A(063), as revealed during the interview: A(063)P :
Besides that plan, what else?

A(063)S : (S pays attention to problems and theorems) I think that's enough Sir

Therefore S is aware in monitoring what is being done in preparing a problem-solving plan.

Based on the following interview data A(064) and A(065)

A(064)P : Can your plan be used to solve the problem?
A(064)S : I think so, Sir
A(065)P : Why?
A(065)S : Because I am sure I can use that method

S looks back at the plan that has been prepared, S states that the plan that has been prepared is sufficient and believes the plan that has been drawn up can be used to solve the problem, so that S no other way is considered to arrange the given problem. This shows that S with his awareness to re-check the truth of the things he did in planning [20, 21].

3.3. The stage of implementing the plan

When going to carry out the plan S estimates that it can solve the problem according to the resolution plan that has been prepared previously and estimates the time it will take to solve the problem or implement the problem-solving plan is about 15 minutes and is sure to solve that time by reason of the steps the solution is clear and pre-planned. In planning to solve the problem according to the steps for solving it previously described and realizing the importance of estimating the time that will be used in implementing the problem solving plan [19, 21]. Therefore, S is aware of planning to solve the problems according to the plan that has been prepared. This was revealed in interviews A(072), A(073), and A(074) as follows:

A(072)P : The time you need?
A(072)S : About 15 minutes, Sir
A(073)P : Can it be?
A(073)S : Yes, Sir

A(074)P : Sure?
A(074)S : Because it was planned beforehand, so I'm sure I can, Sir

The involvement of metacognitive knowledge (declarative knowledge, procedural knowledge, and conditional knowledge) when implementing the problem solving plan, S monitors the implementation of problem solving, where he monitors the problem solving steps to carry out in detail by paying attention to the results of work that has been done implemented, which was revealed in interviews A(100) and A(101):

A(100)P : Are the results obtained in accordance with the objectives of the questions?
A(100)S : (S checks her work)
Yes, Sir
A(101)P : Why?
A(101)S : (S rereading his work)
Because I have double-checked and the results are as expected or what was asked

Therefore, when implementing the plan, S is aware of monitoring the completion of the questions according to the plan that has been prepared or not.

Based on interview data A(102), A(103), and A(104):

A(102)P : When you finished solving the problem, did you read again?
A(102)S : Re-check the results so that they don't miss the plan, Sir.
A(103)P : Continue.
A(103)S : As well as re-seeing who knows what is wrong, Sir
A(104)P : Do you have another way?
A(104)S : Yes, but I haven't thought about it, Sir

This shows that, after solving the problem, S deliberately looks back at the progress achieved after solving the problem, by reviewing the results of the problem solving that has been done, by re- checking the results of his work, whether they are in accordance with the desired results, seeing the possibility of errors in calculating and evaluating the suitability of the steps that have been planned as a basis for concluding that what has been planned has been achieved, so that S can state that the implementation of problem solving is in accordance with the objectives of the problem and state that there is no other way that can be done to carry out the completion plan problem. This shows that S is aware in checking the suitability of the original plan with what is obtained from the problem solving process [19, 20].

3.4. Check back stage

In the plan of checking back, S predicts the time to be used is about 8 minutes, with the reason that this time is sufficient to re-check because it is just a matter of check the results of the implementation of problem solving and the results are as expected. In this case S deliberately examines the steps of completion, checks what is known and what is asked, the theorem used in determining the minimum value, as expressed during the interview A(108), A(109), dan A(110): A(108)P : How long?
A(108)S : About 8 minutes, Sir A(109)P
: Is it enough time like that?
A(109)S : Hopefully enough, Sir
A(110)P : Why?
A(110)S : Because it only checked the results, Sir, was it what was expected or not

Thus S is conscious in planning to check completion steps.

Based on data from interviews A (115) and A (116):

A(115)P : How do you do it?
A(115)S : I checked, starting from how to plan to the end, Sir

A(116)P : You mean last?
A(116)S : I mean up to the way I solve the problem
S realizes that what is planned to check the completion steps is already on the right track or not, where S deliberately looks back on whether the completion step is correct and the result is in accordance with the plan, checking for possible errors in the calculation in determining the value of x that makes the minimum t value. Based on this, it shows that S is aware in monitoring the plan to check the steps for solving the questions [5-7].
After planning to examine the completion steps, where S re-examines the results of the strategies that have been used in examining problem-solving steps, namely by looking back and deciding that what has been planned is appropriate and can be continued, as in interview A(117) and A(118): A(117)P
: Is your way the right way?
A(117)S : (S looking back)
Yes, Sir
A(118)P : What's new again?
A(118)S : I will see the results I got Sir
This suggests that S consciously re-examines the plan to check problem-solving steps, in which the subject deliberately revisits the results and thought processes [19, 20].

4. Conclusion
The results of data analysis showed that students with high mathematical abilities in solving contextual problems involved their metacognitive skills, starting when planning, monitoring, and evaluating their thinking processes and results when solving contextual problems. It is hoped that the subject of this research can improve their cognitive abilities both in learning and in solving problems in everyday life.

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