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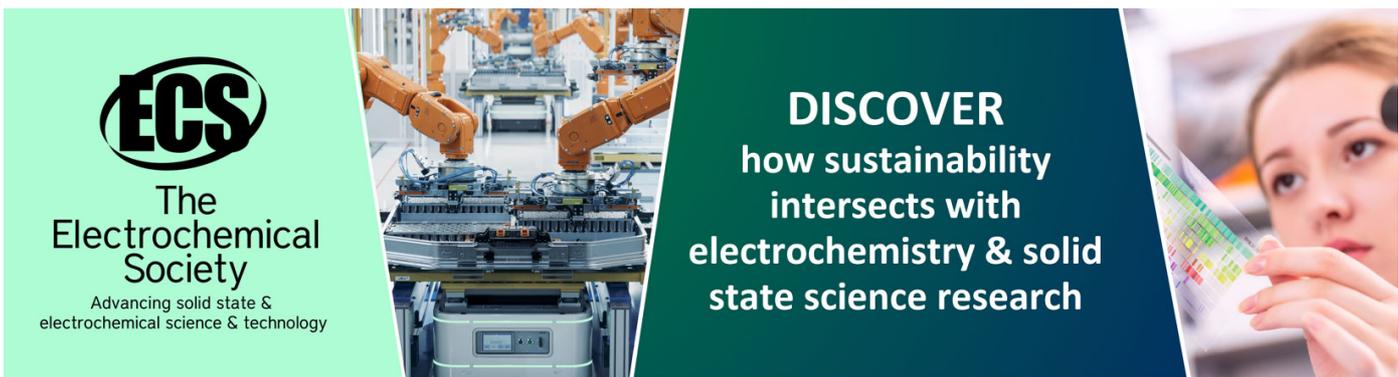
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Application of the Darcy and Richards equations for modelling of water capillary rise in building materials

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Abstract. This paper presents the application of the Darcy and Richards equations, describing water flow and dynamic changes of the water content in porous medium, for numerical modeling of water capillary rise in the selected building materials. The numerical simulations of unsaturated and variably saturated water flow in the studied materials were performed in FEFLOW commercial computational software, by DHI-Wasy, Germany. The developed model represented a wall section constructed of aerated concrete blocks, covered by mortar prepared using blended cement with biomass ash addition. The physical and water retention characteristics of the modeled materials were based on the previously performed laboratory measurements and literature research. The assumed initial conditions reflected air dry wall with limited moisture, while the boundary conditions allowed modeling of capillary rise due to direct contact of the studied materials with water with variable pressure head. The results of numerical calculations allowed observation of the time-dependent changes in the moisture content of building materials in relation to their physical properties and variable boundary conditions.

1. Introduction

Numerical modeling of saturated, variably saturated and unsaturated water flow in soils with various physical and hydraulic characteristics, based on finite elements method (FEM), is highly useful in many branches of science and engineering [1-4]. Such simulations allow the prediction of the groundwater flow in aquifers, water balance of catchment or aquifer, dynamics of infiltration and capillary rise in unsaturated zone of soil or artificial earthen liner, water availability for plants, efficiency of wells, irrigation and drainage, pollutants transport originated from point and surface sources, heat transport [5-7]. The popularity of numerical simulation in the above mentioned cases is highly related to costly and time consuming in-situ measurements and monitoring.

The simulations of moisture transfer in porous construction materials and barriers have been conducted for many years. Within this topic, several models of water transport have been elaborated [8-10]. The most popular models use various physical quantities as driving forces of water transportation. The Fick's law [11], Luikov model [9], Künzel model [12] and Liu model [13] should be mentioned among them. Modeling of water transport in building barriers allows the prediction of the building behavior prone to the water influence that may appear from the environment [14-17]. It enables to forecast the building areas threatened with destruction by the negative impact of water presence, such as mechanical deterioration [18, 19] or biological infestation [20,21].



Generally, development and applications of finite elements numerical models of water flow in porous media consist of three phases: 1) preprocessing, at which geometrical model of the modeled domain is built, discretized into finite elements/volumes, assignment of input data, covering the time related data and soil hydraulic characteristics, determination of boundary conditions (see figure 1, figure 2) solving the simulation using the solver; and 3) postprocessing of the simulation results [7,22,23]. Thus, field and laboratory measurements in most studies are required only at the stage of data gathering to prepare the input data, in the simplest cases mainly saturated and unsaturated hydraulic conductivity as well as the water retention characteristics. The required water conductivity characteristics may be measured in-situ or under the laboratory conditions by means of several available methods, mainly based on constant and falling head permeameters. On the other hand, the water retention characteristics, necessary to simulate the unsaturated flow in porous medium may be obtained during laboratory measurements using several available techniques, including extraction pressure chambers with ceramic plates, sand or sand/kaolin boxes [24,25].

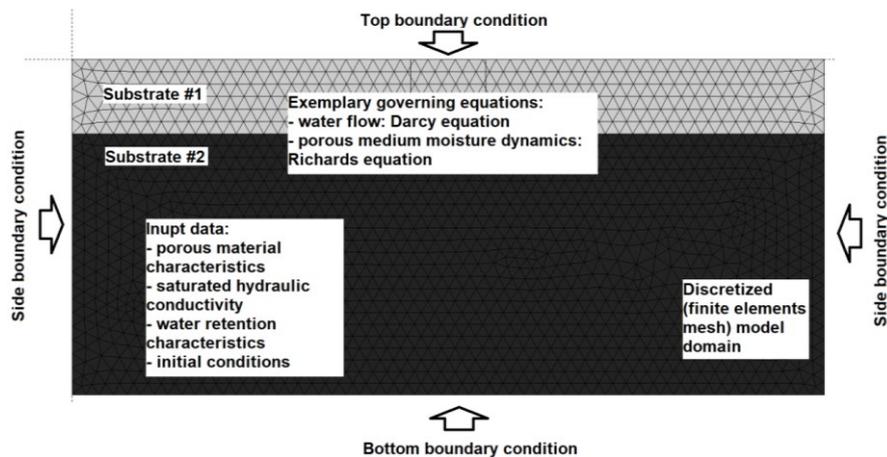


Figure 1. Simplified scheme of the FEM model of saturated, variably saturated or unsaturated water flow in porous medium solving the Darcy and Richards equations.

2. Materials and methods

The presented 2D numerical modeling of capillary rise of water inside the multi-layered wall consisting of aerated concrete blocks, cement mortar, reference coating/plaster mortar and repair coating/plaster mortar utilizing biomass ash as sustainable admixture was performed in the FEFLOW 6.0 commercial simulation software, Wasy-DHI, Germany, especially popular in soil and environmental science [4, 26-28] but also successfully tested in the calculation of water flow and heat transfer through building materials [29, 30]. The developed model consisted of 5130 elements and 2669 nodes and reflected the cross section of the standard aerated concrete wall of dimensions 0.30 m × 0.79 m. Our calculations were performed in two hypothetical scenarios, with two different boundary conditions, for two compositions of the modeled wall: 1) wall consisting of aerated concrete block connected with cement mortar and covered by the reference plaster mortar, 2) the same construction of the wall but one layer (left) of reference plaster mortar replaced by repair mortar containing biomass ash. Figure 2 shows the developed FEM model of the tested wall.

Numerical simulations of the 2D water flow in variably saturated porous media in FEFLOW were based on the standard forms of Darcy's and Richards' equations [31-33]:

$$q_i = -K_{ij} \frac{\partial h}{\partial x_j} \quad (1)$$

$$\frac{\partial \theta}{\partial t} = -\frac{\partial q_i}{\partial x_i} \mp Q \quad (2)$$

where: q_i – water flux vector, (m/s); h – water pore pressure head, (m); t – time, (s); K_{ij} – hydraulic conductivity tensor, $i, j = 1, 2$, (m/s); θ – volumetric water content, (m^3/m^3), Q – sink or source term, (1/s).

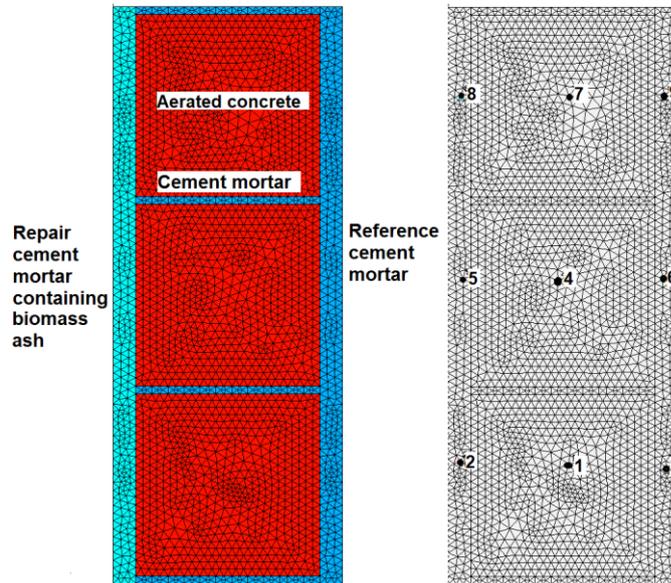


Figure 2. Finite elements mesh of developed 2D numerical model, with marked reference points.

The van Genuchten [34] model of water retention curve was applied to the simulations in FEFLOW in the form:

$$S_a = \frac{S_s - S_r}{[1 + (Ah)^n]^m} + S_r \quad (3)$$

where: S_a – actual degree of saturation, (-); S_s – saturated degree of saturation, assumed $S_s=1$, (-); S_r – residual degree of saturation, (-); h – water pore pressure head, (m); A – fitting parameter, (1/m); n, m – dimensionless fitting parameters: $m = 1 - n^{-1}$.

Hydraulic conductivity coefficient of unsaturated soils K was calculated according to the formula by van Genuchten [34]:

$$K = K_s S_e^l \left[1 - \left(1 - S_e^{\frac{1}{m}} \right)^m \right]^2 \quad (4)$$

where: K_s – coefficient of saturated hydraulic conductivity, (m/s); l – fitting parameter: $l = 0.5$ [7, 18], (-); S_e – dimensionless effective saturation understood as:

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \quad (5)$$

where: θ – volumetric water content, (m^3/m^3), θ_r – residual volumetric water content, (m^3/m^3); θ_s – saturated volumetric water content, (m^3/m^3).

The required input data covering the hydraulic and retention characteristics of the applied building materials i.e. coefficients of saturated hydraulic conductivity, saturated and residual volumetric water contents as well as water retention curve fitting parameters, were based on the previous research and are presented in table 1 [30, 35, 36]. The maximum available saturation for saturated condition was assumed as $S=1$, the saturated volumetric water content θ_s was assumed as equal to porosity.

The required initial conditions for time $t=0$ covered the uniform value of volumetric water content $\theta=0.05$ [m^3/m^3]. The two types of the boundary conditions were assumed in the presented model, to reflect two possible ways of water entrance, through the bottom of the wall and through the section of

its side. Thus, bottom boundary condition Head=0 m at the bottom of modeled domain and side boundary condition Head=0 m at length of approx. 5 cm were assumed for both tested variants. The duration of the simulation amounted to one year, i.e. 365 days.

Table 1. Water transport and retention characteristics of materials assumed to modeling.

Material	K_s (m/s)	θ_s (m ³ /m ³)	θ_r (m ³ /m ³)	Water retention curve fitting parameters	
				A (1/m)	n (-)
Aerated concrete 600	6.0E-7	0.349	0.0025	1.980	1.164
Cement mortar	1.65E-11	0.220	0.0025	0.149	1.019
Reference mortar	3.79E-10	0.336	0.0025	0.0237	1.431
Repair mortar containing ash	8.13E-11	0.435	0.0025	0.276	1.389

3. Results

Figure 3 presents the calculated distributions of porous media saturation for three selected time steps, i.e. 100, 200 and 365 days, for both assumed sections of the wall with bottom boundary condition, allowing capillary rise through the bottom edge of the model. The application of the repair mortar containing biomass ash on time-related distribution of saturation is clearly visible, water capillary rise in this layer is definitely slower than in the reference plaster and in aerated concrete, which resulted in asymmetric distribution of material moisture, in relation to the reference wall.

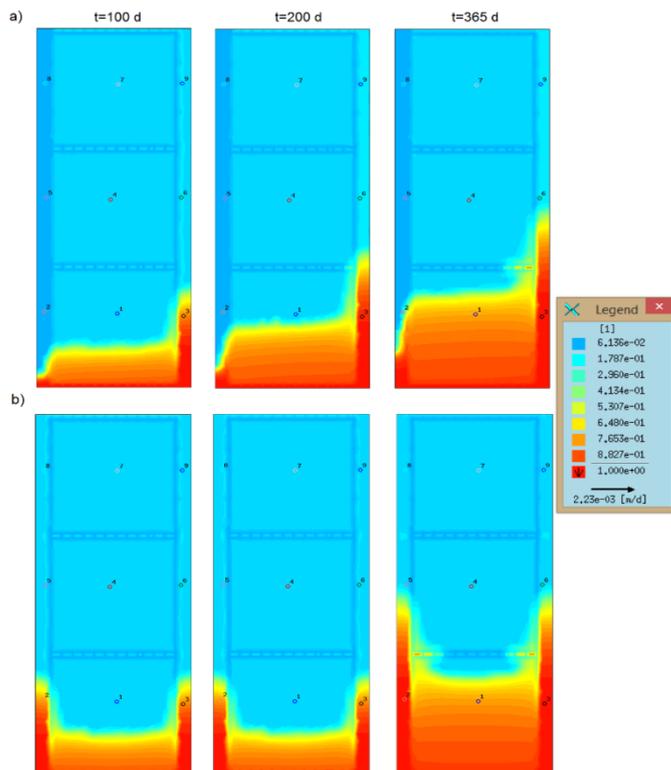


Figure 3. Modeled distribution of dimensionless porous materials saturation by water supplied through the bottom edge of the model: a) wall with repair mortar containing ash, b) wall with reference mortar.

The results discussed above are also visible in the graphs presenting the time-related changes of material saturation determined in reference points No. 1-3 (points No. 4-9 were excluded from the comparison) presented in Figure 4. The obtained results show that the application of the ash-based repair

mortar clearly affected the saturation determined in reference point No. 3 (for the reference mortar, the curves representing saturation in points No. 2 and 3 are almost identical). Nearly full saturation of porous medium in point No. 3 located in the reference cement mortar may be observed close to the final duration of simulation, value of 0.9 was reached after approx. 150 days. At the final time step of simulation, the saturation of repair mortar in point No.3 reaches the value of approx. 0.12.

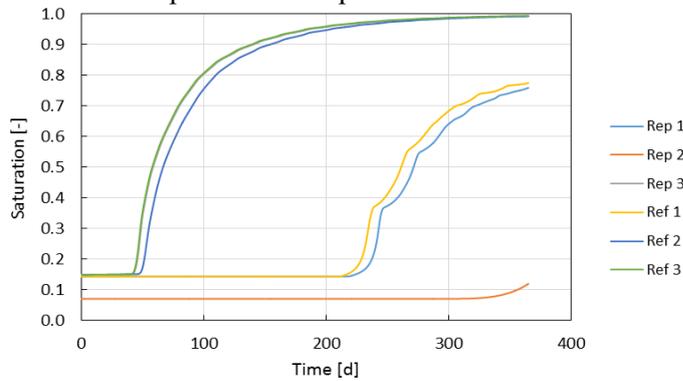


Figure 4. Time-related saturation of porous medium calculated in reference points presented in figure 3.

The results of saturation obtained for the second tested scenario, i.e. for both assumed sections of the wall with side boundary condition allowing capillary rise through the approx. 5 cm long bottom section of the left edge of the modeled domain, are presented in Figure 5 for three selected time steps. The determined time-related distribution of saturation shows clearly that the application of the repair mortar containing ash significantly limits the range of capillary rise in the modeled wall during the assumed duration of simulation.

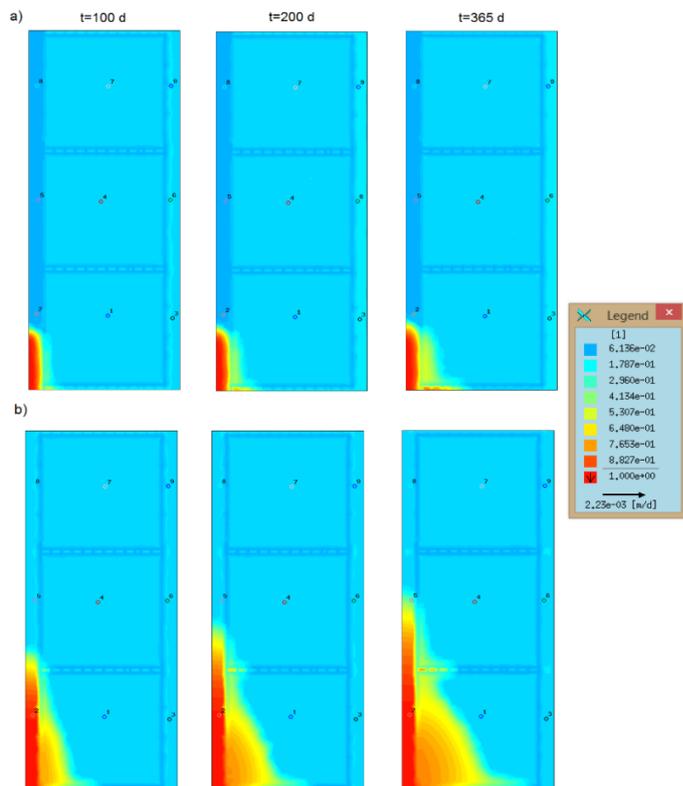


Figure 5. Modeled distribution of dimensionless porous materials saturation by water supplied through the bottom section of the left edge of the model: a) wall with the repair mortar containing ash, b) wall with the reference mortar.

Figure 6 presents the time-related changes of material saturation determined in reference points No. 2 and 5 (again, points with no observed significant changes in saturation by water were excluded from the comparison). It is clearly visible that the changes in saturation in reference points No. 2 and 5 were noted only for the reference coating/plaster mortar, the value of dimensionless saturation reached the level >0.9 after approx. 25 days. No changes in saturation were observed in these points for the wall covered by the repair mortar containing biomass ash.

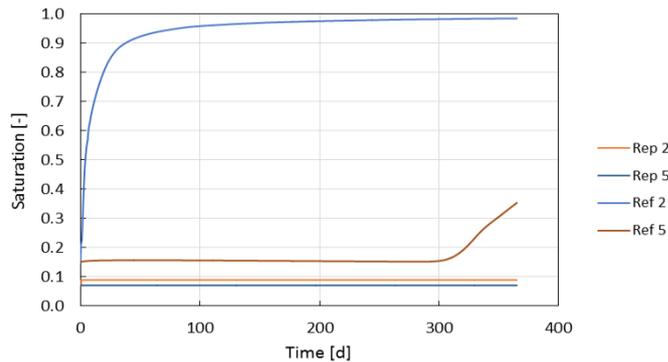


Figure 6. Time-related saturation of porous medium calculated in reference points presented in figure 5.

4. Conclusions

The presented exemplary numerical modeling of saturated and unsaturated water flow in multi-layered rigid porous wall consisting of several building materials of variable permeability and water retention characteristics, based on the standard set of the Darcy and Richards equations, showed its potential in predicting the time-related saturation changes under different boundary conditions. The observed changes in saturation distribution were closely related to the physical and hydraulic characteristics of porous materials selected for modeling. The application of the repair cement mortar containing biomass ash significantly affected the water flow inside the modeled wall. This is clearly visible in the second scenario in which the ash-based mortar limited the water inflow through the left side boundary of the studied wall.

However, to fully assess the application of numerical modeling based on the Darcy and Richards equations for the calculation of saturated, variably saturated and unsaturated water flow in porous building materials, it should be considered that the presented model was neither calibrated nor validated. Moreover, collecting the necessary input data for modeling of the tested materials which were not characterized earlier, covering saturated water conductivity and water retention curve characteristics, is time consuming and requires professional equipment for conducting hydraulic measurements.

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References

- [1] Zhang H and Song W 2007 The current status and prospect of research on groundwater numerical simulation *Water Sci. Technol.* **13** 794–6
- [2] Zhou Y and Li W 2011 A review of regional groundwater flow modeling *Geosci. Front.* **2** 205–14
- [3] Singh A 2014 Groundwater resources management through the applications of simulation modeling: A review *Sci. Total Environ.* **499** 414–23
- [4] Zdehlik R 2016 A review of applications for numerical groundwater flow modeling *International Multidisciplinary Scientific GeoConference: SGEM* **3** 11–18

- [5] Xue Y-Q, Ye S and Xie C-H 2004 Application of multi-scale finite element method to simulation of groundwater flow *J. Hydraul. Eng.* **37** 1–8
- [6] Diersch H-J G and Kolditz O 2002 Variable-density flow and transport in porous media: approaches and challenges *Adv. Water Resour.* **25** 899–944
- [7] Kazda I 2012 *Finite element techniques in groundwater flow studies: with applications in hydraulic and geotechnical engineering* (Amsterdam:Elsevier Science)
- [8] Berger D and Pei D C T 1973 Drying of hygroscopic capillary porous solids —A theoretical approach *Int. J. Heat Mass Transf.* **16** 293–302
- [9] Luikov A V 1975 Systems of differential equations of heat and mass transfer in capillary-porous bodies (review) *Int. J. Heat Mass Transf.* **18** 1–14
- [10] Häupl P, Grunewald J, Fechner H and Stopp H 1997 Coupled heat air and moisture transfer in building structures *Int. J. Heat Mass Transf.* **40** 1633–42
- [11] Dong W, Chen Y, Bao Y and Fang A 2020 A validation of dynamic hygrothermal model with coupled heat and moisture transfer in porous building materials and envelopes *J. Build. Eng.* **32** 101484
- [12] Künzel HM 1995 *Simultaneous Heat and Moisture Transport in Building Components* (Suttgart: Fraunhofer IRB Verlag)
- [13] Liu X, Chen Y, Ge H, Fazio P and Chen G 2015 Numerical investigation for thermal performance of exterior walls of residential buildings with moisture transfer in hot summer and cold winter zone of China *Energy Build.* **93** 259–68
- [14] Roels S, Sermijn J and Carmeliet J 2002 Modelling unsaturated moisture transport in autoclaved aerated concrete: a microstructural approach. ed A Gustavsen *6th Nord. Symp. Build. Phys. (NSB 2002), June 17-19, 2002, Trondheim, Norw.* 167-174 BT-Proceedings of the 6th Symposium on Building Physics in the Nordic Countries : Trondheim
- [15] Mendes N, Winkelmann F, Lamberts R and Philipp P 2003 Moisture effects on conduction loads *Energy Build.* **35** 631–44
- [16] Liu Y, Wang Y, Wang D and Liu J 2013 Effect of moisture transfer on internal surface temperature *Energy Build.* **60** 83–91
- [17] Moon H J, Ryu S H and Kim J T 2014 The effect of moisture transportation on energy efficiency and IAQ in residential buildings *Energy Build.* **75** 439–46
- [18] Barnat-Hunek D, Góra J, Andrzejuk W and Łagód G 2018 The Microstructure-Mechanical Properties of Hybrid Fibres-Reinforced Self-Compacting Lightweight Concrete with Perlite Aggregate *Materials (Basel)*. **11** 1093
- [19] Barnat-Hunek D, Siddique R, Klimek B and Franus M 2017 The use of zeolite, lightweight aggregate and boiler slag in restoration renders *Constr. Build. Mater.* **142** 162–74
- [20] Kočí V, Kočí J, Maděra J, Žák J and Černý R 2020 Computational Prediction of Susceptibility to Biofilms Growth: Two-Dimensional Analysis of Critical Construction Details *Energies* **13** 293
- [21] Suchorab Z, Frąc M, Guz Ł, Oszust K, Łagód G, Gryta A, Bilińska-Wielgus N and Czerwiński J 2019 A method for early detection and identification of fungal contamination of building materials using e-nose ed P Karlovsky *PLoS One* **14** e0215179
- [22] Wang F and Anderson M 1995 *Introduction to groundwater modeling: finite difference and finite element methods* (Cambridge, Massachusetts: Academic Press)
- [23] Vellando P, Juncosa R, Padilla F and García-Rábade H 2020 Numerical evaluation of groundwater flows: MODFLOW vs. FE models *J. Porous Media* **23** 545–62
- [24] Baroni G, Facchi A, Gandolfi C, Ortuani B, Horeschi D and van Dam J C 2010 Uncertainty in the determination of soil hydraulic parameters and its influence on the performance of two hydrological models of different complexity *Hydrol. Earth Syst. Sci.* **14** 251–70
- [25] Widomski M K, Broichsitter S B, Zink A, Fleige H, Horn R and Stepniewski W 2015 Numerical modeling of water balance for temporary landfill cover in North Germany *J. Plant Nutr. Soil Sci.* **178** 401–12
- [26] Mazzia A and Putti M 2006 Three-dimensional mixed finite element-finite volume approach for

- the solution of density-dependent flow in porous media *J. Comput. Appl. Math.* **185** 347–59
- [27] Trefry M G and Muffels C 2007 FEFLOW: A Finite-Element Ground Water Flow and Transport Modeling Tool *Ground Water* **45** 525–8
- [28] Widomski M K, Broichsitter S B, Zink A, Fleige H, Horn R and Stępniewski W 2015 Numerical modeling of water balance for temporary landfill cover in North Germany *J. Plant Nutr. Soil Sci.* **178** 401–12
- [29] Widomski M K, Majerek D and Iwanek M 2018 Numerical modeling of water and heat transport in porous building material *AIP* **020048** 020048
- [30] Widomski M K, Pawelec I, Kumar A and Łagód G 2019 Numerical modelling of water percolation through mortar-based connection of concrete pipes ed M Kulisz, M Szala, M Badurowicz, W Cel, M Chmielewska, Z Czyż, K Falkowicz, J Kujawska and T Tulwin *MATEC Web Conf.* **252** 05014
- [31] Diersch H-J G 2014 *FEFLOW* (Berlin, Heidelberg: Springer Berlin Heidelberg)
- [32] Richards L A 1931 Capillary conduction of liquids through porous mediums *Physics (College. Park. Md.)* **1** 318–33
- [33] Raats P A C 2001 Developments in soil–water physics since the mid 1960s *Geoderma* **100** 355–87
- [34] van Genuchten M T 1980 A Closed-form Equation for Predicting the Hydraulic Conductivity of Unsaturated Soils *Soil Sci. Soc. Am. J.* **44** 892–8
- [35] Suchorab Z, Widomski M, Lagod G and Sobczuk H 2010 Capillary rise phenomenon in aerated concrete. monitoring and simulations *Proceedings of ECOpole* **4** 285–90
- [36] Pavlíková M, Zemanová L, Záleská M, Pokorný J, Lojka M, Jankovský O and Pavlík Z 2019 Ternary Blended Binder for Production of a Novel Type of Lightweight Repair Mortar *Materials* **12** 996.