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Functional materials under elasto-plastic deformation

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Functional materials under elasto-plastic deformation

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Abstract. A theoretical study of the factors affecting the workpiece accuracy under conditions of local plastic deformation has been carried out. The study has shown that when a two-support workpiece is bent by the force applied in the center along its length, plastic deformation occurs only at 1/3 of its length, and 2/3 remains under the elastic deformation. The paper analyzes the technologies and features of transient local plastic deformation, the factors influencing geometry fidelity are determined. The author gives the load function values under which the elastic deformation occurs as well as the carrying capacity loss load for the ideally plastic material unhardened. The function for calculating the values of elastic and plastic deformation zones is determined, which makes it possible to eliminate the need for additional calculations of the plastic deformation zone boundaries.

1. Introduction

The complexity of calculations for elasto-plastic deformation stems from the fact that the dependences of stress σ on deformation ε in the areas of elastic and plastic deformations are different, besides it is necessary to determine the boundaries between the elastic and plastic zones.

For the entire range of deformations, R. Hill [1] used a unified function $\sigma = \sigma_{\text{T}}\text{th}\left(\frac{E\varepsilon}{\sigma_{\text{T}}}\right)$ where E is modulus of elasticity; σ_{T} is yield point; $\sigma(\varepsilon)$ is nonlinear function under plastic deformation.
At small values of deformation ε, the function takes the form of Hooke's law: $\sigma = E\varepsilon$. As ε increases, the function differs slightly from σ_{T}.

The functional relationship of stress and deformation can be expressed in terms of coefficients a_i, b_i which are determined by the experimental data: $\sigma(\varepsilon) = \sum_{a_i}^{n}\frac{b_i}{a_i}[1 - \exp(-a_i\varepsilon)]$.
In the operations of drawing cylindrical workpieces, metal flows radially to the axis and the workpiece compression deformation occurs in the tangential direction, the circle radius of the disk workpiece decreases from R_0 to R_1. In this case, tensile stresses act in the radial direction along the "r" axis. Let us put stresses as $\sigma_r, \sigma_\theta, \sigma_z, \tau_{rz}$, then we can conclude that $\sigma_r > 0$, and $\sigma_\theta < 0$ while the tangential stress τ_{rz} for deformation of thin-sheet blanks is set to zero. At $R_1<R_0$, there occur tensile stresses $\sigma_z > 0$ created by the deformation tool. On his earlier occasions [2] L.A. Shofman gave the condition of instability in the form of $\frac{R_0-R_1}{h} \geq C$ where R_0 is workpiece outer radius, R_1 is radius of the formed cylindrical part, h is sheet thickness, C is a constant depending on the workpiece material properties and the lubrication norms. Usually "C" is equal to 43-46, therefore, for example, a sheet with thickness of h = 0.5 mm can produce a cylindrical workpiece with radius of R_1= 50 mm only of limited length l_{max}. Value $(R_0 - R_1)$ should not exceed value Ch. With C = 45, $(R_0 - R_1)$ should not exceed 45·0.5 = 22.5 mm, i.e. $(R_0 - R_1) < Ch$, R_0- 50 < 22.5 or R_0 <72.5 mm.

2. Theoretical bases of plastic deformation of functional materials

Elasto-plastic bending is used to straighten metal sheets and produce curved blanks, for example, profile bending. Thereby, in the area of elastic deformation, the curvature K is determined by the Hooke's law and the flat cross-section hypothesis [3-5], according to which the bending deformation is equal to $\varepsilon= K \cdot y$, with y being the distance to neutral axis. Since $\sigma = \mathrm{E} \cdot k \cdot y$, the bending moment for symmetrical rectangular sections with dimensions bxh is taken as:

$$\mathrm{M}(x) = 2\int_0^{0.5h} bEky^2 dy = EKI \qquad (1)$$

where I is the workpiece second area moment.

For a rectangular section it is:

$$\mathrm{M(x)} = \frac{Ebh^3}{12} K \text{ или } K = \frac{6Px}{Ebh^3}$$

Let us introduce two dimensionless parameters

$$\mathrm{a} = \frac{\sigma_\text{т} l}{Eh}, \ \mathrm{m} = \frac{Pl}{4\sigma_\text{т} bh^2}. \qquad (2)$$

The first of them characterizes the ratio of the elastic and plastic metal properties, while the second one is load intensity relative to plastic deformation resistance.

Then, curvature in the elastic region is

$$K = \frac{24amx}{l^2} \qquad (3)$$

With $m = \frac{1}{6}$, plastic deformation begins when $x = 0.5l; y = \pm 0.5h;\ \sigma = \sigma_m$. At $x > \frac{l}{12m}$ the curvature in the plastic deformation zone is:

$$K = \frac{2a}{l\sqrt{3}\sqrt{1-4m}} \qquad (4)$$

The curvature reaches its maximum value at $\frac{x}{l} = 0.5$, when $K(0{,}5l) = \frac{2a}{l\sqrt{3}\sqrt{1-4m}}$. It is apparent that if $m \to 0{,}25, K(0{,}5l) \to \infty$ and this corresponds to the loss of workpiece load-carrying capacity for the material unhardened. The advantages of using parameters "a" and "m" are that the curvature linearly depends on parameter "a" in both elastic and plastic deformations. Plastic deformation nonlinearity is peculiar only to the curvature dependence on load parameter "m".

If bending moment is defined as function $M(x)$ and the deformation corresponding to bending $\varepsilon = Ky$, where K is neutral line curvature [6-9], then we obtain the equation:

$$M(x) = 2\int_0^{0.5h} yb\sigma \mathrm{d}y \qquad (5)$$

where $b(y)$ is the width of a symmetrical section workpiece, h is its maximum thickness.

For a rectangular section workpiece $b \times h$:

$$M(x) = \frac{\sigma_\text{т} bh^2}{4} - \frac{2b}{K^2}\left(\frac{b_1}{a_1^3} + \frac{b_2}{a_2^3}\right)$$
$$+ \frac{bh}{K}\left[\frac{b_1}{a_1^2}\left(1 + \frac{2}{a_1 Kh}\right)\exp(-0.5a_1 Kh) + \frac{b_2}{a_2^2}\left(1 + \frac{2}{a_2 Kh}\right)\exp(-0.5a_2 Kh)\right]$$

Let us introduce the dimensionless characteristic of curvature $\varphi = \frac{Kl}{a}$ where $a = \frac{\sigma_T l}{Eh}$, l is length of the workpiece to be deformed:

$$\frac{4M(x)}{\sigma_T bh^2} = 1 - \frac{8}{\varphi^2} + \frac{4}{\varphi}\left(1 + \frac{2}{\varphi}\right)\exp(-0.5\varphi) \qquad (6)$$

For a two-support workpiece subjected to bending by force P applied at the center of its length, $M(x) = 0.5Px$, we get:

$$2m\frac{x}{l} = f(\varphi) \qquad (7)$$

where $m = \frac{Pl}{\sigma_T bh^2}$ is a dimensionless parameter representative of load. As a rule, if $\leq \frac{2}{3}$, there occurs only elastic deformation, and a limiting value $m = 1.0$ corresponds to the loss of carrying capacity for the ideally plastic material unhardened. When $\frac{4M(x)}{\sigma_T bn^2} \to 1$, $\varphi \to \infty$, if $K \approx \frac{d^2V}{dx^2}$, with $V(x)$ being deflection.

Consequently, when we calculate the deformation of elasto-plastic bending of workpieces, in a number of cases, it is possible to use a unified function both for elastic and plastic deformation zones in order to determine the dependence of curvature on bending moment. In actual practice, all the analytical representation values are used with some deviations.

For example, the function of variables $x_1, x_{2,} \dots x_n$; $f(x_1, x_2, x_3 \dots x_n)$ experiences changes $f(x_i \pm \Delta x_i)$. For small values of Δx_i $(i = 1,2 \dots n)$, the possible deviation of the function from its nominal value equal to $f(x_{i0})$ is $\Delta f = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i} \Delta x_i$, where $\frac{\partial f}{\partial x_i}$ are partially differentiable functions. For the value of curvature (4), maximum deviation of this value under load is

$$\Delta K_m = \frac{\partial K_m}{\partial a}\Delta a + \frac{\partial K_m}{\partial l}\Delta l + \frac{\partial K_m}{\partial m}\Delta m$$

and in partial derivatives it is

$$\Delta K_m = \frac{2}{l \cdot 3^{\frac{1}{2}}(1-4m)^{\frac{1}{2}}}\Delta a - \frac{2a}{l^2 3^{\frac{1}{2}}}\frac{\Delta l}{(1-4m)^{\frac{1}{2}}\sqrt{1-4m}} + \frac{4a\Delta m}{l \cdot 3^{\frac{1}{2}}(1-4m)^{\frac{1}{2}}}$$

or in the values of relative deviations of the function and its arguments:

$$\frac{\Delta K_m}{K_m} = \frac{\Delta a}{a} - \frac{\Delta l}{l} + \frac{2\Delta m}{(1-4m)} \qquad (8)$$

$$\frac{\Delta K_m}{K_m} = C_i\frac{\Delta x_i}{x_i} = C_1\frac{\Delta a}{a} + C_2\frac{\Delta l}{l} + C_3\frac{\Delta m}{m} \qquad (9)$$

The maximum possible ratio error of $K_m$value is equal to the sum of ratio errors of arguments that determines the upper limit of possible deviation values C_1 =1, C_2 = -1, $C_3 = \frac{2m}{m}$. For example, if $\frac{\Delta a}{a}$ = 0.08; $\frac{\Delta l}{l}$ = - 0.06, then in worst-case scenario these values should be summed up and the maximum deviation $\frac{\Delta K_m}{K_m}$ caused by such fluctuations in both Δa and Δl is 0.08 + 0.06 = 0.14.

3. Results and discussion

The graph of function $C_3(m)$ in Figure1 shows a tendency towards an increase in possible curvature fluctuations with the increasing load. The possible error rises sharply with the approach of parameter *m* to its limiting value $m = 0.25$, and the function $C_3(m)$ has a minimum at m = 0.225, a minor error of load parameter $\frac{\Delta m}{m}$ of 0.01-0.02 may cause 50 to 100-fold accuracy errors in residual curvature values.

Let us consider the assessment of possible changes in the dimensionless parameters a and m. If $a = \frac{\sigma_T l}{Eh}$, then $\frac{\Delta a}{a} = \frac{\partial \sigma_T}{\sigma_T} + \frac{\Delta l}{l} - \frac{\Delta E}{E} - \frac{\Delta h}{h}$. If $\frac{\Delta \sigma_T}{\sigma_T}$ = 0.05; $\frac{\Delta l}{l}$ = 0.05; $\frac{\Delta E}{E} = -0.02$; $\frac{\Delta h}{h}$ =–0.02, then $\frac{\Delta a}{a}$ = 0.14

The dimensionless parameter determining the load value $m = \frac{Pl}{4\sigma_T b h^2}$ has a relative deviation

$$\frac{\Delta m}{m} = \frac{\Delta P}{P} + \frac{\Delta l}{l} - \frac{\Delta \sigma_T}{\sigma_T} - \frac{\Delta b}{b} - \frac{2}{h}\,\Delta h.$$

If $\frac{\Delta P}{P} = \frac{\Delta l}{l} = 0.05$; $\frac{\Delta \sigma_T}{\sigma_T}$ = – 0.05; $\frac{\Delta b}{b} = \frac{\Delta h}{h}$ = 0.01, the maximum relative deviation of parameter m is $\frac{\Delta m}{m}$ = 0.17.

If the deviations are equal to $\frac{\Delta a}{a} = 0.14$ and $\frac{\Delta m}{m} = 0.17$, $\frac{\Delta l}{l} = 0.05$, then the relative deviation of maximum value of curvature K_m at m = 0.2 is $\frac{\Delta K_m}{K_m}$ = 0.14 + 0.05 + $\frac{2\cdot 0{,}2}{0.2}$0.17 = 0.53,the coefficient $C_3 = 76.43$, a and the relative change in curvature is significant:

$$\frac{\Delta K_0}{K_0} = 0.14 + 0.05 + 76.43 \cdot 0.17 = 13.2,$$

that is an order of magnitude higher than the relative changes in maximum curvature, i.e. $\frac{\Delta K_0}{K_0} \gg \frac{\Delta K_m}{K_m}$.

For the case of $\frac{\Delta a}{a} = 0.10$; $\frac{\Delta l}{l} = -0.03$; $\frac{\Delta m}{m} = 0.15$; $\frac{\Delta l_0}{l_0} = -0.03$; $\frac{\Delta K_0}{K_0} = 0.22$ the limiting value of elastic deformation is $m_{min} = \frac{l}{12 l_0}$. In addition, K_0 = 0, and $\frac{\Delta K_0}{K_0} \to \infty$.

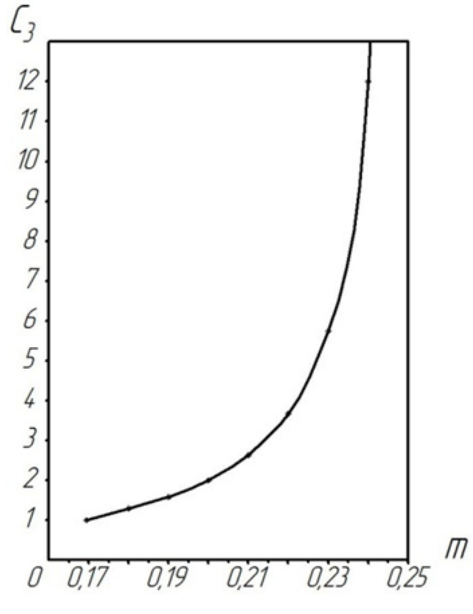

Figure 1. Graph of functions $C_3(m)$ for curvature function K_0

The maximum load value $m_{max} = \frac{l}{8 l_0}$ and $K_0 \to \infty$, as well as C_2, C_3 и C_4 tend to infinity.

Figure 2 shows graphs for a two-support beam loading by two equal forces. One can see that the values of C_2, C_3 and C_4 increase in magnitude if $\frac{l_0}{l} > 0.15$ and this increase grows with load and curvature. Therefore, the higher the degree of bending deformation, the rawer errors are, consequently, the lower dimensional accuracy of workpieces.

When $\frac{l_0}{l} = 0.5$, i.e. the beam is bent by one force applied in the middle of a workpiece length, its dimensional accuracy decreases.

When elastic-plastic deformation of the bend of the workpieces, flat sections remain in them. They must also be deformed to ensure that the cross-section curvature is constant.

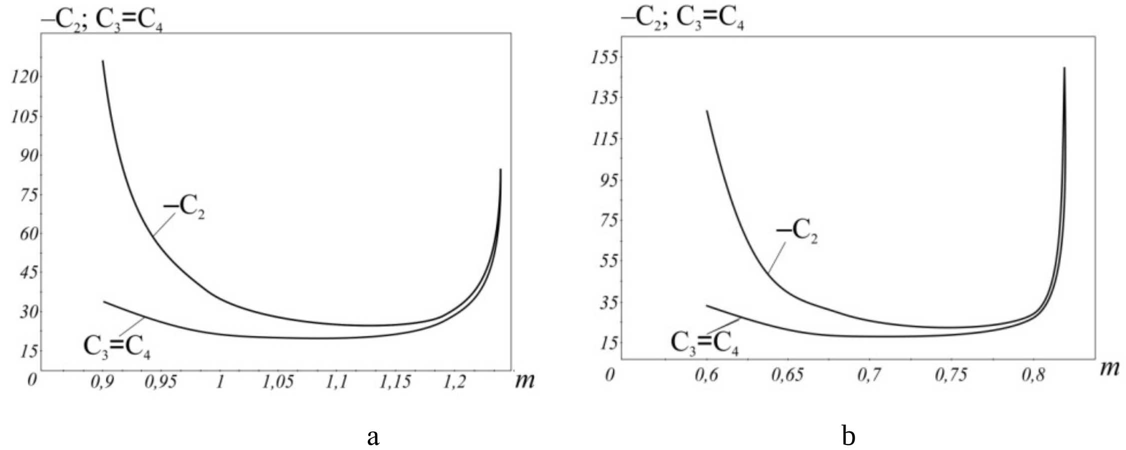

Figure 2. Graphs of coefficient functions $C_1 - C_4$ with different *m* values and $\frac{l_0}{l}$ for bending the workpiece by two equal forces: a - $\frac{l_0}{l} = 0.15$; b - $\frac{l_0}{l} = 0.20$

4. Conclusion

The estimation technique is developed for assessing the deformation forces influence on the curvature accuracy of a workpiece made of functional materials under conditions of elastic-plastic bending. Based on calculations, the author shows that when a workpiece is bent by one force applied between two supports, plastic deformation cannot occur beyond the 1/3 of the workpiece length; bending by two forces is preferable to one force applied in the middle of the span.

When developing the technology of plastic functional materials, the values of forces, residual stresses and the influence of deformation modes on them are determined. Residual stresses during bending with elastic-plastic deformation the residual stresses are combined with the working stresses that occur when the product is loaded during operation, this reduces the durability and can lead to a significant deterioration in product quality. It is shown that deviations of the function that characterizes the load of the order of 0.01-0.02 can cause an error in the values of the residual curvature 50 -100 times higher.

5. References

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