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The analysis of Venus' physical surface using methods of fractal geometry

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The analysis of Venus' physical surface using methods of fractal geometry

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Abstract. In this paper, the work on investigating fractal structures on Venus was performed on the basis of observations taken by the "Magellan" spacecraft (NASA). The uncertainties in some data produced by "Magellan" were filled by the information that had been collected before – in "Venera 15", "Venera 16", and "Pioneer" missions. During the implementation of the work a digital map of Venus' surface was built, and its spatial model was created. It is worth noting that the choice of basic level surface on Venus is defined by a certain value of potential or a point on its surface through which the geoid passes. The model of Venus' physical surface was created use the harmonic expansion into spherical functions of altimetry data the "Magellan" mission. In the present paper, for determining and analyzing fractal dimensions the Minkowski mathematical algorithm, which is a simplified option of Hausdorff-Besicovitch dimension and provides high reliability and accuracy, was used. As a result, fractal correlations of Venus' geoid anomalies in both longitude and latitude as well as the mean value of fractal dimensions were calculated. The following values of mean fractal dimension for Venus surface are obtained: in latitude – D_β = 1.003; in longitude – D_λ = 0.98. Based on these values, we may conclude that the topographic model of Venus' physical surface is close to spherical figure. The comparison between the obtained Venus fractal parameters with the ones of the Earth shows the good agreement.

1. Introduction

Currently, the main approaches to the study and description of processes in planetary systems are statistical and fractal methods. In particular, the robust method allows investigating the structure of complex objects taking into account their specific character, while fractal geometry allows studying not only the structure, but also the connection between structure and processes of its formation [1]. In this respect, the problem of developing methods for recognizing fractal structures of planetary objects is relevant. As variations of Venus' physical surface represent a complex multi-parameter system [2], its analysis should be conducted by means of complex physics methods, one of whose directions is fractal analysis [3]. For the studies of Venus, data from the Magellan spacecraft (NASA) [4] were used. The equipment of this artificial satellite allowed scanning almost the entire surface of Venus using a radar with synthesized aperture of S-band (12 cm) and a microwave radiometer as well as investigating topography by the special radar – altimeter [5]. The comparison between the obtained

Venus fractal parameters with the ones of the Earth confirms the conclusion in [6], in which the geological evolution of Earth and Venus are analyzed.

Some of the main methods available for analyzing complex astronomical systems are the robust methods [7–11]. It is necessary to take into account that such methods cannot be used in all cases, as not all celestial structures are of stochastic nature (including shape, physical parameters etc.). To study complex systems, fractal analysis can be used. For complex objects, the fractal method allows to determine values of fractal dimension (FD) and self-similarity coefficients. On the basis of these parameters it is possible to study connections between a celestial body's evolutionary parameters and its structure. Therefore, the study of celestial objects by means of fractal geometry methods is the relevant and modern task. Both the structure of Venus and its gravitational field relate to complex multiparameter systems. For studying such systems, it is necessary to use the theory of complex physics, which includes the fractal geometry methods [3]. In this work, the regression model of Venusian structure was created on the basis of altimetry data of "Magellan" mission (NASA) [13]. The general aim of "Magellan" mission was the study of Venusian chemical parameters, its inner and outer structure, and planetary properties [5].

2. Building a model of Venus' physical surface using harmonic analysis

To develop the topographic model of Venus, the harmonic expansion of altimetry data from the "Magellan" mission into spherical functions was implemented using a formula [15, 16] as follows:

$$h(\lambda,\beta) = \sum_{n=0}^{N}\sum_{m=0}^{n}(\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda)\cdot\bar{P}_{nm}(\cos\beta) + \varepsilon, \quad (1)$$

where $h(\lambda, \beta)$ – altitude function dependent on longitude and latitude;
λ, β – longitude, latitude (known parameters);
$\bar{C}_{nm}, \bar{S}_{nm}$ – harmonic normalized amplitudes;
$\bar{P}_{nm}$ – Legendre functions, normalized and associated;
ε – regression error, random.

This mathematical expression was also used for the processing of other astronomical observations [17].

Similarly, one may explore Venusian gravity anomalies [18, 19].

The average profiles were created by modeling slices of the Venus surface through cutting Venus sphere by meridian planes for every 20° in longitude from 0° to 360° and for each of such longitude values, the latitude changed from –60° to +70° angular degree. As an example, such a model of a slice of the surface of Venus by the meridian plane for 180° in longitude is shown in figure 1.

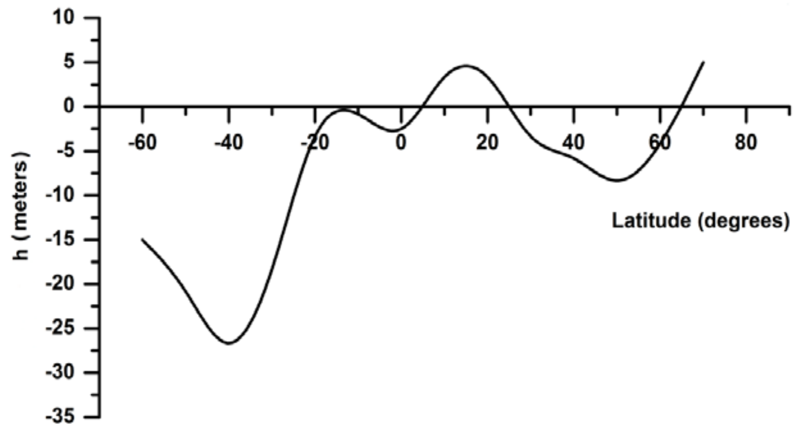

Figure 1. Average Venusian profile of the physical surface (longitude is 180°).

The Venusian gravity potential models (GPM) were created similarly to the topography models (1). For GPM were created averaged Venusian profiles for every value in longitude in 20° interval (from 0° to 360°) and for each of such fixed values of longitude, the latitude changed from –70° to +70°. As an example, such a model of the profile of the Venusian gravity anomalies in the meridian plane for 30° in longitude is shown in figure 2.

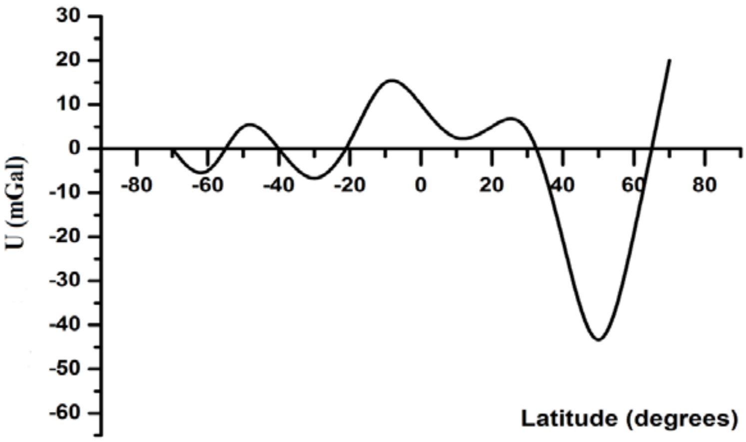

Figure 2. Average profile of Venusian gravity anomalies (longitude is 30°).

According to equation (2), for each of the averaged profile models the values of D were obtained. In equation (2) N is the number of cubes with set size that could be placed into the averaged profile model. For N to be integer, the division into cubes with set size starts with the value of 24. The calculated values for the structure and gravity models are shown in figure 3.

$$D = \dim M = \lim_{\sigma \to 0} \frac{\ln N(\sigma)}{\ln \frac{1}{\sigma}}. \tag{2}$$

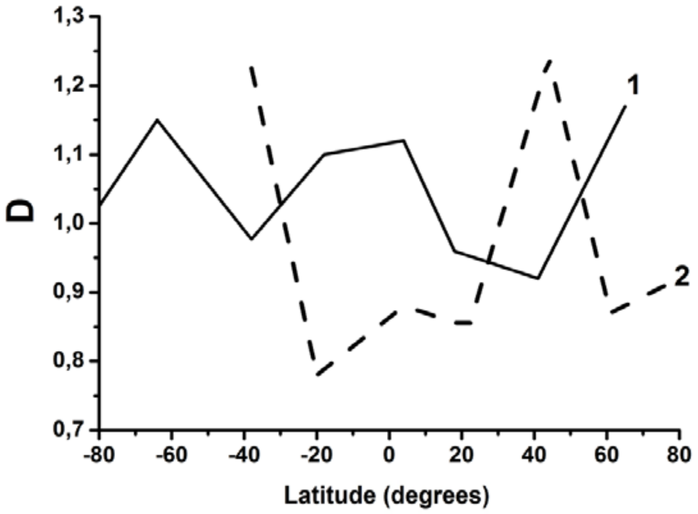

Figure 3. Comparison of FD. 2 diagrams of the values of FD for topographic (1) and gravity (2) models of Venus. Each point in diagram corresponds to value of FD for profile model built for a fixed value of latitude and covering longitude from 0° to 360°.

According to figure 3, the values of FD for topographic model of Venus are between 0.92 and 1.23, and the values of FD for gravity anomaly model are between 0.75 and 1.24.

3. Distribution of fractal similarity coefficient along Venus' surface

The structure of an object under consideration may be presented as an ordered set $A(N^2)$, where N^2 is the number of elements a_{ij} of the set $a_{ij} \in A(N^2)$, where $i, j = 1 \dots N$.

Figure 1 gives the differences in longitudes and latitudes after taking into account the secular acceleration, without taking precession into account from the planets. Figure 2 shows the discrepancies in longitude and latitude after joint consideration of the precession from the planets and secular acceleration.

Table 1 lists all the numerical values of the series used to convert the reference frame. These series were taken from [8], except for the last two arguments, which were taken from [9].

Unfortunately, there is ambiguity in the values of some arguments. The contribution of these quantities leads to a discrepancy by an amount not exceeding 2 arc seconds.

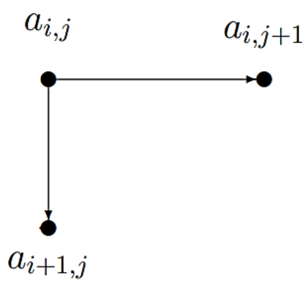

Figure 4. Hasse diagram.

A partial order in a finite set is defined by a Hasse diagram (figure 4). Elements of a set have some properties $H_\xi(a)$ (size, colour, volume, shape, etc.) inherent only to the elements of this set $\forall a_{ij}(a_{ij} \in \{a \middle| H_\xi(a)\}$. If there are more than 1 ($\xi > 1$) common properties, a set could be described by several fractal properties.

Let us represent the set $A(N^2)$ as

$$A(N^2) = Q^{(1)}(n^2) \cup Q^{(2)}(n^2) \cup \dots \cup Q^{(\alpha^2)}(n^2), \tag{3}$$

where $Q^{(k)}(n^2)$ – disjoint subsets of the set $A(N^2)$

$$Q^{(k)}(n^2) \cap Q^{(k')}(n^2) = \emptyset, \tag{4}$$

where α and n are integers.

Then α and n represent sets $\forall \alpha \in \{\alpha_\gamma\}$ and $\forall n \in \{n_\gamma\}$. Let us note that $n_{max} = sup\{n_\gamma\} \in \{n_\gamma\} = N$, $n_{min} = inf\{n_\gamma\} \in \{n_\gamma\} = 1$, $\alpha_{max} = sup\{\alpha_\gamma\} \in \{\alpha_\gamma\} = N$, $\alpha_{min} = inf\{\alpha_\gamma\} \in \{\alpha_\gamma\} = 1$. For instance, if $N = 24$, $\{\alpha_\gamma\} = \{24, 12, 8, 6, 4, 3, 2, 1\}$ and $\{n_\gamma\} = \{1, 2, 3, 4, 6, 8, 12, 24\}$.

There are upper and lower borders of the $A(N^2)$ set according to the properties $H_\xi(a)$:

$$AG_\xi = \sup A(N^2) \tag{5}$$

$$g_\xi = \inf A(N^2), \tag{6}$$

while $G_\xi \in A(N^2)$ and $g_\xi \in A(N^2)$.

The fractal property D_ξ of the $A(N^2)$ set according to the $H_\xi(a)$ property is defined by an angle dependence coefficient of $\log \Gamma_\xi(n^2)$ on $\log s_\xi n^2$, where $\Gamma_\xi(n^2)$ is the number of discontiguous cubes' surfaces covering the $Q^{(k)}(n^2)$ set:

$$D_\xi = \sum_\gamma \frac{\log \Gamma_\xi(n_{\gamma+1}^2) - \log \Gamma_\xi(n_\gamma^2)}{abs(\log S_\xi(n_{\gamma+1}^2)) - abs(\log S_\xi(n_\gamma^2))} * \left(\frac{\alpha_{\gamma+1} - \alpha_\gamma}{N - 1}\right). \tag{7}$$

The self-similarity coefficient K_ξ is defined as

$$K_\xi = \frac{D_\xi^0}{D_\xi} \tag{8}$$

where D_ξ^0 is a fractal dimension of a self-similar set:

$$D_\xi^0 = \frac{\log \Gamma_\xi(N^2) - \log \Gamma_\xi(1)}{abs(\log S_\xi(N^2)) - abs(\log S_\xi(1))}. \tag{9}$$

Figure 5 presents a distribution of the self-similarity coefficient on the surface of Venus divided into square areas with a resolution of 15°×15° in λ_i and β_i topographic coordinates.

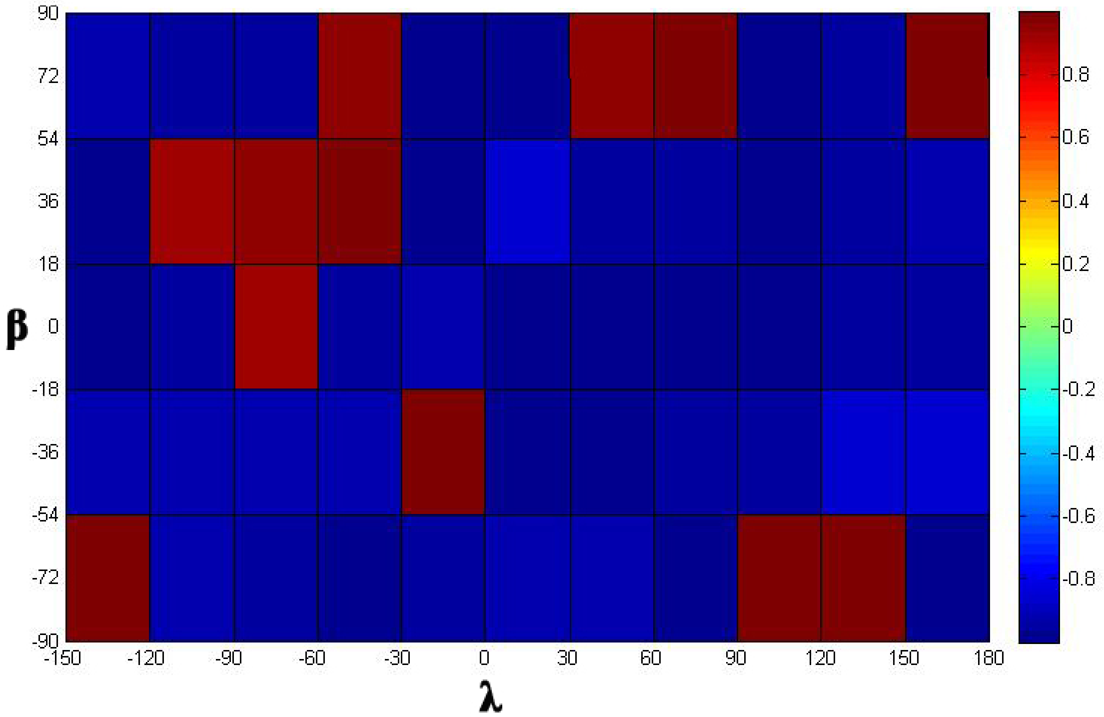

Figure 5. Distribution of self-similarity coefficient for various zones of Venus' surface.

An analysis of figure 5 shows that the distribution of self-similarity coefficient for various zones of the Venus' surface varies from –0,8 to +0,8, which points to a fractality of Venus' surface and also to the change in its structure from one zone to another.

It is worth emphasizing that the following models were investigated: in paragraph 2 – the averaged profiles of Venus (i.e. plane models), while in paragraph 3 – the surface of Venus (i.e. 3D model).

4. Summary and conclusions

The analysis of modern methods for solving problems of Venus topography on the basis of the data produced in space missions is conducted. In particular, altimetry was investigated. A multiple processing of space dataset was found necessary due to the constant improvement of processing methods based on which global models of Venus are constructed. This direction has become particularly important after the appearance of observational data based on space measurements.

As a result, the fractal dimensions in both latitude and longitude were obtained for Venusian physical surface model. The values of averaged fractal dimensions were calculated: averaged fractal dimension of Venusian physical surface model in longitude was D = 1.039, in latitude – D = 1.063.

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