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Investigation of the geometric meaning of the derivative using mathematical packages

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Abstract. The paper suggests that in the process of training mathematical disciplines in technical universities in the context of digitalization of education, applied software packages to solve practice-oriented problems are used. It is advisable to reveal basic definitions, concepts, theorems, laws on practical examples, following the wise Latin aphorism “Plus usus sine doctrina, quam c. usum doctrina valet Q / Practice without theory is more valuable than theory without practice”. The more practical examples will be used in the process of mastering the theoretical material, the stronger and more fundamental the mathematical knowledge of students will be. The paper examines the stages of investigation the geometric meaning of the derivative using the MathCad computer system. The geometric meaning of the derivative will be the most accessible and memorable when students study intersecting parabolas.

1. Introduction

Nowadays, due to a sharp increase in the amount of information that a student of a technical university must receive, understand, realize and be able to apply in his professional activity in the future. One of the main tasks is to find optimal ways of training. The learning process begins with mathematics as one of main significant disciplines in higher educational institutes of a technical profile. Exactly, the level of formation of professional competencies of future specialists of a technical university in the field of fire service and civil protection largely depends on the quality of mathematical training.

In the process of training fire service and civil protection specialists within the framework of digitalization of the educational environment, it is urgent to use applied software packages at lessons in mathematics while solving practice-oriented problems [1-5].

2. Statement of the problem

The paper shifts the focus to the study of the geometric meaning of the derivative using the MathCad computer system.

Consider the formulation of the investigation problem.

Find the angle between the parabolas $y = x^2 + 7$ and $y = x^2 + 6x - 5$ at the point of their intersection. Plot graphs of parabolas and their tangents at the intersection.

Note: recall that the angle between two curves $y = f_1(x)$ and $y = f_2(x)$ at the point of their intersection $M_0(x_0, y_0)$ is an angle between the tangents to these curves at the point M_0 ; its tangent is calculated by the formula:



$$\operatorname{tg}(\varphi) = \frac{f_2'(x_0) - f_1'(x_0)}{1 + f_1'(x_0) \cdot f_2'(x_0)}.$$

The stages of solving the given research problem are presented on the MathCad worksheet.

1. Set functions $f_1(x)$, $f_2(x)$ and find coordinates of the point of their intersection M_0 .

It is necessary to solve the system of nonlinear equations in order to find coordinates of the point M_0 :

$$\begin{cases} y = x^2 + 7, \\ y = x^2 + 6x - 5. \end{cases}$$

In Mathcad, one can use the **Given/Find** computational block to solve systems of linear and nonlinear equations. The **given/Find** block consists of three parts going sequentially one after another:

- **Given** is a keyword;
- a system written by logical operators in the form of equalities and, possibly, inequalities;
- **Find** (x_1, \dots, x_n) is a built-in function for solving a system of equations for the variables x_1, \dots, x_n .

The peculiarity of using the **Given/Find** block is that the Boolean equality operator located on the Boolean algebra panel (or the <Ctrl + => key combination on the keyboard) but not the numerical calculation operator (a <=> key on the keyboard) uses it to write equations. It is necessary to execute the chain of commands **Insert - Function - Solve equations - Find** to insert a **Find** function.

The equation shows the given functions $f_1(x)$ and $f_2(x)$, as well as a **Given/Find** computational unit. Listing Mathcad–Finding coordinates of the point M_0 :

$$\begin{aligned} f_1(x) &:= x^2 + 7 & f_2(x) &:= x^2 + 6x - 5 \\ \text{Given} & & & \\ y &= x^2 + 7 & & \\ y &= x^2 + 6x - 5 & & \\ \text{Find}(x, y) &\rightarrow \begin{pmatrix} 2 \\ 11 \end{pmatrix} \end{aligned}$$

Thus, a point M_0 has coordinates (2; 11).

2. Calculate the derivatives of the functions $f_1(x)$ and $f_2(x)$ at the point $x_0 = 2$. Listing Mathcad–Calculation $f_1'(x_0)$ and $f_2'(x_0)$:

$$x := 2 \quad \frac{d}{dx} f_1(x) = 4 \quad \frac{d}{dx} f_2(x) = 10$$

3. To find the angle between the parabolas, use the **atan(z)**, function; it calculates the arctangent, i.e., the value of the angle (in radians) whose tangent is z . To insert the **atan** function, follow the command chain **Insert - Function - Trigonometric - atan**. After entering the function, press <=> (equal) key on the keyboard. The result is shown below (the expression in brackets is the tangent of the angle φ). Listing Mathcad–Calculation of the angle φ (in radians):

$$\phi := \operatorname{atan}\left(\frac{10 - 4}{1 + 4 \cdot 10}\right) = 0.145$$

As it has already been noted, the **atan** function calculates the angle in radians. To switch to degrees, we will use the convenient Mathcad functionality for setting units of measurement for dimensional values. For this, click on the block with the **atan** function and place the cursor at the end of the block

after the black marker. Listing Mathcad– Preparation for introduction the units of measurement of the Angle ϕ :

$$\phi := \operatorname{atan}\left(\frac{10 - 4}{1 + 4 \cdot 10}\right) = 0.145$$

Run the chain of commands **Insert - Unit of measurement**. Select *angle* as dimension and *degrees* as an unit of measurement in the window **Insert Units** (figure 1).

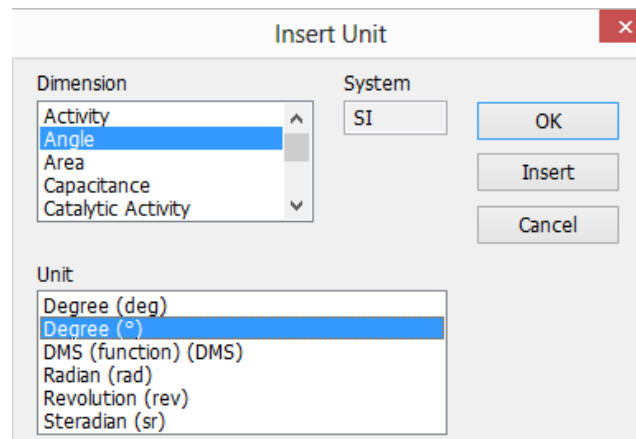


Figure 1. Insert Unit window.

The result is shown in the form of Listing Mathcad– Calculation of the angle ϕ (in degrees):

$$\phi := \operatorname{atan}\left(\frac{10 - 4}{1 + 4 \cdot 10}\right) = 8.326^\circ$$

4. For a graph of functions construction $f_1(x)$ and $f_2(x)$ redefine them through another variable, for example z in representation of functions $f_1(x)$ and $f_2(x)$ using a variable z

$$f1(z) := z^2 + 7 \quad f2(z) := z^2 + 6z - 5$$

5. Write down the equations for tangents $y_1(z)$ and $y_2(z)$, also using a variable z .

6. Construct graphs of functions $f_1(z)$, $f_2(z)$ and tangents $y_1(z)$, $y_2(z)$ (figure 2).

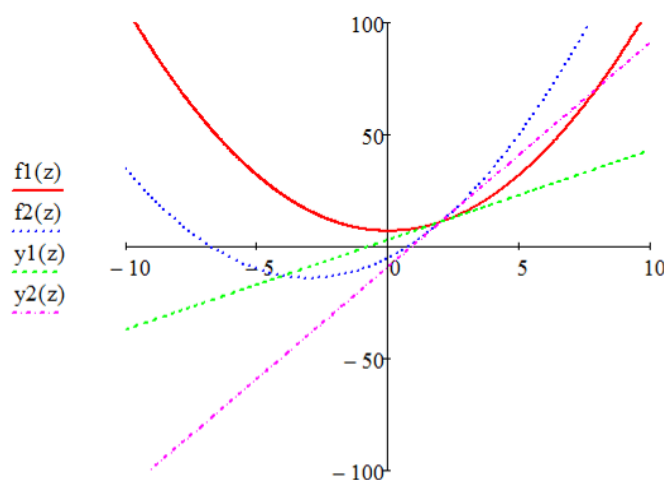


Figure 2. Graphs of functions $f_1(z)$, $f_2(z)$ and tangents $y_1(z)$, $y_2(z)$.

The resulting graphs on the MathCad worksheet help students to make a conclusion about the definition of the geometric meaning of the derivative.

The geometric meaning of the derivative is as follows if a function $y=f(x)$ is constructed in a Cartesian rectangular coordinate system (figure 2), then a derivative of this function at a given point is equal to the tangent of the slope between the tangent to the curve drawn through this point and the abscissa axis.

3. Conclusion

The investigation approach contributes to a more qualitative learning mathematical concepts, definitions, theorems, formulas and the competent application of theoretical knowledge in solving practice-oriented problems in the process of training mathematical disciplines [6-8].

The use of the applied software packages in the context of digitalization of mathematical education contributes to an increase in the level of mathematical training of future fire service and civil protection specialists, as well as the formation and development of professional motivation.

References

- [1] Artamonov V S, Ivanov A Y, Sharapov S V, Trofimets E N and Trofimets V Ya 2017 Information systems and processes in the analytical training of management scholars *Espacios* **38(25)** 18
- [2] Batkovskiy A M and Trofimets V Ya 2015 Decision support systems with modules of applied mathematical models and methods *Problems of radio electronics* **9** 253-75
- [3] Bulgakov S V 2015 Application of multiagent systems in information systems *Prospects for Science and Education* **5(17)** 136-40
- [4] Trofimets V Ya and Trofimets E N 2008 *Optimization in Excel* (Yaroslavl State Technical University, Yaroslavl) p 104
- [5] Trofimets E N 2012 Information technologies of mathematical modeling in economic universities *Educational technologies and society* **15(1)** 414-23
- [6] Batkovskiy A M, Semenova E G, Trofimets E N, Trofimets V Ya and Fomina A V 2016 Computer Modeling of Leasing Operations *Indian Journal of Science and Technology* **9(28)** 2-12
- [7] Batkovskiy A M, Semenova E G, Trofimets E N, Trofimets V Ya and Fomina A V 2016 Method for Adjusting Current Appropriations under Irregular Funding Conditions *Journal of Applied Economic Sciences, Romania: European Research Centre of Managerial Studies in Business Administration* **5(43)** 828-41
- [8] Artamonov V S, Maximov A V, Motorygin Y D, Trofimets E N and Trofimets V Ya 2016 Management of the Formation of Rating Preferences of Economic Entities upon Collective Choice *International Journal of Economics and Financial Issues* **6(4)** 1956-64