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On resolving perfect dominating number of comb product of special graphs

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Abstract. A set of vertices $D \subseteq V(G)$ is the dominating set of graph G if every vertex on graph G is dominated by dominators. The dominating set of D on graph G is a perfect if every point of a graph G is dominated by exactly one vertex on D. For each vertex $v \in V(G)$, the k-vector $r(v|W)$ is called the metric code or location W, where $W = \{w_1, w_2, \ldots, w_k\} \subseteq V(G)$. An ordered set $W \subseteq V(G)$ is called the resolving set of graph G if each vertex $u, v \in V(G)$ has a different point representation with respect to the W where $r(u|W) \neq r(v|W)$. The ordered set $W_{rp} \subseteq V(G)$ is called the resolving perfect dominating set on graph G if W_{rp} is the resolving set and perfect dominating set of graph G. The minimum cardinality of the resolving perfect dominating set is called the resolving perfect dominating number which is denoted by $\gamma_{rp}G$. In this study, we analyzed the resolving perfect dominating number of comb product operations between two connected graphs, such as $K_n \triangleright P_2$, $K_n \triangleright P_3$, $Btn \triangleright P_2$, $Bt_n \triangleright P_3$, and $Bt_n \triangleright C_3$.
Keywords: Perfect dominating set, Resolving perfect dominating set, Resolving perfect dominating number

1. Introduction

All graphs used in this paper are connected, simple, and undirected. A graph G is written with $G = (V, E)$ where $V(G)$ is a set of vertex on graph G can be written with $V(G) = \{v_1, v_2, ..., v_n\}$ and $E(G)$ is a set of edge on graph G can be written with $E(G) = \{e_1, e_2, ..., e_n\}$ where $E(G)$ is the set which can be empty of unsorted pairs (v_1, v_2) of the vertex $(v_1, v_2) \in V(G)$. See [1,2] for a more detailed graph definition.

Haynes introduced the concept of dominating number with the symbol $\gamma(G)$ [3]. A dominating set on graph G is a set of vertices $D \subseteq V(G)$ where each vertex is dominated exactly once by the part of (D). A dominating set of D of a graph G is perfect if each vertex of G not in D is adjacent to exactly one vertex of D [4]. A set of D_p is called the perfect dominating set of graph G, and the perfect dominating number is denoted by $\gamma p(G)$. On Biggs [5] paper, early concept of the perfect d-dominating appears with the term perfect d-code to indicate the perfect d-dominating set.

K-vector $r(v|W) = ((d(v, w1), d(v, w2), \ldots, d(v, wk))$ is a metric representation of vertex v with respect to W, where $W = w_1, w_2, \ldots, W_k$ is ordered subset of vertices on connected graph G [6]. An ordered set $W \subseteq V(G)$ is called the resolving set of graph G if each vertex $u, v \in V(G)$ has a different vertex representation with respect to W where $r(u|W) \neq r(v|W)$. The metric dimension of the graph G is the minimum cardinality of the resolving set of graph G which is denoted by $dim(G)$. The ordered set $W_{rp} \subseteq V(G)$ is called the resolving perfect dominating set on graph G if W_{rp} is the resolving set and perfect dominating set of graph G. The minimum cardinality of the resolving perfect dominating set is called the resolving perfect dominating number which is denoted by $\gamma_{rp}(G)$. The definition of the resolving perfect dominating set is illustrated as follows:

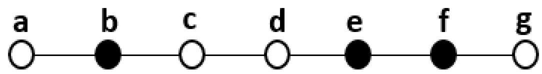

Figure 1. b, e, f is Perfect Dominating Set with $\gamma_p(G)$

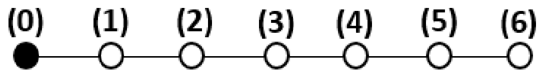

Figure 2. Metric Dimension

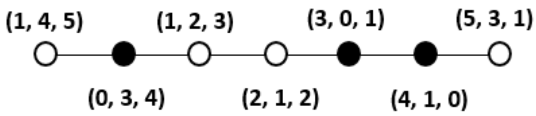

Figure 3. Resolving Perfect Dominating Set with $\gamma_{rp}(G)$

Perfect dominating has been widely studied by many researchers, among them are Young Soo Kwon, and Jaeun Lee [7], studied perfect domination sets in Cayley graphs, Marilynn and Quentin [4] studied perfect dominating set, and Italo [8] studied perfect domination in regular grid graphs. Then, some of the results of the metric dimensions can be checked on the paper [6], [9], [10], [11], and resolving set has been studied widely by Brigham et. al.[14] studied on resolving dominating in graph, Hayyu, et al.[13] studied resolving domination number of helm graph and it's operation, and Wangguway, et.al [19] studied on resolving domination number of special family of graphs.

In this study, the graph used is a connected graph with the comb product graph operation. Let graphs G and H are two connected graphs, where the comb product of G and H is denoted by $G \triangleright H$ which is obtained from taking a copy of graph G as many vertex on graph H and then attaching the copy of graph G to the vertices of graph H. The definition of the comb product can be seen in Suhadi, et. al. [15]. In this paper, the completion of perfect dominance numbers is examined in the comb product operation between two connected graphs such as $K_n \triangleright P_2$, $K_n \triangleright P_3$, $Btn \triangleright P_2$, $Btn \triangleright P_3$, and $Btn \triangleright C_3$.

2. Results

In this section we describe the resolving perfect dominating number γ_{rp} of a connected graph with the result of the comb product operation, and the set resolving sets can be seen in black vertices. The following is the theorem about the perfect dominating set in this study.

Theorem 1. *Let $K_n \triangleright P_2$ be a comb product graph order $n \geq 3$ $\gamma_{rp}(K_n \triangleright P_2) = n$. $K_n \triangleright P_2$ be a comb product graph order $n \geq 3$.*

Proof. Graph $K_n \triangleright P_2$ be a comb product graph order $n \geq 3$, with a set of vertex $V(K_n \triangleright P_2) = \{\{x_{i,j}; 1 \leq i \leq n, j = 1,2\}\}$, and a set of edge $E(K_n \triangleright P_2) = \{\{x_i x_j; 1 \leq i \leq n-1, i+1 \leq j \leq n\} \bigcup \{x_{i,1}x_{i,2}; 1 \leq i \leq n\}\}$. The vertex cardinality of the graph $K_n \triangleright P_2$ is $|V(K_n \triangleright P_2)| = 2\text{n}$, and the edge cardinality of the graph $K_n \triangleright P_2$ is $|E(K_n \triangleright P_2)| = \frac{1}{2}(n^2 + n)$. Let $D(K_n \triangleright P_2) = \{x_{i,1}; 1 \leq i \leq n\}$, then $|D(K_n \triangleright P_2)| = n$. A set of D is perfect dominating set if each vertex $V(K_n \triangleright P_2) \cap D$ dominates each vertex $x_{i,1}$, or $V(K_n \triangleright P_2) - D$ is dominated by each vertex $x_{i,2}$, then we get $|N(V) \cap D| = 1$. Then, a set of D is the perfect dominating set.

We show that D is a resolving set of graph $K_n \triangleright P_2$ if each vertex $v \in V(K_n \triangleright P_2)$ has a different vertex representation with respect to W where $r(u|W) \neq r(v|W)$ is described below.

Table 1. Representation of vertex $v \in V(K_n \triangleright P_2)$ respect to W

v	$r(v \mid W)$	$Condition$
$x_{i,1}$	$(\underbrace{2....2}_{i-1}, 1, \underbrace{2....2}_{n-i})$	$1 \leq i \leq n, n \geq 3$
$x_{i,2}$	$(\underbrace{3....3}_{i-1}, 0, \underbrace{3....3}_{n-i})$	$1 \leq i \leq n, n \geq 3$

Table 1 shows that $v \in V(K_n \triangleright P_2)$ has a different vertex representation with respect to D, because having different vertex representations, it can be accomplished that D is the resolving perfect dominating set.

Additionally, we proved that the minimum cardinality of the resolving perfect dominating is W. For example, $\gamma_{rp}(K_n \triangleright P_2) < n$, take $|W| = n-1$, so that

(i) If $W = \{x_{i,1}; 1 \leq i \leq n-1\}$ then, there is vertex that is not dominated by W, that is $x_{n,1} \in V(K_n \triangleright P_2)$, and $x_{n,2}$ dominated twice by W, so D is not the perfect dominating set of the graph $K_n \triangleright P_2$.

(ii) If $W = \{x_{i,2}; 1 \leq i \leq n-1\}$ then, there are vertices that are not dominated by W, among others $x_{n,1}, x_{n,2} \in V(K_n \triangleright P_2)$, so D is not the perfect dominating set of the graph $K_n \triangleright P_2$.

We can conclude that the minimum cardinality of the perfect dominating set on the graph $(K_n \triangleright P_2$ with $\gamma_{rp}(K_n \triangleright P_2) = n$ is W. As an example, resolving perfect dominating set of $K_n \triangleright P_2$ can be seen in Figure 4 and the function can see in Table 1.

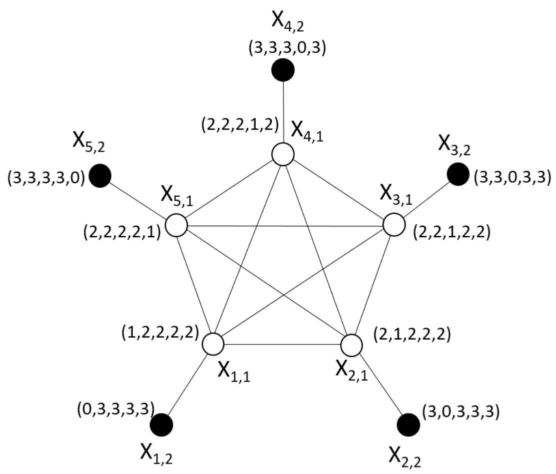

Figure 4. $\gamma_{rp}(K_5 \triangleright P_2) = 5$

Theorem 2. *Let $K_n \triangleright P_3$ be a comb product graph order $n \geq 3$ $\gamma_{rp}(K_n \triangleright P_3) = n$.*

Proof. Graph $K_n \triangleright P_3$ be a comb product graph order $n \geq 3$, where $x_{i,1}$ as the sticking point on graph K_n, with a set of vertex $V(K_n \triangleright P_3) = \{\{x_{i,j}; 1 \leq i \leq n, j = 1,2,3\}\}$, and a set of edge $E(K_n \triangleright P_3) = \{\{x_{1,i}x_{1,i+1}; i = 1,2\} \bigcup \{x_{i,j}x_{i,j+1}; 1 \leq i \leq n, j = 1,2\}\}$. The vertex cardinality of the graph $K_n \triangleright P_3$ is $|V(K_n \triangleright P_3)| = 3$n, and the edge cardinality of the graph $K_n \triangleright P_3$ is $|E(K_n \triangleright P_3)| = \frac{1}{2}(n^2 + 3n)$. Let $D(K_n \triangleright P_3) = \{x_{i,2}; 1 \leq i \leq n\}$, then $|D(K_n \triangleright P_3)| = n$. A set of D is perfect dominating set if each vertex $V(K_n \triangleright P_3) \cap D$ dominates each vertex $x_{i,1}$, and $x_{i,3}$, or $V(K_n \triangleright P_3) - D$ is dominated by each vertex $x_{i,2}$, then we get $|N(V) \cap D| = 1$. Then, a set of D is the perfect dominating set.

We show that D is a resolving set of graph $K_n \triangleright P_3$ if each vertex $v \in V(K_n \triangleright P_3)$ has a different vertex representation with respect to W where $r(u|W) \neq r(v|W)$ is described below.

Table 2. Representation of vertex $v \in V(K_n \triangleright P_3)$ respect to W

v	$r(v \mid W)$	*Condition*
$x_{i,1}$	$(\underbrace{2....2}_{i-1}, 1, \underbrace{2....2}_{n-i})$	$1 \leq i \leq n, n \geq 3$
$x_{i,2}$	$(\underbrace{3....3}_{i-1}, 0, \underbrace{3....3}_{n-i})$	$1 \leq i \leq n, n \geq 3$
$x_{i,3}$	$(\underbrace{4....4}_{i-1}, 1, \underbrace{4....4}_{n-i})$	$1 \leq i \leq n, n \geq 3$

Table 2 shows that $v \in V(K_n \triangleright P_3)$ has a different vertex representation with respect to D, because having different vertex representations, it can be accomplished that D is the resolving perfect dominating set.

Additionally, we proved that the minimum cardinality of the resolving perfect dominating is W. For example $\gamma_{rp}(K_n \triangleright P_3) < n$, take $|W| = n - 1$, so that

(i) If $W = \{x_{i,1}; 1 \leq i \leq n-1\}$ then, there are vertices that are not dominated by W, among others $x_{n,2}, x_{n,3} \in V(K_n \triangleright P_3)$, and $x_{n,1} \in V(K_n \triangleright P_3)$ dominated twice by W, so D is not the perfect dominating set of the graph $K_n \triangleright P_3$.

(ii) If $W = \{x_{i,2}; 1 \leq i \leq n-1\}$ then, there are vertices that are not dominated by W, among others $x_{n,1}, x_{n,2}, x_{n,3} \in V(K_n \triangleright P_3)$, so D is not the perfect dominating set of the graph $K_n \triangleright P_3$.

(iii) If $W = \{x_{i,3}; 1 \leq i \leq n-1\}$ then, there are vertices that are not dominated by W, among others $x_{i,1}, x_{n,1}, x_{n,2}, x_{n,3} \in V(K_n \triangleright P_3)$, so D is not the perfect dominating set of the graph $K_n \triangleright P_3$.

We can conclude that the minimum cardinality of the perfect dominating set on the graph $K_n \triangleright P_3$ with $\gamma_{rp}(K_n \triangleright P_3) = n$ is W. As an example resolving perfect dominating set of $K_n \triangleright P_3$ can be seen in Figure 5 and the function can see in Table 2.

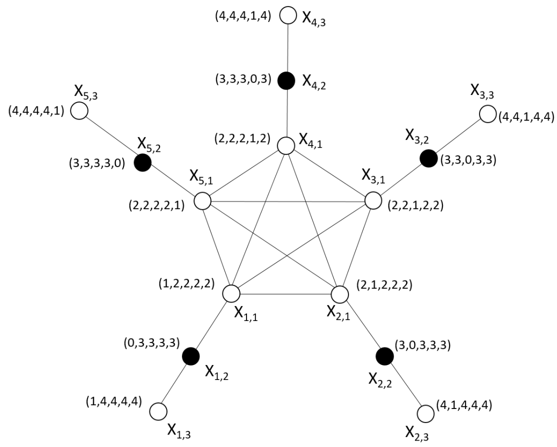

Figure 5. $\gamma_{rp}(K_5 \triangleright P_3) = 5$

Theorem 3. *Let $Bt_n \triangleright P_2$ be a comb product graph order $n \geq 2$ $\gamma_{rp}(Bt_n \triangleright P_2) = n + 2$.*

Proof. Graph $Bt_n \triangleright P_2$ be a comb product graph order $n \geq 2$, with a set of vertex $V(Bt_n \triangleright P_2) = \{\{x_{i,j}; i = 1,2, j = 1,2\} \bigcup \{y_{i,j}; 1 \leq i \leq n, j = 1,2\}\}$, and a set of edge $E(Bt_n \triangleright P_2) = \{\{x_{,j}ix_{i,j+1}; i = 1,2, j = 1\} \bigcup \{x_{i,1}x_{i+1,1}; i = 1\} \bigcup \{x_{i,1}y_{j,1}; i = 1,2, 1 \leq j \leq n\} \bigcup \{y_{i,j}y_{i,j+1}; 1 \leq i \leq n, j = 1\}\}$. The vertex cardinality of the graph $Bt_n \triangleright P_2$ is $|V(Bt_n \triangleright P_2)|$ = 2n+4, and the edge cardinality of the graph $Bt_n \triangleright P_2$ is $|E(Bt_n \triangleright P_2)|$ = 3n+3. Let $D(Bt_n \triangleright P_2) = \{\{x_{i,1}; i = 1,2\} \bigcup \{y_{i,1}; 1 \leq i \leq n\}\}$, then $|D(Bt_n \triangleright P_2)| = n + 2$. A set of D is perfect dominating set if each vertex $V(Bt_n \triangleright P_2) \cap D$ dominates each vertex $x_{i,2}$, and $y_{i,2}$, or $V(Bt_n \triangleright P_2) - D$ is dominated by each vertex $x_{i,1}$, $y_{i,1}$ and then we get $|N(V) \cap D| = 1$. Then, a set of D is the perfect dominating set.

We show that D is a resolving set of graph $Bt_n \triangleright P_2$ if each vertex $v \in V(Bt_n \triangleright P_2)$ has a different vertex representation with respect to W where $r(u|W) \neq r(v|W)$ is described below.

Table 3. Representation of vertex $v \in V(Bt_n \triangleright P_2)$ respect to W

v	$r(v \mid W)$	*Condition*
$x_{i,1}$	$(\underbrace{1, ..., 1}_{i-1}, 0, \underbrace{1, ..., 1}_{n-i})$	$i = 1, 2, , n \geq 2$
$x_{i,2}$	$(\underbrace{2, ...2}_{i-1}, 1, \underbrace{2....2}_{n-i})$	$i = 2, n \geq 2$
$y_{i,1}$	$(1, 1, \underbrace{2, ..., 2}_{i-1}, 0, \underbrace{2, ..., 2}_{n-i})$	$1 \leq i \leq n, n \geq 2$
$y_{i,2}$	$(2, 2, \underbrace{3, ..., 3}_{i-1}, 1, \underbrace{3, ..., 3}_{n-i})$	$1 \leq i \leq n, n \geq 2$

Table 3 shows that $v \in V(Bt_n \triangleright P_2)$ has a different vertex representation with respect to D, because having different vertex representations, it can be accomplished that D is the resolving perfect dominating set.

Additionally, we proved that the minimum cardinality of the resolving perfect dominating is W. For example $\gamma_{rp}(Bt_n \triangleright P_2) < n$, take $|W| = n + 1$, so that

(i) If $W = \{\{x_{i,1}; i = 1, 2\} \bigcup \{y_{i,1}; 1 \leq i \leq n - 1\}\}$ then, there is vertex that is not dominated by W, that is $x_{n,2} \in V(Bt_n \triangleright P_2)$, and $x_{n,1}$ dominated twice by W, so D is not the perfect dominating set of the graph $Bt_n \triangleright P_2$.

(ii) If $W = \{\{x_{i,2}; i = 1, 2\} \bigcup \{y_{i,2}; 1 \leq i \leq n+1\}\}$ then, there are vertices that are not dominated by W, among others $x_{n,1}, x_{n,2} \in V(Bt_n \rhd P_2)$, so D is not the perfect dominating set of the graph $Bt_n \rhd P_2$.

We can conclude that the minimum cardinality of the perfect dominating set on the graph $(K_n \rhd P_2$ with $\gamma_{rp}(Bt_n \rhd P_2) = n + 2$ is W. As an example, resolving perfect dominating set of $K_n \rhd P_2$ can be seen in Figure 6 and the function can see in Table 3.

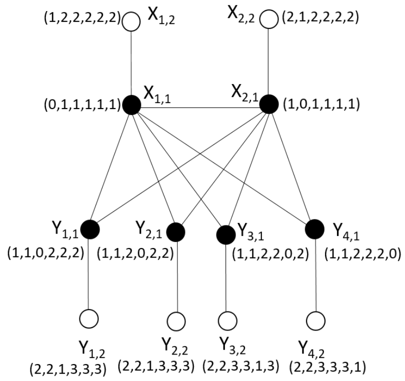

Figure 6. $\gamma_{rp}(Bt_4 \rhd P_2) = 6$

Theorem 4. *Let $Bt_n \rhd P_3$ be a comb product graph order $n \geq 2$, $\gamma_{rp}(Bt_n \rhd P_3) = n + 2$.*

Proof. Graph $Bt_n \rhd P_3$ be a comb product graph order $n \geq 2$, where $x_{i,1}$, and $y_{i,1}$ as the sticking point on graph Bt_n, with a set of vertex $V(Bt_n \rhd P_3) = \{\{x_{i,j}; i = 1, 2, j = 1, 2, 3\} \bigcup \{y_{i,j}; 1 \leq i \leq n$, j=1,2,3$\}\}$ and a set of edge $E(Bt_n \rhd P_3) = \{\{x_{i,j}x_{i,j+1}; i = 1, 2, j = 1, 2\} \bigcup \{x_{i,1}x_{i+1,1}; i = 1\} \bigcup \{x_{i,1}y_{j,i}, i = 1, 2, 1 \leq j \leq n\} \bigcup \{y_{i,j}y_{i,j+1}\}\}$. The vertex cardinality of the graph $Bt_n \rhd P_3$ is $|V(Bt_n \rhd P_3)| =$ 3n+6, and the edge cardinality of the graph $Bt_n \rhd P_3$ is $|E(Bt_n \rhd P_3)| = 4n + 5$. Let $D(Bt_n \rhd P_3) = \{\{x_{i,2}; i = 1, 2\} \bigcup \{y_{i,2}, 1 \leq i \leq n\}\}$, then $|D(B_n \rhd P_3)| = n + 2$. A set of D is perfect dominating set if each vertex $V(B_tn \rhd P_3) \cap D$ dominates each vertex $x_{i,1}, x_{i,3}, y_{i,1}$, and $y_{i,3}$, or $V(Bt_n \rhd P_3) - D$ is dominated by each vertex $x_{i,2}$, and $\{y_{i,2}\}$, then we get $|N(V) \cap D| = 1$. Then, a set of D is the perfect dominating set.

Table 4. Representation of vertex $v \in V(Bt_n \rhd P_3)$ respect to W

v	$r(v \mid W)$	*Condition*
x_i	$(\underbrace{....2}_{i-1}, 1, \underbrace{2....}_{n-i+2})$	$i = 1, 2, n \geq 2$
$x_{i,1}$	$(\underbrace{....3}_{i-1}, 0, \underbrace{3....}_{n-i+2})$	$i = 1, 2, n \geq 2$
$x_{i,2}$	$(\underbrace{....4}_{i-1}, 1, \underbrace{4....}_{n-i+2})$	$i = 1, 2, n \geq 2$
y_i	$(2, 2, \underbrace{3....3}_{i-1}, 1, \underbrace{3....}_{n-i})$	$1 \leq i \leq n, n \geq 2$
$y_{i,1}$	$(3, 3, \underbrace{4...4}_{i-1}, 0, \underbrace{4...4}_{n-i})$	$1 \leq i \leq n, n \geq 2$
$y_{i,2}$	$(4, 4, \underbrace{5....5}_{i-1}, 1, \underbrace{5....5}_{n-i})$	$1 \leq i \leq n, n \geq 2$

We show that D is a resolving set of graph $Bt_n \rhd P_3$ if each vertex $v \in V(Bt_n \rhd P_3)$ has a different vertex representation with respect to W where $r(u|W) \neq r(v|W)$ is described below.

Table 4 shows that $v \in V(K_n \triangleright P_3)$ has a different vertex representation with respect to D, because having different vertex representations, it can be accomplished that D is the resolving perfect dominating set. The function can see in Table 4.

Additionally, we proved that the minimum cardinality of the resolving perfect dominating is W. For example $\gamma_{rp}(Bt_n \triangleright P_3) < n$, take $|W| = n+1$, so that

(i) If $W = \{\{x_{i,1}; i = 1,2\} \bigcup \{y_{i,1}, 1 \leq i \leq n11\}\}$ then, there are vertices that are not dominated by W, among others $y_{n,2}, x_{y,3} \in V(Bt_n \triangleright P_3)$, and $x_{n,1} \in V(Bt_n \triangleright P_3)$ dominated twice by W, so D is not the perfect dominating set of the graph $Bt_n \triangleright P_2$.

(ii) If $W = \{\{x_{i,2}; i = 1,2\} \bigcup \{y_{i,2}; 1 \leq i \leq n-1\}\}$ then, there are vertices that are not dominated by W, among others $x_{n,1}, x_{n,2}, x_{n,3} \in V(Bt_n \triangleright P_3)$, so D is not the perfect dominating set of the graph $Bt_n \triangleright P_3$.

(iii) If $W = \{x_{i,3}; 1 \leq i \leq n-1\} \bigcup \{y_{i,3}; 1 \leq i \leq n-1\}\}$ then, there are vertices that are not dominated by W, among others $x_{i,1}, x_{n,1}, x_{n,2}, x_{n,3}, x_{n,3} \in V(Bt_n \triangleright P_3)$, so D is not the perfect dominating set of the graph $Bt_n \triangleright P_3$.

We can conclude that the minimum cardinality of the perfect dominating set on the graph $Bt_n \triangleright P_3$ with $\gamma_{rp}(Bt_n \triangleright P_3) = n+2$ is W. As an example, resolving perfect dominating set of $Bt_n \triangleright P_3$ can be seen in Figure 7.

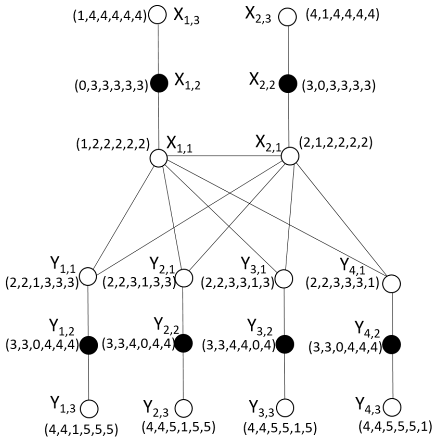

Figure 7. $\gamma_{rp}(Bt_n \triangleright P_3) = 5$

Theorem 5. *Let $Bt_n \triangleright C_3$ be a comb product graph order $n \geq 2$ $\gamma_{rp}(Bt_n \triangleright C_3) = n+2$.*

Proof. Graph $Bt_n \triangleright C_3$ be a comb product graph order $n \geq 2$, with a set of vertex $V(Bt_n \triangleright C_3) = \{\{x_i; i = 1,2\} \bigcup \{x_{1,i}; i = 1,2\} \bigcup \{x_{2,i}; i = 1,2\} \bigcup \{y_i; 1 \leq \ i \leq n\} \bigcup \{y_{i,j}; 1 \leq i \leq n, j = 1,2\}\}$ and a set of edge of $E(Bt_n \triangleright P_3) = \{\{x_i x_{i+1}; i = 1\} \bigcup \{x_i y_j; i = 1,2, 1 \leq j \leq n\} \bigcup \{x_1 x_{1,i}, i = 1,2\} \bigcup \{x_2 x_{2,i}, i = 1,2\} \bigcup \{x_{i,j} x_{i,j+1}; i = 1,2, j = 1\} \bigcup \{y_i y_{i,j}; i \leq i \leq n, j = 1,2\} \bigcup \{y_{i,1} y_{i,2}; 1 \leq i \leq n\}\}$. The vertex cardinality of the graph $Bt_n \triangleright C_3$ is $|V(Bt_n \triangleright C_3)| = 3n+6$, and the edge cardinality of the graph $Bt_n \triangleright P_2$ is $|E(Bt_n \triangleright C_3)| = 5n+7$. Let $D(Bt_n \triangleright C_3) = \{\{x_{i,1}; i = 1,2\} \bigcup \{y_{i,1}, 1 \leq i \leq n\}\}$, then $|D(B_n \triangleright C_3)| = n+2$. A set of D is perfect dominating set if each vertex $V(B_t n \triangleright C_3) \cap D$ dominates each vertex $x_i, x_{i,2}, y_i$, and $y_{i,2}$, or $V(Bt_n \triangleright C_3) - D$ is dominated by each vertex $x_{i,1}$, and $\{y_{i,2}\}$, then we get $|N(V) \cap D| = 1$. Then, a set of D is the perfect dominating set.

Additionally, we proved that the minimum cardinality of the resolving perfect dominating is W. For example $\gamma_{rp}(Bt_n \triangleright C_3) < n$, take $|W| = n+1$, so that

(i) If $W = \{\{x_{i,1}; i = 1, 2\} \bigcup \{y_{i,1}, 1 \le i \le n-1\}\}$ then, there are vertices that are not dominated by W, among others $y_n, y_{n,1}, x_{y,2} \in V(Bt_n \triangleright C_3)$, so D is not the perfect dominating set of the graph $Bt_n \triangleright C_3$.

(ii) If $W = \{\{x_i; i = 1, 2\} \bigcup \{y_i; 1 \le i \le n-1\}\}$ then, there are vertices that are not dominated by W, among others $y_{n,1}, x_{y,2} \in V(Bt_n \triangleright P_3)$, and $x_n \in V(Bt_n \triangleright C_3)$ dominated twice by W, so D is not the perfect dominating set of the graph $Bt_n \triangleright C_3$.

(iii) If $W = \{\{x_{i,2}; i = 1, 2\} \bigcup \{y_{i,2}, 1 \le i \le n-1\}\}$ there are vertices that are not dominated by W, among others $y_n, y_{n,1}, x_{y,2} \in V(Bt_n \triangleright C_3)$, so D is not the perfect dominating set of the graph $Bt_n \triangleright C_3$.

We show that D is a resolving set of graph $Bt_n \triangleright C_3$ if each vertex $v \in V(Bt_n \triangleright C_3)$ has a different vertex representation with respect to W where $r(u|W) \ne r(v|W)$ is described below. Table 5 shows that $v \in V(K_n \triangleright P_3)$ has a different vertex representation with respect to D, because having different vertex representations, it can be accomplished that D is the resolving perfect dominating set. The function can see in Table 5.

Table 5. Representation of vertex $v \in V(Bt_n \triangleright C_3)$ respect to W

v	$r(v \mid W)$	$Condition$
x_i	$(\underbrace{....2}_{i-1}, 1, \underbrace{2....}_{n-i+2})$	$i = 1, 2, n \ge 2$
$x_{i,1}$	$(\underbrace{....3}_{i-1}, 0, \underbrace{3....}_{n-i+2})$	$i = 1, 2, n \ge 2$
$x_{i,2}$	$(\underbrace{....3}_{i-1}, 1, \underbrace{3....}_{n-i+2})$	$i = 1, 2, n \ge 2$
y_i	$(2, 2\underbrace{3....3}_{i-1}, 1, \underbrace{3....}_{n-i})$	$1 \le i \le n, n \ge 2$
$y_{i,1}$	$(3, 3, \underbrace{4...4}_{i-1}, 0, \underbrace{4...4}_{n-i})$	$1 \le i \le n, n \ge 2$
$y_{i,2}$	$(3, 3, \underbrace{4....4}_{i-1}, 1, \underbrace{4....4}_{n-i})$	$1 \le i \le n, n \ge 2$

We can conclude that the minimum cardinality of the perfect dominating set on the graph $Bt_n \triangleright C_3$ with $\gamma_{rp}(Bt_n \triangleright C_3) = n + 2$ is W. As an example, resolving perfect dominating set of $Bt_n \triangleright P_3$ can be seen in Figure 8, and the function can see in Table 5.

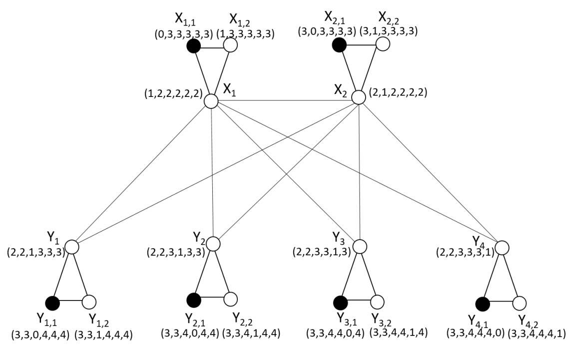

Figure 8. $\gamma_{rp}(Bt_4 \triangleright C_3) = 6$

3. Conclusion

The resolving perfect dominating number was analyzed in this study, based on the research obtained $K_n \triangleright P_2$ with $n \ge 3$, $K_n \triangleright P_3$ with $n \ge 3$, $B(2, n) \triangleright P_2$ with $n \ge 2$, $B(2, n) \triangleright P_3$

with $n \geq 2$, $Bt_n \triangleright C_3$ with $n \geq 2$, and $Bt_n \triangleright B(2,2)$. Base on research $\gamma_{rp}(K_n \triangleright P_2) = n$, $\gamma_{rp}(K_n \triangleright P_3) = n$, $\gamma_{rp}(Bt_n \triangleright P_2) = n + 2$, $\gamma_{rp}(Bt_n \triangleright P_2) = n + 2$, and $\gamma_{rp}(Bt_n \triangleright C_3) = n + 2$.

Open Problem

Find the resolving perfect dominating number for other graph operations.

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