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# Time Dependent Stabilization of a Hamiltonian System

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# Time Dependent Stabilization of a Hamiltonian System

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**Abstract.** In this paper we consider the unstable chaotic attractor of a Hamiltonian system with Toda lattice potential and stabilize it by an integral form control. In order to obtain stability results, we use a control function in an integral form: $u(t) = \int_0^t k(t,s)X(s)ds$, in which all the back story of the process X(t) is taken into consideration. Using the exponential kernel $k(t,s) = e^{-\beta(t-s)}$, we replace the study of integro-differential system of order 4 with an analysis of $5^{th}$ order system of ordinary differential equations (without integrals). Numerical solution of the resulting system leads to the asymptotically stabilization of the unstable fixed point.

## 1. Introduction.

The presence of chaos in physical systems has been extensively demonstrated and is very common. In practice, however, it is often desired that chaos should be avoided and that the performance of a system should be stabilized. A review of various methods of controlling chaos is presented in [1].

Various stabilization approaches are based on a delayed feedback control scheme [19], where dynamical time-delayed feedback control is used. By this method, "next step" state of a system is controlled by the "previous step" state, that is, by the "history" of the system, thus allowing fine tuning and stabilization i.e. the control signal $u(t)$ depend on the difference between the current state of the system $x(t)$ and the earlier state $x(t-\tau)$ of the system. Examples of the use of this method can be found in electronic chaos oscillators [20], magneto-elastic chaos [9], lasers [3], the low-dimensional chaos arising from the nonlinear interaction between the two different types of ionization waves [13], chemical systems [15], [11], plasma [7], [17] and the Lorenz attractor [18]. Various modifications of this method have been proposed in [22], where an extended delayed feedback controller, using information about many previous states of the system, was suggested. More examples can be found in a review by M.A.F. Sanjuan and C. Grebogi [21]. Yahalom et al. [23] have indicated that chaos is not a result of the state of the system at one point in time but has to do with the state of the system over and extended duration defined by chaos uncertainty relation.

Mathematically the method proposed by Pyragas [19] is based on stabilization of $\tau$-periodic orbit of a non-linear system by a feedback law:

$$u(t) = -KF(t),\ F(t) = x(t) - x(t-\tau) + RF(t-\tau), \tag{1}$$

where $x(t)$ is output variable, $\tau$ is a period of the stabilized periodic orbit, $F(t)$ is the control feedback force and K is gain coefficient. Parameter $R$ satisfied the inequality $|R| < 1$.

Since the chaotic attractors have periodic solutions with different periods, the problem of finding reasonable $\tau$ is still open. The second of important problems is the problem of finding an analytic and constructive method for selecting the control feedback force and gain coefficient.

Another suggestion to solve with delays is a reduction model approach, also designated "the finite spectrum assignment technique", which created in the works of [2, 10, 12, 16]. Recent developments of this concept are discussed in [14] (see also the references therein). A development of this idea is to choose $u(t)$ in the form of the sum:

$$u(t) = \sum_{i=1}^{m} K_i X(t - \tau_i). \tag{2}$$

In the limit of continuous $\tau_i$ we may choose the control $u(t)$ in the form:

$$u(t) = \int_0^t k(t, s) X(s) ds. \tag{3}$$

in which all the backstory of the process $X(t)$ is taken into consideration. It will be appropriate for our stabilization goal to choose

$$k(t, s) = e^{-\beta(t-s)}. \tag{4}$$

The method proposed here avoids the difficulties associated with the use of control in an integral form. Using the exponential kernels, we reduce the study of integro-differential system of the order $n$ to analysis of $(n + m)$-th order system of ordinary differential equations [5, 8].

**Example**. Consider the following scalar equation

$$x'(t) = a_1 x(t) \tag{5}$$

$a_1$ is a constant. If $a_1 > 0$, then solution of equation (7): $x = 0$ is unstable. Let us use the control $u(t)$ in the form (3), where $k(t, s) = e^{-\beta(t-s)}$ and consider the integro-differential equation:

$$x'(t) - a_1 x(t) + \alpha u(t) = x'(t) - a_1 x(t) + \alpha \int_0^t e^{-\beta(t-s)} x(s) ds = 0, \ t \in [0, +\infty). \tag{6}$$

where $\alpha$ and $\beta$ are control parameters.

In accordance with the Leibnitz rule of differentiation under the sigh of an integral depending on a parameter and limits of integration depend on the differentiation variable:

$$\frac{d}{dy} \int_{a(y)}^{b(y)} f(x, y) dt = \int_{a(y)}^{b(y)} \frac{\partial}{\partial y} f(x, y) dx + b'(y) f(y, b(y)) - a'(y) f(y, a(y)) \tag{7}$$

we obtain:

$$u'(t) = -\beta \int_0^t e^{-\beta(t-s)} x(s) ds + e^{-\beta(t-t)} x(t) - 0 = -\beta u(t) + x(t) \tag{8}$$

Using Leibnitz rule, we can write the corresponding system in the form:

$$x'(t) = a_1 x(t) - \alpha u(t), \quad u'(t) = -\beta u(t) + x(t) = 0, \quad t \in [0, +\infty), \quad u(0) = 0. \tag{9}$$

Its characteristic equation is the following:

$$\lambda^2 + (\beta - a_1)\lambda + \alpha - \beta a_1 = 0. \tag{10}$$

The condition:

$$\beta - a_1 > 0 \text{ and } \ \alpha - \beta a_1 > 0 \tag{11}$$

is necessary and sufficient for the exponential stability of the system (9). If condition (11) is fulfilled, then the solution $x(t) = 0$ of the equation (6) is exponentially stable.

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## 2. Hamiltonian system with Toda potential.

The state of a Hamiltonian system can be described by N generalized momenta $p \equiv (p_1, ..., p_N)$ and the same number $N$ generalized coordinates $q \equiv (q_1, ..., q_N)$. Here $N$ designates the number of a system's degrees of freedom. The evolution of $p$ and $q$ in time is determined by the equations of motion:

$$\dot{p_i} = -\frac{\partial H}{\partial q_i}; \quad \dot{q_i} = \frac{\partial H}{\partial p_i}; \quad (i = 1, ..., N)\,, \tag{12}$$

which become concrete only with the Hamiltonian:

$$H = (H, p, q, t)\,. \tag{13}$$

The Hamiltonian is given in $2N$ - dimensional phase space $(p, q)$ and may also be an explicit function of time. Pairs of variables $(p_i, q_i)$ are called canonically conjugate pairs and the equations (12) are canonical equations. We consider the two-dimensional motion may be defined by the Hamiltonian

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + V(x, y), \quad m = 1, \tag{14}$$

where $V(x, y)$ is Toda lattice potential:

$$V(x, y) = \frac{1}{2}(x^2 + y^2) + x^2 y - \frac{1}{3}y^3 + \frac{3}{2}x^4 + \frac{1}{2}y^4. \tag{15}$$

We rewrite system (12) in the form:

$$\dot{x} = p_x, \quad \dot{y} = p_y, \quad \dot{p_x} = -\frac{\partial H}{\partial x}, \quad \dot{p_y} = -\frac{\partial H}{\partial y}. \tag{16}$$

or in more concrete form:

$$\dot{x} = p_x, \quad \dot{y} = p_y, \quad \dot{p_x} = -x - 2xy - 6x^3, \quad \dot{p_y} = -y - x^2 + y^2 - 2y^3. \tag{17}$$

We represent this system in the time dependent explicit form:

$$\begin{aligned} x'(t) &= p(t), \quad p'(t) = -x(t) - 2x(t)y(t) - 6x^3(t), \\ y'(t) &= q(t), \quad q'(t) = -y(t) - x^2(t) + y^2(t) - 2y^3(t). \end{aligned} \tag{18}$$

System (18) is autonomous and homogeneous. Studying the stability of this system, we consider its linear approximation of the original nonlinear system at the equilibrium point: $x = 0, y = 0, p = 0, q = 0$ according to Lyapunov's linearization method:

$$x'(t) = p(t), \quad p'(t) = -x(t), \quad y'(t) = q(t), \quad q'(t) = -y(t). \tag{19}$$

We rewrite system (19) in the vector form

$$\begin{bmatrix} x'(t) \\ p'(t) \\ y'(t) \\ q'(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ p(t) \\ y(t) \\ q(t) \end{bmatrix} \tag{20}$$

let us designate:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \tag{21}$$

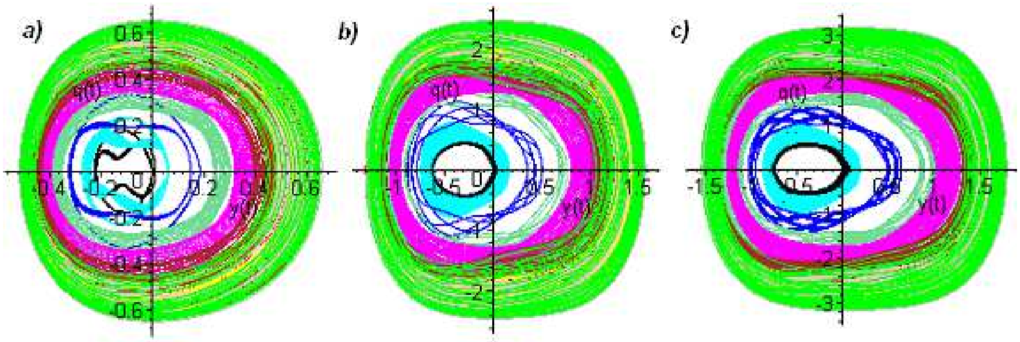

**Figure 1.** The phase portrait of the solution of system (20) in the $(y, q)$ plane for a) $E = 0.214$, b) $E = 3.0$ and c) $E = 5.0$ with initial conditions: $x(0) = 0$, $y(0) = 0$, $p(0) = \sqrt{2E}\sin\left(\frac{a\pi}{2}\right)$, $q(0) = \sqrt{2E}\cos\left(\frac{a\pi}{2}\right)$. For $a = 0$ (gray), $a = 0.1$ (green), $a = 0.2$ (pink), $a = 0.3$ (yellow), $a = 0.4$ (red), $a = 0.5$ (brown), $a = 0.6$ (magenta), $a = 0.7$ (auqamarine), $a = 0.8$ (blue), $a = 0.9$ (cyan), $a = 1.0$ (black)

as the (constant) matrix of system (20). The determinant:

$$det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 & 0 & 0 \\ -1 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & -1 & -\lambda \end{vmatrix} \tag{22}$$

leads to the characteristic equation $det(A - \lambda I) = 0$ which will take the form:

$$\lambda^4 + 2\lambda^2 + 1 = (\lambda^2 + 1)^2 = (\lambda - i)^2(\lambda + i)^2 = 0 \tag{23}$$

and its characteristic roots are

$$\lambda_1 = \lambda_2 = i, \quad \lambda_3 = \lambda_4 = -i. \tag{24}$$

Now if:

$$det(A - \lambda I) = (\lambda - \lambda_1)^{m_1}(\lambda - \lambda_2)^{m_2}...(\lambda - \lambda_i)^{m_i}...(\lambda - \lambda_n)^{m_n} \tag{25}$$

then elementary divisors are $(\lambda - \lambda_1)^{m_1}$ , . . . , $(\lambda - \lambda_i)^{m_i}$ , . . . . Their number is the same as the number of the Jordan blocks of $A$ and the elementary divisor $(\lambda - \lambda_i)^{m_i}$ corresponds to a Jordan blocks of order $m_i$. If $m_i = 1$ then $(\lambda - \lambda_i)$ is a simple elementary divisor. It follows that the Jordan form of (21) consists of two blocks corresponding to each of the characteristic roots.

In accordance with the theorem, which was proved in the monograph [4], a linear homogeneous system with constant matrix A is stable if and only if all of its eigenvalues have non-positive real parts, and the eigenvalues with zero real part may have only simple elementary divisors. That is, the corresponding Jordan cells are reduced to one element.

So the linear system (19) is chaotic behavior and hence also the nonlinear system (18) is chaotic. The chaotic behaviour is demonstrated in Figure 1 where one can easily see than trajectories filling the area (see also [23]).

### 3. Stabilization of Hamiltonian system by feedback control in integral form

Let us use the control $u(t)$ in the form [6]:

$$u(t) = \int_0^t e^{-\beta(t-s)} q(s) ds, \tag{26}$$

in which all the history of the process q(t) is taken into account [5]. We apply stabilization to the second equation of system (18) by feedback delay control, because for a mechanical system control is normally applied to acceleration.

$$\begin{aligned} x'(t) &= p(t), \quad p'(t) = -x(t) - 2x(t)y(t) - 6x^3(t) - \alpha \int_0^t e^{-\beta(t-s)} p(s)ds, \\ y'(t) &= q(t), \quad q'(t) = -y(t) - x^2(t) + y^2(t) - 2y^3(t) \end{aligned} \tag{27}$$

where $\alpha$ and $\beta$ are control parameters. We can rewrite the system (27) in a form of the system of ordinary differential equations according to Leibnitz rule (7):

$$\begin{aligned} x'(t) &= p(t), \quad p'(t) = -x(t) - 2x(t)y(t) - 6x^3(t) - \alpha u(t), \\ y'(t) &= q(t), \quad q'(t) = -y(t) - x^2(t) + y^2(t) - 2y^3(t), \quad u'(t) = p(t) - \beta u(t). \end{aligned} \tag{28}$$

System (28) is autonomous and homogeneous. Studying the stability of this system, we consider its linear approximation of the original nonlinear system at the equilibrium point $x = 0, y = 0, p = 0, q = 0, u = 0$:

$$x'(t) = p(t), \quad p'(t) = -x(t) - \alpha u(t), \quad y'(t) = q(t), \quad q'(t) = -y(t), \quad u'(t) = p(t) - \beta u(t) \tag{29}$$

The constant matrix of the system (29) is:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -\alpha \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -\beta \end{bmatrix} \tag{30}$$

Which leads to the characteristic equation:

$$det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 & 0 & 0 & 0 \\ -1 & -\lambda & 0 & 0 & -\alpha \\ 0 & 0 & -\lambda & 1 & 0 \\ 0 & 0 & -1 & -\lambda & 0 \\ 0 & 1 & 0 & 0 & -\lambda - \beta \end{vmatrix} = 0 \tag{31}$$

The characteristic equation can be also written in the form:

$$\begin{gathered} -\left(\lambda^5 + \lambda^4\beta + (\alpha + 2)\lambda^3 + 2\beta\lambda^2 + (\alpha + 1)\lambda + \beta\right) = \\ -\left(\lambda^3 + \lambda^2\beta + \lambda(1 + \alpha) + \beta\right)\left(\lambda^2 + 1\right) = 0 \end{gathered} \tag{32}$$

and

$$\lambda_1 = \frac{C}{6} - \frac{2D}{C} - \frac{\beta}{3}; \quad \lambda_{2,3} = -\frac{C}{12} + \frac{D}{C} - \frac{\beta}{3} \pm i\sqrt{3}\left(\frac{C}{6} + \frac{2D}{C}\right); \quad \lambda_{4,5} = \pm i. \tag{33}$$

where

$$\begin{aligned} &A = 12 + 36\alpha + 24\beta^2 + 36\alpha^2 - 60\alpha\beta^2 + 12\beta^4 + 12\alpha^3 - 3\alpha^2\beta^2; \\ &B = -72\beta + 36\alpha\beta - 8\beta^3; C = \left(B + 12\sqrt{A}\right)^{\frac{1}{3}}; \ D = \tfrac{1}{3}\left(1 + \alpha - \tfrac{\beta}{3}\right). \end{aligned} \tag{34}$$

For $\alpha > 0$, $\beta > 0$ $\lambda_1$ and real parts of $\lambda_2$ and $\lambda_3$ are negative. According to J. L. Massera's theorem, this is a stability indication for nonlinear ODE systems [4]. Massera's theorem states that given a nonlinear homogeneous system:

$$\frac{dy}{dt} = A(t)y + f(t, y); \ f(t, 0) \equiv 0, \tag{35}$$

the function $y$ is limited, If:
**1)**

$$\|f(t,y)\| \leq \psi(t)\|y\|^m, \qquad (m>1) \tag{36}$$

where $\psi(t)$ is a positive function such that:

$$\lim_{x\to\infty}\frac{1}{t}ln|\psi(t)|=0 \tag{37}$$

**2)** for Lyapunov characteristic exponents $a_1, ...a_n$ of the linear approximation of a nonlinear system:

$$\frac{dy}{dt}=A(t)y \tag{38}$$

the inequality:

$$max_k(a_k) < -\frac{\kappa}{m-1} \leq 0 \tag{39}$$

is fulfilled, where:

$$\kappa=\sum_{k=1}^{n}a_k-\lim_{x\to\infty}\frac{1}{t}\int_0^t tr\{A(t_1)\}dt_1 \tag{40}$$

then the solution $y \equiv 0$ of a nonlinear system is asymptotically stable (Lyapunov) for $t \to \infty$. In the neighborhood of $y = 0$, which is the equilibrium position of the system (28), nonlinear terms are smaller then linear ones. System (28) is a system with constant coefficients (autonomous), therefore conditions (36) and (37) are satisfied, i.e. condition **1)** fulfilled.

Since the characteristic exponents $a_j (j = 1, ..., n)$ of solutions of a linear system

$$\frac{dy}{dt}=Ay(t) \tag{41}$$

with a constant matrix $A$ are the real parts of the characteristic roots of the matrix $A$, i.e.

$$a_j = Re\lambda_j(A) \quad (j=1,...,n) \tag{42}$$

where $\lambda_j = \lambda_j(A)$ are roots of the equation $det(A - \lambda E) = 0$ [4] then condition **2)** is fulfilled as

$$max_k(a_k) = max(\lambda_1, Re\lambda_{2,3}) < 0, \tag{43}$$

$$\sum_{k=1}^{3}a_k=\lambda_1+2Re\lambda_{2,3}=-\beta \tag{44}$$

and

$$\lim_{x\to\infty}\frac{1}{t}\int_0^t tr\{A\}dt_1=-\beta. \tag{45}$$

Thus

$$\kappa=\sum_{k=1}^{3}a_k-\lim_{x\to\infty}\frac{1}{t}\int_0^t tr\{A\}dt_1=0 \tag{46}$$

and inequality (39) is fulfilled. The stabilized solution is demonstrated in figure 2 where the trajectories are periodic and are not surface filling.

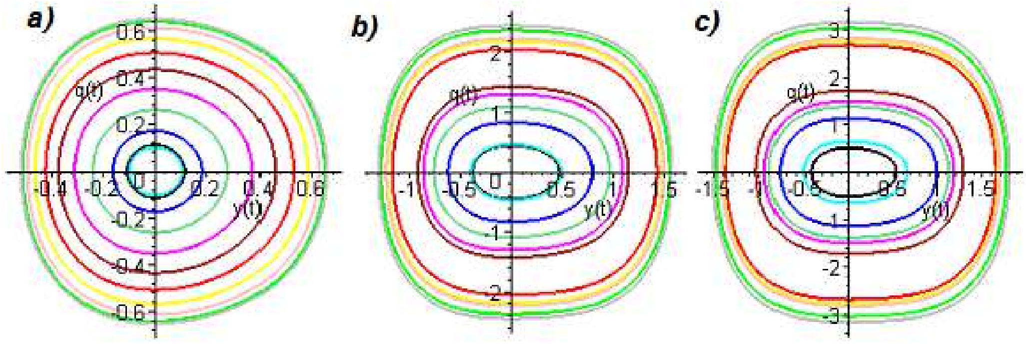


**Figure 2.** The phase portrait of the solution of system (19) in the $(y, q)$ plane for a) $E = 0.214$, b) $E = 3.0$ and c) $E = 5.0$ with initial conditions: $x(0) = 0$, $y(0) = 0$, $p(0) = \sqrt{2E}\sin\left(\frac{a\pi}{2}\right)$, $q(0) = \sqrt{2E}\cos\left(\frac{a\pi}{2}\right)$, $\alpha = 2.0, \beta = 1.0$, $40 \leq t \leq 300$. For $a = 0$ (gray), $a = 0.1$ (green), $a = 0.2$ (pink), $a = 0.3$ (yellow), $a = 0.4$ (red), $a = 0.5$ (brown), $a = 0.6$ (magenta), $a = 0.7$ (auqamarine), $a = 0.8$ (blue), $a = 0.9$ (cyan), $a = 1.0$ (black)

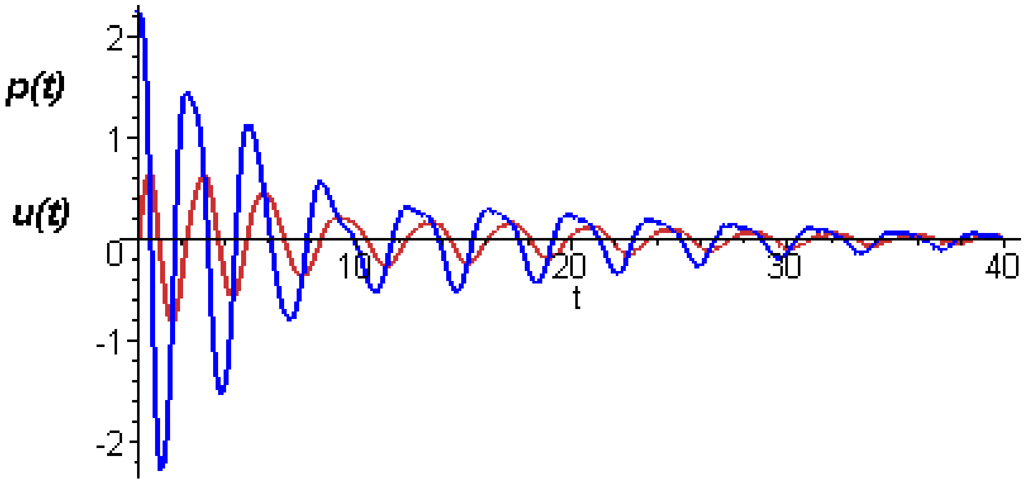


**Figure 3.** The control function $u(t)$ (red) as compared to the stabilized process $p(t)$ (blue). $E = 5.0, \alpha = 2.0, \beta = 1.0, a = 0.5$

## 4. Conclusion

This model satisfies the rapid convergence of the process to a limit cycle with a small control function $u(t)$.

This model differs from the methods described in publications of other authors by the absence of an adjustable parameters and rough approximations in determining of the control function.

It should be noted that the period of limit cycle obtained by above-described method is not associated with the "period" of a chaotic motion but is automatically obtained by the substitution of the values of the control parameters $\alpha$ and $\beta$ to the nonlinear differential equations system.

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