Journal of Physics: Conference Series



# An ambiguity function transformation characteristic for Radar signal sorting

To cite this article: Jun Han *et al* 2021 *J. Phys.: Conf. Ser.* **1738** 012034

# An ambiguity function transformation characteristic for Radar signal sorting

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**Abstract.** The sorting rate of current methods is not high and too sensitive to the signal noise ratio (SNR). In order to solve this problem, a novel characteristic for sorting radar emitter signal is proposed. The ambiguity function Radon curve of received signal is extracted, and then a rectangle pulse sequence and a triangle pulse sequence are constructed. Furthermore, resemblance coefficients of the ambiguity function Radon curve with two pulse sequences are extracted, and are used as the sorting parameters. The advantage of this new characteristic is validated by simulation result, and the lowest sorting rate is 90%at SNR=10dB.

## 1. Introduction

With the use of more and more new complex radar systems, more and more problems need to be solved. Most of the existing sorting algorithms are based on the analysis of conventional parameters such as arrival time, arrival direction, carrier frequency and pulse width of intercepted signals. PRI sorting algorithm is a popular sorting method, including sequence difference histogram, sequence difference histogram, PRI variable conversion method and improved PRI transform algorithm [1][2]. However, these algorithms have some limitations and are difficult to adapt to the complex electromagnetic environment. Literature [3] classified traditional radar radiation source signals according to arrival direction, carrier frequency, pulse width and pulse amplitude. However, when the carrier frequency and pulse width change frequently, the separation rate will be greatly reduced.

Pulse internal characteristic is one of the most important characteristic parameters of radar emitter signal. Although some of the conventional parameters of radar emitter signals change frequently, they are stable. At present, some scholars have classified radar emitter signals by using intra-pulse characteristic parameters such as entropy, similarity coefficient and complexity, and achieved some success [4-6], but these characteristic parameters are more sensitive to noise. This paper puts forward a new sorting algorithm: the ambiguity function Radon curve of received signal is extracted, and then a rectangle pulse sequence and a triangle pulse sequence are constructed. Furthermore, resemblance coefficients of the ambiguity function Radon curve with two pulse sequences are extracted, and are used as the sorting parameters. Simulation result shows that this method can sort different modulation radar emitter signals at low SNR exactly.

## 2. Ambiguity function

*2.1. Ambiguity function transformation*

The ambiguity function which lies on the signal wave is an effective tool for analyzing the radar emitter signal and designing wave [7]. Owing to the ambiguity function describes signal structure information inextenso, difference between different signal structure information which is useful for sorting signal can be obtained. Following, the definition of ambiguity function and its two important qualities are given.

Ambiguity function of $x(t)$ is

$$\chi(\tau, f_d) = \int_{-\infty}^{\infty} u(t)u^*(t+\tau)\mathrm{e}^{\mathrm{j}2\pi f_d t}\mathrm{d}t \tag{1}$$

Where $\tau$ is time delay, $f_d$ is the Doppler frequency shift, $u(t)$ is the complex envelope.

The ambiguity function has two important qualities.

(1) The cubage immutable

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\chi(\tau, f_d)|^2 \mathrm{d}\tau \mathrm{d}f_d = |\chi(0,0)|^2 = (2\mathrm{E})^2 \tag{2}$$

(2) Uniqueness

Assuming $\chi_1(\tau, f_d)$ and $\chi_2(\tau, f_d)$ are the ambiguity functions of signal $x_1(t)$ and $x_2(t)$ respectively. The difference between $x_1(t)$ and $x_2(t)$ is equal to constant c which modulus is equal to 1 if $\chi_1(\tau, f_d) = \chi_2(\tau, f_d)$ namely

$$x_1(t) = cx_2(t) \quad |c| = 1 \tag{3}$$

From (1), we could know that the ambiguity function is Doppler Frequency Shift form of signal $x(t)$ match filter essentially, and it is the two-dimension (2-D) time-frequency expression in plane $\tau$ and $f_d$. The ambiguity function provides the similarity measure between the signal and the expression of delay and frequency shift, and reflects the internal structure information of the signal. (2) It can be seen that the fuzzy area is only related to the signal power and has nothing to do with the signal expression, but different signal expressions will distribute the fuzzy area in different forms. It can be seen from (3) that different signals have different fuzzy degree functions. Therefore, the fuzzy degree difference function is selected in this paper to classify radar emitter signals.

*2.2. Extracting radon curve*

As ambiguity function is three-dimension (3-D) characteristic and it is not easy to sort signal, so we need to predigest it to 2-D characteristic. In the process of predigesting, we need to hold two principles, one is to reduce computation as soon as possible for ensuring the operating speed and the other is the predigested characteristic can reflect the characteristic of the ambiguity function as soon as possible. This paper selects Radon transformation which can realize the full angle observation of the image.

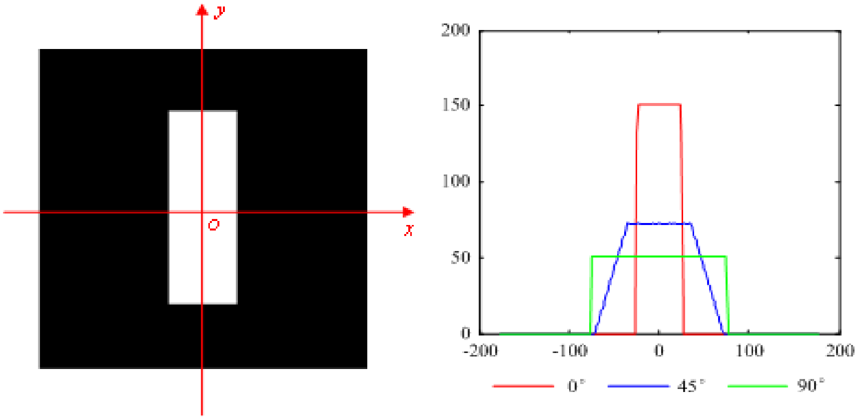
Radon transformation is based on the idea of projective integrals, it takes the line integral in a certain direction and projects the integral onto the transformation plane, and then the Radon curve is got. The curve of the graph along the $\theta$ direction is defined as

$$g(s,\theta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(x\cos\theta + y\sin\theta - s)\mathrm{d}x\mathrm{d}y \tag{4}$$

Where, $f(x,y)$ is the original image, $g(s,\theta)$ is the Radon curve, $\delta(\bullet)$ is impact function. From (4), we could know Radon curve $g(s,\theta)$ can be got while $f(x,y)$ takes line integral along the $l(s = x\cos\theta + y\sin\theta)$ .

Take a binary rectangle block of 150×50 as an example, the white area is 1, and the other area is 0. The Radon transform simulation is shown in figure1. Where, figure (a) takes the center of the rectangular block as the origin, and established the coordinate system xOy with the horizontal and vertical direction as the horizontal and vertical axes respectively. The horizontal axis of figure (b) is the coordinate value s when the corresponding projection Angle is $\theta$, and the vertical axis is the Radon transform value.

It can be seen from figure 1 that the Radon curve of different projection angles are different, indicating that the rectangular blocks have different integral values in different direction. Therefore, Radon transform can be used to simplify the processing of 3d graphics.




(a)Original image (b)Radon transformation in different direction
Figure 1. Simulation diagram of Radon transformation

In order to make the simplified Radon curve of the ambiguity function can distinguish the difference types of radar emitter signals to the greatest extent. The following two definitions are given to measure the degree of separation between signals.

the difference $B_{i,j}$ between signal $i$ and signal $j$ is defined as

$$B_{ij} = \| X_i - Y_j \|_2 \tag{5}$$

Where, $X_i$ and $Y_j$ respectively are the normalized ambiguity function Radon curve of signal $i$ and signal $j$, $X_i = \{x_{i1}, x_{i2}, \cdots, x_{iN_i}\}$, $N_i$ and $N_j$ respectively are the sequence length of normalized ambiguity function Radon curve of signal $i$ and signal $j$, $\|\bullet\|_2$ means 2 norm.

if the type of radar emitter signal used for feature extraction is H, then the separation degree of H signals is defined as

$$f = \frac{2}{H(H-1)} \sum_{i=1}^{H-1} \sum_{j=i+1}^{H} B_{ij} \tag{6}$$

According to (6), the larger f is, the better separation degree between signals is.

When the projection angel is different, the degree of separation between signals is different, we need to find the best projection angel to distinguish the signal to the greatest extent, considering the Radon transform with symmetry and periodicity, therefore, only the projection angel of 0 to 90 every 10 division, through a large number of simulations, the signal separation degree is different while the projection angel is different in each SNR, and when the projection angel is 90, the overall maximum signal separation degree, therefore using projection angel for 90 to simplify the Radon transform processing.

In order to visualize the difference of ambiguity function Radon curve of different radar emitter signal, CW, LFM, FSK, BPSK, QPSK, LFM-BPSK, FSK-BPSK, NLFM are given in the case of no noise. The ambiguity function Radon curve is shown in figure 2.

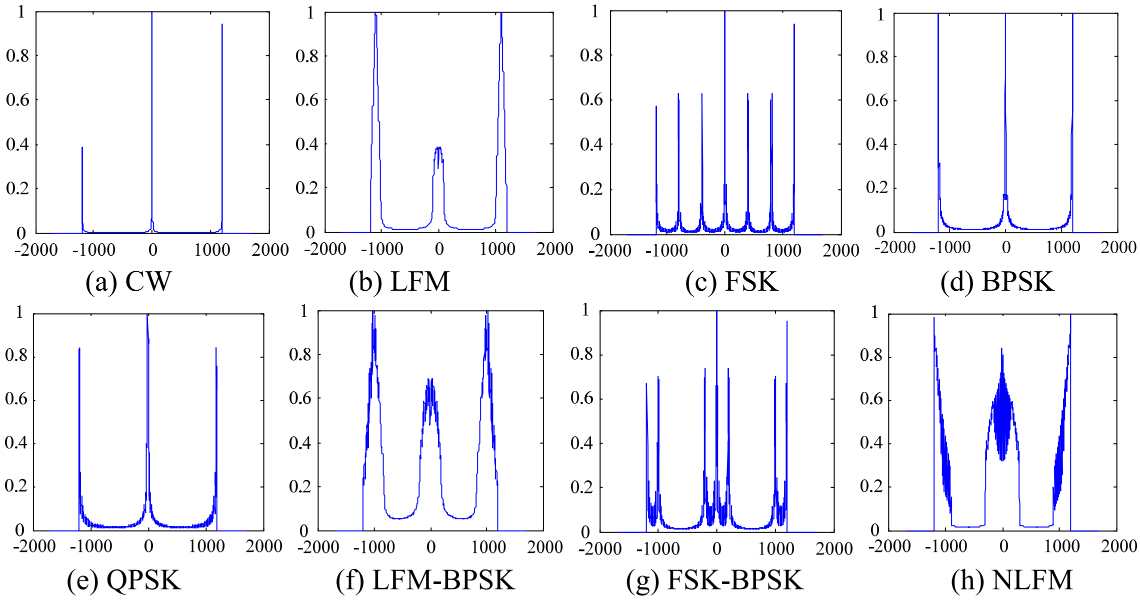



Figure 2. The ambiguity function Radon curve of class 8 radar emitter signals

It can be seen from figure 2 that the ambiguity functions Radon curve of different radar emitter signals have certain difference, which can be used to characterize the signal energy feature and distinguish each other. In order to reflect the influence of noise on the ambiguity functions Radon curve, LFM and FSK-BPSK are taken as example.

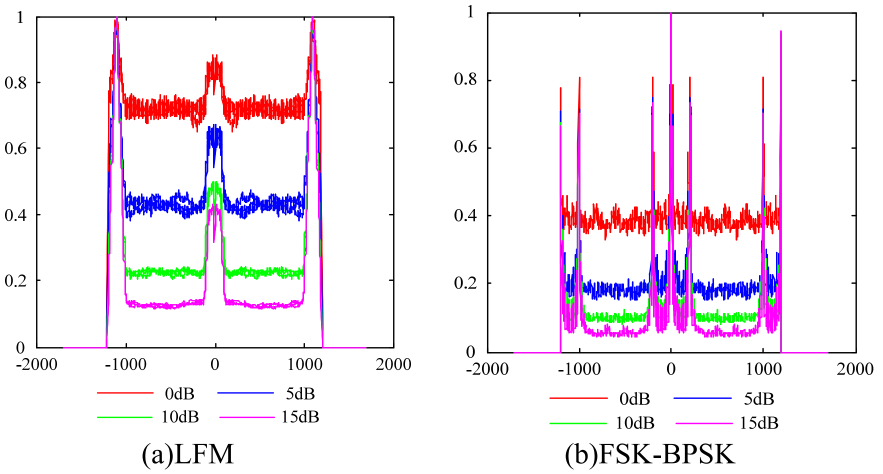



Figure 3. The ambiguity function Radon curve of LFM and FSK-BPSK in different SNR

As can be seen from figure 3, when the noise are superimposed in the signal, the middle region of the ambiguity function Radon curve has a position offset, and there is little change except that, and the greater the noise, the greater the position offset. Therefore, the ambiguity function Radon curve not only guarantees the difference between the different radar emitter signals, but also has good anti-noise performance, and can realize the time-frequency analysis of radar emitter signals.

## 3. Extracting Resemblance Coefficient

From the analysis above, the ambiguity function Radon curve distinguishes different radar emitter signals, but its dimension is too big and not good to accomplish rapid sorting, so the dimension reduction of ambiguity function Radon curve is needed. According to Fig. 2, ambiguity function Radon curve of different radar emitter signals have different geometric scales and sparse characteristics, so this paper considers constructing a rectangle pulse sequence and a triangle pulse sequence and calculating similar degree between ambiguity function Radon curve and two pulse sequences, namely resemblance coefficient. The definition of resemblance coefficient is as follows: If two one-dimensional discrete non-negative signals are $\{S_1(i)\}_{i=1}^{N}$ and $\{S_2(i)\}_{i=1}^{N}$, the coefficient of them is defined as:

$$C_r = \frac{\sum_{i,j=1}^{N} S_1(i)S_2(j)}{[\sum_{i=1}^{N} S_1^2(i)\sum_{j=1}^{N} S_2^2(j)]^{1/2}} \tag{7}$$

Where, $C_r$ is the resemblance coefficient of signal sequence $\{S_1(i)\}_{i=1}^{N}$ and $\{S_2(i)\}_{i=1}^{N}$. According to Cauchy-Schwarz inequality [8]: $(\sum_{i=1}^{n} a_i b_i)^2 \le \sum_{i=1}^{n} a_i^2 \sum_{i=1}^{n} b_i^2$, (7) has the following relation:

$$0 \le C_r \le 1 \tag{8}$$

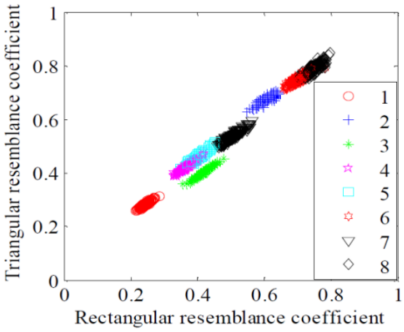
According to (7) and (8), when two signal sequences have the linear relation, namely $S_1(i) = kS_2(j)(k>0)$, $C_r$ equals to 1, when two signal sequences are orthogonal and not related to each other, $C_r$ equals to 0, two signal sequences are more similar, the resemblance coefficient is bigger.

In the paper, rectangle pulse sequence $rect(k)$ and triangle pulse sequence $tri(k)$ are defined as (9) and (10), where N is length of ambiguity function Radon curve sequence.

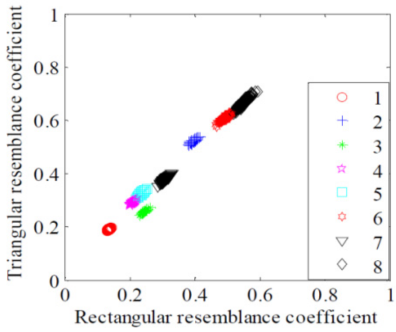
$$rect(k) = \begin{cases} 1 & 1 \le k \le N \\ 0 & else \end{cases} \tag{9}$$

$$tri(k) = \begin{cases} 2k/N & 1 \le k \le N/2 \\ 2-2k/N & N/2 < k \le N \end{cases} \tag{10}$$

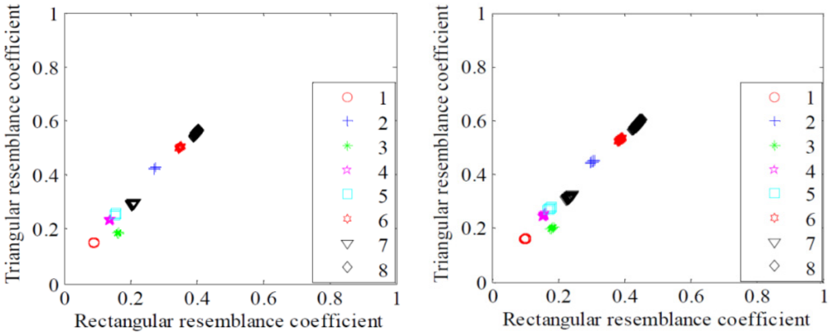
At the SNR of 0dB, 5dB, 10dB and 15dB, resemblance coefficients of ambiguity function Radon curve with two pulse sequences are calculated. Each kind of radar emitter signals is taken 200 signals. Two-dimensional histograms of resemblance coefficient of each kind at corresponding SNR are shown in Fig. 4. In Fig. 4, number one to eight separately stands for the signal of CW, LFM, FSK, BPSK, QPSK, LFM-BPSK, FSK, BPSK and NLFM.




(a)SNR=0dB




(b)SNR=5dB




(c)SNR=10dB (d)SNR=15dB

Figure 4. Two-dimensional histograms of resemblance coefficient

According to Fig. 4, when SNR is low, part signals exist overlapping phenomenon and signals are scattered. When SNR is high, signals are distinguished easily and concentrated. For one thing, it explains that resemblance coefficients of different radar emitter signals exist differences. Characteristic parameters have good separability and the greater SNR, the better separability. For another, it explains that characteristic parameters have good stability and the greater SNR, the better stability.

## 4. Simulations

In order to analyze the performance of ambiguity function transform characteristic, KFCM is used for the sorting experiments [9][10]. Eight kinds of radar emitter signals are used in the simulation, which are CW, LFM, FSK, BPSK, QPSK, LFM-BPSK, and NLFM. The signal of FSK has two frequency points of 20MHz and 40MHz, the signal of FSK-BPSK has two frequency points of 20MHz and 40MHz, the rest have the carrier frequency of 30MHz, pulse width of l0us, and sampling frequency is 120MHz. The band width of LFM signal is 2MHz, the encoding law of FSK signal is [100110], the encoding law of BPSK signal is [11100010010], the encoding law of QPSK signal is [01230312211300112012], the band width of LFM-BPSK signal is 5MHz and encoding law is [11100010010], both the carrier frequency and encoding law of the FSK-BPSK signal are [11100010010], signal of NLFM is sine signal. In the SNR of 5dB, l0dB, 15dB and 20dB, create 100 signals of each kind.

KFCM algorithm is used to sort the eight kinds of radar emitter signals. A simulation experiment is made in each SNR. The characteristics used are resemblance coefficients of ambiguity function Radon curve. Oriental cluster number is c=2, largest cluster number is Cmax=8, iteration number T is 50, stop condition is ≤0.001, kernel function is Guassian Radial Basis. Table 1 shows the eight signals' sorting accuracy in different SNR conditions. From Table 1, we can know that when SNR is 20dB or 15 dB, eight signals' sorting accuracy is 100%, with the SNR goes low, sorting accuracy is decreasing at the same time. When the SNR is 10dB, we can know that the characteristics of BPSK and QPSK are partly overlapped, as a result the sorting accuracy decreases, while the rest are still 100%. When the SNR is 5dB, overlapping degree of the eight kinds of signal's characteristics complexity increases, sorting accuracies decrease, lowest of them is 90%, but the degree of separation of some signals are still excellent and sorting accuracy is 100%. From the analysis above, we know the accuracies are still satisfactory in Low SNR.

Table 1. The sorting accuracy of eight kinds of radar emitter signals (%)

| SNR | CW | LFM | FSK | LFM-BPSK | BPSK | QPSK | FSK-BPSK |
|---|---|---|---|---|---|---|---|
| 5dB | 100 | 100 | 100 | 94 | 92 | 90 | 100 |
| 10dB | 100 | 100 | 100 | 100 | 94 | 93 | 100 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 15dB | 100 | 100 | 100 | 100 | 100 | 100 | 100 |
| 20dB | 100 | 100 | 100 | 100 | 100 | 100 | 100 |

## 5. Conclusions

Aiming at some problems in process of sorting radar emitter signal at present, this paper puts forward a novel sorting algorithm. Extracting resemblance coefficient of the ambiguity function Radon curve of the received radar emitter signal, this two characteristic parameters have anti-noise capability and separability, and conquers the disadvantages of the current sorting algorithms. By simulating eight kinds of radar emitter signal, the result proves the new method is feasible and valuable.

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