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Determining the Number of Disconnected Vertices Labeled Graphs of Order Six with the Maximum Number Twenty Parallel Edges and Containing No Loops

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Determining the Number of Disconnected Vertices Labeled Graphs of Order Six with the Maximum Number Twenty Parallel Edges and Containing No Loops

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Abstract. If there exist two vertices on a given graph that are not connected by a path, then we call that graph is disconnected. Given a graph with *n* vertices and *m* edges, then a lot of graphs can be constructed. In this paper, we discuss the number of disconnected vertices labeled graphs of order six (n = 6) with the maximum number of parallel edges is twenty. Moreover, a maximum number of edges that connect different pair of vertices is ten (parallel edges are counted as one) and containing no loops (isomorphic graphs are counted as one).

Keyword: counting graph, disconnected graph, vertices labelled graph.

1. Introduction

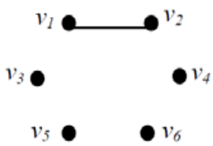
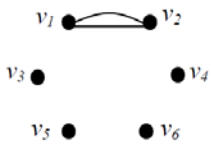
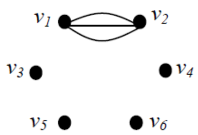
Historically, graph enumeration had been done by Cayley in 1874 [1]. Cayley used the concept of tree in his way to enumerating the number of the isomer of hydrocarbon C_nH_{2n+2} [2]. The basic concept of labeling and the enumerating graph was given by Harary and Palmer [3]. There are many ways to construct the disconnected graph of order *n*, *m* edges, and *t* edges that connecting the different pairs of vertices. In 2016, the number of disconnected vertices labeled graphs containing no parallel edges is observed [4], and in 2017, the number of disconnected vertices labeled graph of order maximum four is discussed [5]. Wamilana et al. continued their research to determine the number of connected vertices labeled graph of order five with the maximum number of parallel edges is five and containing no loops [6].

In this paper, we give the general form of the number of disconnected vertices labeled graphs of order six (n = 6), m edges, and t edges that connecting different pairs of vertices, with $1 \le m \le 20$, $1 \le t \le 10$, and containing no loops (isomorphic graphs are counted as one).

2. Construction and Observation

Given a graph G of order six (n = 6) with m edges, $1 \le m \le 20$, and the number of edges that connect the different vertices or points is t (the number of parallel edges that connect the same pair of points is counted as one), with $1 \le t \le 10$. Some of the patterns of a disconnected graph of order 6, m edges and t edges are given in the following table:

Table 1. The number of disconnected graph with n = 6, with $1 \le m \le 20$, $1 \le t \le 10$.

	Pattern	The number of graphs
$n = 6, t = 1, m = 1$		15
$n = 6, t = 1, m = 2$		15
$n = 6, t = 1, m = 3$		15

The following table shows the number of disconnected graph or order six (n = 6), m edges, and t edges that connecting the different pair of vertices, with $1 \le t \le 5$.

Table 2. The number of disconnected graph with n = 6, with $1 \le m \le 20$, $1 \le t \le 5$

m	The numbers of disconnected graphs				
	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
1	15	-	-	-	-
2	15	105	-	-	-
3	15	210	365	-	-
4	15	315	1095	975	-
5	15	420	2190	3900	957
6	15	525	3650	9750	4785
7	15	630	5475	19500	14355
8	15	735	7665	34125	33495
9	15	840	10220	54600	66990
10	15	945	12140	81900	120582
11	15	1050	16425	117000	200970
12	15	1155	20075	160875	315810
13	15	1260	24090	214500	473715
14	15	1365	28470	278850	315810
15	15	1470	33215	354900	957957
16	15	1575	38325	443625	1306305
17	15	1680	43800	546000	1741740
18	15	1785	49640	663000	2277660

19	15	1890	55845	795600	2928420
20	15	1995	62415	944775	3709332

The following table shows the number of disconnected graph or order six (n = 6), m edges, and t edges that connecting the different pair of vertices, with $6 \leq t \leq 10$.

Table 3. The number of disconnected graph with n = 6, $6 \leq m \leq 20$, $6 \leq t \leq 10$

m	The numbers of disconnected graphs				
	t = 6	t = 7	t = 8	t = 9	t = 10
6	715	-	-	-	-
7	4290	345	-	-	-
8	15015	2415	210	-	-
9	40040	9660	1680	60	-
10	90090	28980	7560	540	6
11	180180	72450	25200	2700	60
12	330330	159390	69300	9900	330
13	566280	318780	166320	29700	1320
14	920205	592020	360360	77220	4290
15	1431430	1036035	720720	180180	12012
16	2147145	1351350	1351350	386100	30030
17	3123120	2402400	2402400	772200	68640
18	4424420	4084080	4084080	1458600	145860
19	6126120	6683040	6683040	2625480	291720
20	8314020	10581480	10581480	4534920	554268

The number of disconnected graph with n = 6, with $1 \leq m \leq 20$, $1 \leq t \leq 5$ in Table 2 can be represented as follow:

Table 4. The number of disconnected graph with n = 6, with $1 \leq m \leq 20$, $1 \leq t \leq 5$

m	The numbers of disconnected graphs				
	t = 1	t = 2	t = 3	t = 4	t = 5
1	1 × 15	-	-	-	-
2	1 × 15	1 × 105	-	-	-
3	1 × 15	2 × 105	1 × 365	-	-
4	1 × 15	3 × 105	3 × 365	1 × 975	-
5	1 × 15	4 × 105	6 × 365	4 × 975	1 × 957
6	1 × 15	5 × 105	10 × 365	10 × 975	5 × 957
7	1 × 15	6 × 105	15 × 365	20 × 975	15 × 957
8	1 × 15	7 × 105	21 × 365	35 × 975	35 × 957
9	1 × 15	8 × 105	28 × 365	56 × 975	70 × 957
10	1 × 15	9 × 105	36 × 365	84 × 975	126 × 957
11	1 × 15	10 × 105	45 × 365	120 × 975	210 × 957
12	1 × 15	11 × 105	55 × 365	165 × 975	330 × 957
13	1 × 15	12 × 105	66 × 365	220 × 975	495 × 957
14	1 × 15	13 × 105	78 × 365	286 × 975	715 × 957
15	1 × 15	14 × 105	91 × 365	364 × 975	1001 × 957
16	1 × 15	15 × 105	105 × 365	455 × 975	1365 × 957
17	1 × 15	16 × 105	120 × 365	560 × 975	1820 × 957
18	1 × 15	17 × 105	136 × 365	680 × 975	2380 × 957
19	1 × 15	18 × 105	153 × 365	816 × 975	3060 × 957
20	1 × 15	19 × 105	171 × 365	969 × 975	3876 × 957

The number of disconnected graph with n = 6, with 1 ≤ m ≤ 20, 6 ≤ t ≤ 10 in Table 3 can be represented as follow:

Table 5. The number of disconnected graph with n = 6, 6 ≤ m ≤ 20, 6 ≤ t ≤ 10

m	The numbers of disconnected graphs				
	t = 6	t = 7	t = 8	t = 9	t = 10
6	1 × 715	-	-	-	-
7	6 × 715	1 × 345	-	-	-
8	21 × 715	7 × 345	1 × 210	-	-
9	56 × 715	28 × 345	8 × 210	1 × 60	-
10	126 × 715	84 × 345	36 × 210	9 × 60	1 × 6
11	252 × 715	210 × 345	120 × 210	45 × 60	10 × 6
12	462 × 715	462 × 345	350 × 210	165 × 60	55 × 6
13	792 × 715	924 × 345	792 × 210	495 × 60	220 × 6
14	1287 × 715	1716 × 345	1716 × 210	1287 × 60	315 × 6
15	2002 × 715	3003 × 345	3432 × 210	3003 × 60	2002 × 6
16	3003 × 715	5005 × 345	6435 × 210	6435 × 60	5005 × 6
17	4368 × 715	8008 × 345	11440 × 210	12870 × 60	11440 × 6
18	6188 × 715	12376 × 345	19440 × 210	24310 × 60	24310 × 6
19	8568 × 715	18564 × 345	31824 × 210	43758 × 60	48620 × 6
20	11628 × 715	27132 × 345	50388 × 210	75582 × 60	92378 × 6

3. Result and Discussion

Given a graph G with the number of vertex V is n = 6, the number of edges is m, with 1 ≤ m ≤ 20, and the number of edges that connect the different vertices is t (the number of parallel edges that connect the same pair of vertices is counted as one), with 1 ≤ t ≤ 10.

After the construction and observation, we get the following results. The disconnected vertices labelled graph with order n, m edges, and t edges that connect the different pairs of vertices is denoted by $G'(p)_{n,m,t}$. The number of $G'(p)_{n,m,t}$ is denoted by $N\big(G'(p)_{n,m,t}\big)$.

Result 1. *If n = 6, m = 1, 2, ..., 20, and t = 1, then* $N\big(G'(p)_{6,m,1}\big) = 15$.

Proof. Based on Tabel 4 column t = 1, we have the following sequence of number:

1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

The sequence form polynomial of order zero, with general form $a_m = a_0$.

For m = 1, we have $a_m = 15$. Therefore, the number of disconnected vertices labelled graph with $n = 6$, m = 1, 2, …, 20, and t = 1 is $N\big(G'(p)_{6,m,1}\big) = 15$.

Result 2. *If n = 6, m = 2, 3,..., 20, and t = 2, then* $N\big(G'(p)_{6,m,2}\big) = 105 \times C_1^{(m-1)}$.

Proof. Based on Tabel 4 column t = 2, we have the following sequence of number:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

Since the fix difference in the sequence occurs in the first order, the general form of the polynomial is

$$a_m = a_1 m + a_0. \quad (1)$$

By substituting $m = 2$ and 3 to Equation (1), we have:

$$105 = 2a_1 + a_0 \quad (2)$$

$$210 = 3a_1 + a_0 \quad (3)$$

Solving this linearly equation system, we get $a_0 = -105$, $a_1 = 105$ and hence

$N\big(G'(p)_{6,m,2}\big) = 105\, m - 105 = 105(m-1) = 105 \times \mathrm{C}_1^{(m-1)}$.

Result 3. *If n = 6, m = 3, 4, 5, ..., 20, and t = 3, then* $N\big(G'(p)_{6,m,3}\big) = 365 \times C_2^{(m-1)}$.
Proof. Based on Table 4 column t = 3 and m = 3, 4, 5, …, 20, we get the following sequence:

1 3 6 10 15 21 28 36 45 55 66 78 91 105 120 136 153 171
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Since the fix difference in the sequence occurs in the second order, the general form of the polynomial is

$$a_m = a_2 m^2 + a_1 m + a_0. \qquad (4)$$

By substituting $m = 3, 4$ and 5 to Equation (4), we have:

$$365 = 9a_2 + 3a_1 + a_0 \qquad (5)$$
$$1095 = 16a_2 + 4a_1 + a_0 \qquad (6)$$
$$2190 = 25a_2 + 5a_1 + a_0 \qquad (7)$$

Solving this linearly equation system, we get $a_2 = \frac{365}{2}$, $a_1 = -\frac{1095}{2}$, $a_0 = \frac{730}{2}$ and hence
$N\big(G'(p)_{6,m,3}\big) = \frac{365}{2}m^2 - \frac{1095}{2}m + \frac{730}{2} = 365 \times \mathrm{C}_2^{(m-1)}$.

Result 4. *For n = 6, m = 4, 5, ..., 20, and t = 4,* $N\big(G'(p)_{6,m,4}\big) = 975 \times C_3^{(m-1)}$.
Proof.
Based on Table 4 column t = 4 and m = 4, 5, …, 20, we get the following sequence: 1, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286, 364, 455, 560, 680, 816, 969. This sequence is related to arithmetic polynomial of order three.

1 4 10 20 35 56 84 120 165 … 560 680 816 969
3 6 10 15 21 28 36 45 … 120 136 153
3 4 5 6 7 8 9 … 16 17
1 1 1 1 1 1 … 1

Since the fix difference in the sequence occurs in the third order, the general form of the polynomial is

$$a_m = a_3 m^3 + a_2 m^2 + a_1 m + a_0. \qquad (8)$$

By substituting $m = 4, 5, 6$ and 7 to Equation (8), we have:

$$975 = 64a_3 + 16a_2 + 4a_1 + a_0 \qquad (9)$$
$$3900 = 125a_3 + 25a_2 + 5a_1 + a_0 \qquad (10)$$
$$9750 = 216a_3 + 36a_2 + 6a_1 + a_0 \qquad (11)$$
$$19500 = 343a_3 + 49a_2 + 7a_1 + a_0 \qquad (12)$$

Solving this linearly equation system, we get $a_3 = \frac{19500}{12}$, $a_2 = -\frac{11700}{12}$, $a_1 = \frac{21450}{12}$, $a_0 = -\frac{11700}{12}$ and hence
$N\big(G'(p)_{6,m,4}\big) = \frac{19500}{12}m^3 - \frac{11700}{12}m^2 + \frac{21450}{12}m - \frac{11700}{12} = 975 \times \mathrm{C}_3^{(m-1)}$.
In a similar way as the Result 1-4, we can provide the following formulas:
Result 5. For n = 6, m = 5, 6, …, 20, and t = 5, $N\big(G'(p)_{6,m,5}\big) = 957 \times \mathrm{C}_4^{(m-1)}$.
Result 6. For n = 6, m = 6, 7, …, 20, and t = 6, $N\big(G'(p)_{6,m,6}\big) = 715 \times \mathrm{C}_5^{(m-1)}$.
Result 7. For n = 6, m = 7, 8, …, 20, and t = 7, $N\big(G'(p)_{6,m,7}\big) = 345 \times \mathrm{C}_6^{(m-1)}$.
Result 8. For n = 6, m = 8, 9, …, 20, and t = 8, $N\big(G'(p)_{6,m,8}\big) = 210 \times \mathrm{C}_7^{(m-1)}$.
Result 9. For n = 6, m = 9, 10, …, 20, and t = 9, $N\big(G'(p)_{6,m,9}\big) = 60 \times \mathrm{C}_8^{(m-1)}$.
Result 10. For n = 6, m = 10, 11, …, 20, and t = 10, $N\big(G'(p)_{6,m,10}\big) = 6 \times \mathrm{C}_9^{(m-1)}$.

4. Conclussion

From the results, we can conclude that the number of disconnected vertices labeled graphs of order six (n = 6), m edges and t edges that connecting different pairs of vertices, with $1 \le m \le 20$, $1 \le t \le 10$, $t \le m$, and containing no loops is $N\big(G'(p)_{6,m,t}\big) = k_t \times \mathrm{C}_{(t-1)}^{(m-1)}$, where $k_1 = 15,\ k_2 = 105,\ k_3 = 365, k_4 = 975, k_5 = 957, k_6 = 715, k_7 = 345, k_8 = 210, k_9 = 60,$ and $k_{10} = 6$.

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