Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Implementation of Image Resampling Algorithm Based on Compressed Sensing

To cite this article: Denghui Li and Yanhong Wang 2021 *J. Phys.: Conf. Ser.* **1732** 012071

Implementation of Image Resampling Algorithm Based on Compressed Sensing

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Abstract. In this paper, the sampling rate of traditional signal reconstruction should be more than 2 times the maximum frequency of the original signal in order to ensure the non-distortion reconstruction of the signal. The theoretical knowledge of compressed sensing is deeply analysed, and the image signal is reconstructed by block compression sensing method. Experiments show that the sampling rate is more higher, and the reconstruction error is more smaller, but the processing complexity is more higher; conversely, the lower the sampling rate, the lower the processing complexity, the greater the reconstruction error.

1. Introduction

Today's society is a society of information explosion. Every day we receive all kinds of information, among which a lot of information is based on images. Image itself has the characteristics of large amount of data. As people put forward higher requirements for image quality, the amount of image data also becomes very larger. This is a tremendous pressure for sampling and storage. In order to restore the undistorted signal, the sampling frequency must satisfy the requirement that the sampling rate be greater than twice the maximum frequency of the signal. It is difficult for existing equipment to achieve such a high sampling rate, or it will cost a lot of price to achieve such a high sampling rate, but this is obviously not appropriate and not worthwhile. In order to solve these problems, compressed sensing arises at the historic moment.

2. Compressed Sensing

Candes, Tao, Donoho et al. proposed compressed sensing as a new sampling theory in 2006, and we abbreviate compressed sensing as CS. Under the CS's theory, the original signal is firstly transformed into a sparse or compressible signal[1], [2]. Then, the transformed signal is observed by using the sparse basis characteristic and the observation matrix. The signal is projected into a space whose dimension is much smaller than the original dimension, and the key information points are retained. Finally, the original signal is restored by solving linear or non-linear equations.

We first establish a mathematical model, assuming that the signal is sparse and compressible[3], [4], [5], which is denoted as $x \in R^N$, and under sparse base mapping ,x has k-sparse description as:

$$x = \psi s \tag{1}$$

In formula (1), *s* is the sparse representation of *x*, and *s* contains *k* non-zero elements, and $\ll N$.

We can use an $M \times N$ observation matrix to project and observe the signal x, and get the observed signal y:

$$y = \Phi x \tag{2}$$

So y is an $M \times 1$ dimensional vector, and $M < N$.

By combining formula (1) and formula (2), we can get the following formula:

$$y = \Phi x = \Phi \psi s = As \tag{3}$$

In formula (3), A is called a sensing matrix, and satisfies the principle of constrained equidistant. So formula (3) is a problem for solving linear equations of s and y . But due to M < N, the number of equations is less than the number of unknowns, and this problem becomes the problem of solving undetermined equations. Because that s is the sparse representation of x , so y is a linear combination of K column vectors not equal to zero in sparse basis s of sensor matrix A[6], [7]. So if we know the position of K non-zero elements in s, we can list M* K linear equations to solve non-zero terms, and its necessary and sufficient condition is the following formula:

$$1 - \varepsilon \leq \frac{\|Av\|}{\|v\|_2} \leq 1 + \varepsilon \tag{4}$$

In formula (4), ε is called is equidistant constrained constant, and $\varepsilon \in (0,1)$.

The construction of observation matrix is the most critical step in compressed sensing theory. A large number of scholars have found that the observation matrix must satisfy the following conditions: firstly, the column vectors of the observation matrix should be linearly independent; secondly, column vectors of observation matrices must satisfy independent randomness; finally, the solution obtained should be the minimum L_1 norm. After a lot of research, it is found that Gauss random matrix, Bernoulli random matrix and partial Fourier matrix can be used as observation matrices satisfying isometric constraints, but they are not universal and difficult to implement.

3. Block Compression Sensing

In 2009, L proposed a block compression sensing method, which divides the image into several blocks of the same size, and then uses the same observation matrix to process each block image. By using this method, it is easy to find a general and suitable Gauss random observation matrix because of the smaller image blocks, and it also reduces the data storage and improves the accuracy of the algorithm, which makes the implementation of software and hardware relatively simple[8],[9].

We can first divide an image with size N*N into n non-overlapping blocks with size B*B, and $n = (N/B)^2$, if x_i is the vector of the i-th block, the corresponding observation values y_i can be obtained under the action of the appropriate observation matrix Φ_B:

$$y_i = \Phi_B x_i \tag{5}$$

In formula (4), Φ_B is the observation matrix, and it can usually be a Gauss random matrix, its size is $n_B \times B^2$. In order to satisfy the constrained equidistant condition, Φ_B is an orthogonalized Gauss random matrix. All elements in matrix Φ_B obey normal distribution with mean 0 and variance $1/n_B$. When the sampling rate is α, $n_B = [\alpha B^2]$ is satisfied, and The observation matrix Φ can be expressed as follows:

$$\Phi = \begin{bmatrix} \Phi_B & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \Phi_B \end{bmatrix} \tag{6}$$

According to the formula, it can be found that the sampling rate of the whole system can be changed by changing the sampling rate of α, which facilitates the hardware design. The number of blocks is more larger, B is more smaller, the faster the processing speed and the less memory it occupies, but the quality of signal reconstruction is worse.

4. Experimental results

In order to verify the correctness of the algorithm, the image resampling experiment based on compressed sensing is carried out. First, the initial conditions are set. If we set the sampling rate to 0.9, the signal length is 16*16, then the number of measurements is 0.9*16*16. Make up 0 where the blocks are not covered. Then each block is processed, and the two-dimensional image is transformed into one-dimensional image by column vector. The second step is to implement sparse transformation using DCT transform and other matrices. Finally, orthogonal matching pursuit algorithm is used to reconstruct the signal. The original and resampled figures are shown in Figures 1 and Figures2.

Figure 1. the original image

Figure 2. the resampled image

As can be seen from the two images, the quality of the resampled image is not much different from that of the original image, only a little blurred in some details. There is a certain degree of distortion in the details. Although the details are distorted, the compression rate is improved. If we adopt different sampling rates, we can get different mean square error (MSE) and data processing time. These two are inversely proportional, that is to say, when the sampling rate is higher, the processing time is longer, and the root mean square error is smaller. Figure 3 shows the image quality at different sampling rates. The relationship between sampling rate, MSE and processing time is shown in Fig. 4 and Fig.5.

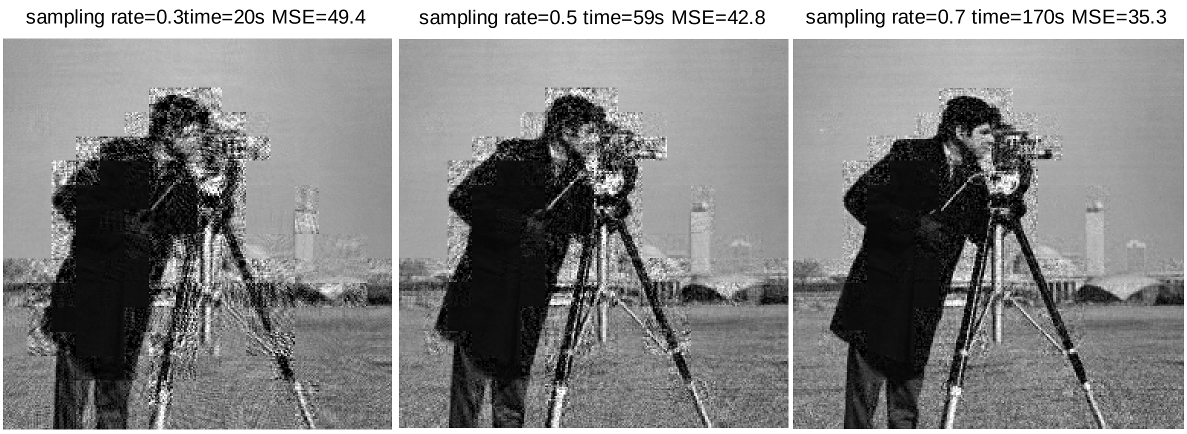

Figure 3. Reconstructed Images at Different Sampling Rates

The relationship between sampling rate, MSE and processing time is shown in Fig. 4 and Fig.5. Figure3 and Figure4 shows that when the compression sampling rate is small enough, the signal can not be reconstructed. With the increase of the number of observations, especially the sampling rate in the (0.1, 0.5) interval, the probability of reconstruction error decreases sharply. In the interval of (0.5, 0.9), the reconstruction error tends to be flat. This shows that the original image can be reconstructed accurately with high probability as long as the sampling rate is above 0.5.

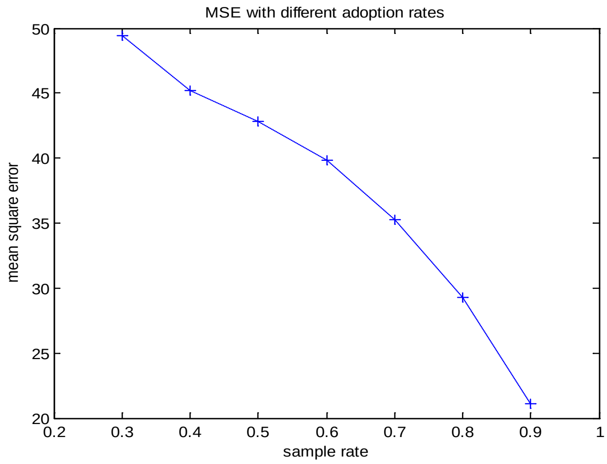

Figure 4. MSE at Different Sampling Rates

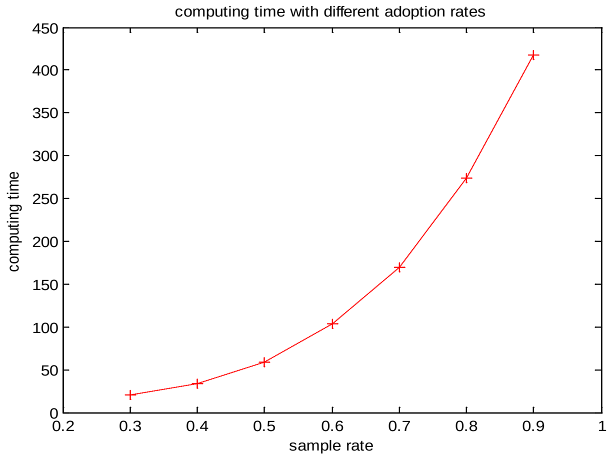

Figure 5. Computing time at different sampling rates

5. Conclusion

This paper mainly studies image reconstruction algorithm based on compressed sensing. The orthogonal matching pursuit algorithm is used to sample and reconstruct the data. The simulation experiment is carried out on the MATLAB platform, and the corresponding conclusions are obtained. As a more advanced theory at present, the research of compressed sensing theory is only in its infancy. In the future, it will have a broader prospect in theory and application.

6. Acknowledgments

This work was supported by Guangxi Key Lab of Wireless Wideband Communication & Signal Processing (CXKL06180105) and Key Laboratory of Cognitive Radio and Information Processing, Ministry of Education (Guilin University of Electronic Technology, and Scientific research project of the Guangxi Education Office(2017KY1350), thanks for their supports.

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