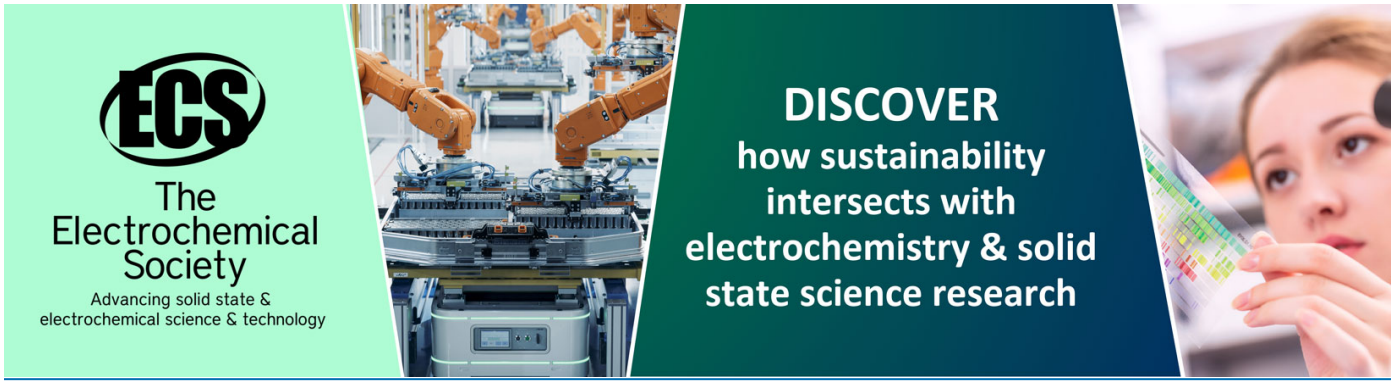
Masses of two Higgs doublets within effective theory with four-quark interactions



Masses of two Higgs doublets within effective theory with four-quark interactions

A A Osipov[1] and M M Khalifa[2,3]

[1] Joint Institute for Nuclear Research, Dubna, Moscow oblast, 141980 Russia
[2] Moscow Institute of Physics and Technology, Dolgoprudnyi, Moscow oblast, 141701 Russia
[3] Department of Physics, Al-Azhar University, Cairo, 11751 Egypt

E-mail: aaosipov@jinr.ru, mkhalifa@phystech.edu

Abstract. The composite Higgs scenario in the context of the effective model with $SU(2)_L \times U(1)_R$ symmetric four-fermion interaction proposed by Miransky, Tanabashi and Yamawaki (MTY) is considered. The low-energy dynamics of the model is described by the effective Lagrangian obtained by the Schwinger-DeWitt technique. The questions of the Nambu sum rule and the spectrum of Higgs states are addressed in detail.

1. Introduction

The top condensation models can be used to explore the origin of mass, for instance, the reason behind the greatness of top quark mass compared with other known quarks [1-10]. In these models, at high energies $\Lambda \gg \Lambda_{EW} \approx 250\ GeV$, the $SU(2)_L \times U(1)_R$ gauge symmetry group of electroweak interactions is dynamically broken by effective four-quark interactions. Owing to a strong coupling, in the fermion spectrum of the theory, a gap appears (the nonzero mass of the *t*-quark) and, as a consequence, the boson condensate is formed predominantly of the third-generation quarks. The collective excitations of the condensate manifest themselves in the form of boson modes associated with composite (quark–antiquark) Higgs bosons, the dynamics of which at low energies $\mu \ll \Lambda$ is described by an effective action which can be found by integrating out the short-distance components of quark fields at leading $1/N_c$ order, where N_c is the number of the color degrees of freedom of quarks. It is supposed that induced four-quark interactions should explain the origin of the Higgs sector of the Standard Model (SM).

The minimal top-condensation models (see, e.g., [8]) do not contain new particles and are entirely consistent with the SM at low energies $\mu \sim \Lambda_{EW}$, with the only difference being that the Higgs field is a composite state. One serious phenomenological problem of this approach is the too high mass of the Higgs particle $m_H = 2m_t$. From the theoretical point of view, this result is understandable, since it is an analogue of the known relation that appears in the Nambu–Jona– Lasinio (NJL) model [11] for the mass of the scalar fermion–antifermion bound state (σ meson) $m_\sigma = 2m_f$, where m_f is the mass of one of the two constituent fermions. The generalized form of this equation is known as the Nambu sum rule [12–14]. According to this hypothesis, the boson modes in a four-fermion-interaction system can be united into pairs, so-called Nambu partners; for each of the pairs, the equality $m_1^2 + m_2^2 =$

$4m_f^2$, which couples the gap of the fermionic spectrum m_f with the corresponding gaps of the bosonic spectrum m_1 and m_2, holds true.

2. The MTY model

Let us consider the model of [6, 7]. Its Lagrangian contains $SU(2)_L \times U(1)_R$ gauge-invariant four-quark interactions that approximate physics unknown to us above a certain $\Lambda \gg \Lambda_{EW} = \left(\sqrt{2}G_F\right)^{-1/2} = 250\ GeV$ scale as

$$\mathcal{L}_{4\psi} = \mathrm{g}_1\left(\bar{\psi}_L^a\psi_R^b\right)\left(\bar{\psi}_R^a\psi_L^b\right) + \mathrm{g}_2\left(\bar{\psi}_L^a\psi_R^b\right)(i\tau_2)^{ac}(i\tau_2)^{be}\left(\bar{\psi}_L^c\psi_R^e\right) + \mathrm{g}_3\left(\bar{\psi}_L^a\psi_R^b\right)\tau_3^{bc}\left(\bar{\psi}_L^c\psi_R^a\right) + h.c.. \quad (1)$$

Here, the summation over the repeated Latin superscripts ($a, b, c, d, e = 1,2$) is supposed, whose superscripts correspond to the flavor degrees of freedom of the quark fields ψ. For the sake of simplicity, only the third-generation quarks are considered, i.e.,

$$\psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = \begin{pmatrix}t\\ b\end{pmatrix}. \quad (2)$$

The projection on the chiral states $\psi_L = P_L\psi$ and $\psi_R = P_R\psi$ is performed by the $P_L = \frac{1}{2}(1-\gamma_5)$ and $P_R = \frac{1}{2}(1+\gamma_5)$ operators. The independent constants g_i are positive and have the dimension $[g_i] = M^{-2}$. The set of the $SU(2)$ matrices is standard, $\tau_i (i = 1,2,3)$ are the Pauli matrices. The theory under consideration is dynamically equivalent to the semi-bosonized theory described by the generating functional

$$Z = \int d\sigma_\alpha d\pi_\alpha d\psi d\bar{\psi}\, expi \int d^4x\left[\bar{\psi}\left(i\gamma^\mu D_\mu + \sigma + i\gamma_5\pi\right)\psi + \mathcal{L}_{4\psi}(\sigma,\pi)\right], \quad (3)$$

into which we introduced the boson variables $\sigma = \sigma_\alpha\tau_\alpha\ and\ \pi = \pi_\alpha\tau_\alpha$ (it is supposed here that the Greek subscript runs through the values $\alpha = 0, 1, 2, 3$ and $\tau_0 = 1$). The expression

$$\mathcal{L}_{4\psi}(\sigma,\pi) = -\frac{1}{\bar{g}^2}\left[(g_1+g_2)\left(\pi_0^2+\sigma_i^2\right) + (g_1-g_2)\left(\sigma_0^2+\pi_i^2\right) - 2\mathrm{g}_3(\pi_0\pi_3 + \sigma_0\sigma_3 - \sigma_1\pi_2 + \sigma_2\pi_1)\right], \quad (4)$$

where $\bar{g}^2 = g_1^2 - g_2^2 - g_3^2$, is the bosonized form of Lagrangian density (1). The form (4) is convenient for the $1/N_c$ expansion of the theory since, already in (3), the integration over the quark fields is possible. The covariant derivative of the quark fields

$$\mathcal{D}_\mu\psi = \left(\partial_\mu - i\Gamma_\mu\right)\psi, \qquad \Gamma_\mu = gT_iA_\mu^iP_L + g'B_\mu(P_LT_3 - Q), \quad (5)$$

contains the gauge fields of the electroweak interactions A_μ^i and B_μ together with corresponding coupling constants g and g'; the generators of the $SU(2)$ group are $T_i = \tau_i/2$; $Q = T_3 + 1/6$ is the quark charge matrix.

3. Schwinger-DeWitt expansion and gap equation

It is known [15–18] that the contribution of one-loop quark diagrams in the real part of the effective action in Euclidean space can be represented in the form of an asymptotic series over the proper time t as

$$ReS_{\mathrm{E}} = -\frac{1}{2}\int \frac{dt}{t^3}\int \frac{d^4x_E}{(4\pi)^2}\sum_{n=0}^{\infty} t^n\, tr(a_n) \tag{6}$$

where a_n is the Seeley–DeWitt coefficients. They are expressed by field functions and, in particular, have the form $a_0 = 1, a_1 = -Y\,, and\ a_2 = Y^2/2 - F_{\mu\nu}^2/12$. We do not need the rest of the coefficients, since only at the coefficients with the subscript $n = 0{,}1{,}2$ do the integrals over the proper time diverge and therefore dominate in asymptotic expansion (6). To regularize them, we introduce two dimensional parameters. They characterize the ultraviolet Λ and infrared μ cutoff scales. As a_0 does not contain fields, we need only two integrals, which we denote by C_1 and C_2 and represent in the form

$$C_1 = \int_{1/\Lambda^2}^{1/\mu^2}\frac{dt}{t^2} = \Lambda^2 - \mu^2, \qquad C_2 = \int_{1/\Lambda^2}^{1/\mu^2}\frac{dt}{t} = ln\frac{\Lambda^2}{\mu^2}. \tag{7}$$

where Λ is the scale on which the physics unknown to us is approximated by effective four-quark interactions (1) and μ is a low-energy scale ($\Lambda \gg \mu$), with respect to which the one-loop contributions are determined. This is done in such a way that, at $\Lambda = \mu$, all contributions induced by the $1/N_c$ expansion vanished, which is usually required [8].

The model in question yields the following analytical expressions for the field functions contained in a_n, which we write in the Minkowski spacetime metric as

$$\begin{gathered} Y = \sigma^2 + \pi^2 + i\gamma_5[\sigma,\pi] - i\nabla_\mu\gamma^\mu(\sigma + i\gamma_5\pi) - \tfrac{i}{4}\big[\gamma_\mu,\gamma_\nu\big]F^{\mu\nu}, \\ F^{\mu\nu} = \partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu - i\big[\Gamma_\mu,\Gamma_\nu\big], \\ \nabla_\mu\varphi = \partial_\mu\varphi - i\big[\Gamma_\mu,\varphi\big]. \end{gathered} \tag{8}$$

The consideration of the contribution leading in $1/N_c$ at low energies results in an additional summand described by the Lagrangian density as

$$\Delta\mathcal{L} = -\tfrac{1}{32\pi^2}\Big[C_1 tr(-Y) + C_2 tr\left(\tfrac{Y^2}{2} - \tfrac{1}{12}F_{\mu\nu}^2\right)\Big]. \tag{9}$$

In this case, the complete low-energy theory of fermions and bosons is described by the $SU(2)_L \times U(1)_R$ gauge-invariant Lagrangian density as

$$\mathcal{L} = \bar{\psi}\big(i\gamma^\mu D_\mu + \sigma + i\gamma_5\pi\big)\psi + \mathcal{L}_{4\psi}(\pi,\sigma) + \Delta\mathcal{L}. \tag{10}$$

The last term does not change the initial theory at high energies Λ, since $\Delta\mathcal{L} = 0$ at $\Lambda = \mu$, but it becomes critical at low energies as it contains the potential of the composite Higgs particles, their couplings with the gauge fields, and kinetic terms of free bosonic fields.

Here, we restrict ourselves to consideration of only the Higgs sector of the model. It can be easily established from (10) that the corresponding Lagrangian density has the form

$$\begin{gathered} \mathcal{L}_{\mathrm{H}} = \bar{\mathrm{C}}_1(\Phi_1^+\Phi_1 + \Phi_2^+\Phi_2) - 2\bar{\mathrm{C}}_2\Big[\tfrac{1}{4}(\Phi_1^+\Phi_1 + \Phi_2^+\Phi_2)^2 + (\Phi_1^+\Phi_1)(\Phi_2^+\Phi_2) - \\ \big(\mathrm{Im}(\Phi_1^+\Phi_2)\big)^2\Big] - \tfrac{1}{\bar{g}^2}[(g_1 - g_2)\Phi_1^+\Phi_1 + (g_1 + g_2)\Phi_2^+\Phi_2 + 2g_3\mathrm{Re}(\Phi_1^+\Phi_2)], \end{gathered} \tag{11}$$

where $\bar{C}_{1,2} = N_C C_{1,2}/(4\pi^2)$ and the scalar and pseudoscalar fields are united into two doublets as

$$\Phi_1 = \begin{pmatrix} \pi_2 + i\pi_1 \\ \sigma_0 - i\pi_3 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \sigma_1 - i\sigma_2 \\ -\sigma_3 + i\pi_0 \end{pmatrix}. \tag{12}$$

Supposing that the vacuum expectations of the σ_0 and σ_3 fields may differ from zero ($\langle\sigma_0\rangle = -m_0$ and $\langle\sigma_3\rangle = -m_3$), we find the minimum potential energy conditions (the gap equations) to determine m_0 and m_3 as

$$\begin{aligned} \mathrm{m}_0(g_1 - g_2) - \mathrm{m}_3 g_3 &= \bar{g}^2 \mathrm{m}_0[\bar{\mathrm{C}}_1 - (m_0^2 + 3m_3^2)\bar{\mathrm{C}}_2], \\ \mathrm{m}_0(g_1 + g_2) - \mathrm{m}_0 g_3 &= \bar{g}^2 \mathrm{m}_3[\bar{\mathrm{C}}_1 - (m_3^2 + 3m_0^2)\bar{\mathrm{C}}_2]. \end{aligned} \tag{13}$$

The nonzero vacuum expectations lead to a gap in the spectrum of fermions. As a consequence, the top and bottom quarks acquire nonzero mass as

$$m_t = m_0 + m_3, \;\; m_b = m_0 - m_3. \tag{14}$$

The mass of the quarks differs greatly, $\mathrm{m}_t \gg \mathrm{m}_b$ therefore, a phenomenologically acceptable solution of the gap equations should be sought near the equal values $m_3 \approx m_0$.

The quadratic form over fields in $\mathcal{L}_H$ is diagonalized by two orthogonal rotations. The first rotation characterized by the angle θ diagonalizes the charged modes. The second rotation by the angle θ' is associated with the diagonalization of the neutral particles. The tangents of the angles are expressed by the vacuum averages m_0 and m_3 and the four-quark interaction constants g_2 and g_3,

$$tan\theta = \frac{m_3}{m_0}, \qquad tan2\theta' = 3tan2\theta - 2\frac{g_3}{g_2}, \tag{15}$$

and the transformations themselves have the forms

$$\Phi_1 = cos\theta H_1 + sin\theta H_2, \tag{16}$$
$$\Phi_2 = cos\theta H_2 - sin\theta H_1, \tag{17}$$

where

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ h_0 - m - i\phi_3 \end{pmatrix}, \;\; H_2 = \begin{pmatrix} h_1 - ih_2 \\ -h_3 + i\phi_0 \end{pmatrix}. \tag{18}$$

Here, $m = \sqrt{m_0^2 + m_3^2}$ and the neutral scalar Higgs fields $\chi_1$andχ_2 are introduced through the rotation

$$h_0 = cos(\theta - \theta')\chi_1 + sin(\theta - \theta')\chi_2, \tag{19}$$
$$h_3 = cos(\theta - \theta')\chi_2 - sin(\theta - \theta')\chi_1, \tag{20}$$

which diagonalizes the quadratic form formed by the fields h_0 and h_3. It should be noted that, in new variables, the nonzero vacuum expectation develops only the field H_1, $\langle H_1\rangle = (0, -m)$. Such behavior is a characteristic of any model with two Higgs states [19].

4. The spectrum and Nambu sum rule

Before writing expressions for the mass of the Higgs particles, we should make two remarks. The first remark refers to the form of the gap equations. They can be rewritten in such a way that the quadratic divergence $\bar{\mathrm{C}}_1$ and the logarithmic divergence $\bar{\mathrm{C}}_2$ are completely separated as

$$\bar{g}^2\bar{C}_1 = g_1 - \frac{2g_2}{cos2\theta} + \frac{2g_3}{sin2\theta}, \tag{21}$$

$$\bar{g}^2 m^2 \bar{C}_2 = -\frac{g_2}{cos2\theta} + \frac{g_3}{sin2\theta}, \tag{22}$$

This form proves to be useful for calculating the spectrum. The second remark refers to the redefinition of the Higgs fields. The point is that the expression for the kinetic part of the free Higgs fields contained in (9) has a nonstandard form as

$$\mathcal{L}_H^{kin} = \frac{1}{2}\bar{C}_2\left(\left|D_\mu H_1\right|^2 + \left|D_\mu H_2\right|^2\right), \tag{23}$$

where the covariant derivative is determined as

$$D_\mu H_{1,2} = \left(\partial_\mu - i\frac{g}{2}T_i A_\mu^i - i\frac{g'}{2}B_\mu\right) H_{1,2}. \tag{24}$$

To make the expression assume the standard form, let us redefine the fields $H_{1,2} \to 1/\sqrt{\bar{C}_2}\, H_{1,2}$. This is supposed in what follows.
Given the aforementioned, from (10) we obtain[20,21]

$$m_{\chi_1}^2 = 4m^2 + \frac{2g_2}{\bar{g}^2\bar{C}_2}\left(\frac{1}{\cos2\theta} - \frac{1}{\cos2\theta'}\right), \tag{25}$$

$$m_{\chi_2}^2 = 4m^2 + \frac{2g_2}{\bar{g}^2\bar{C}_2}\left(\frac{1}{\cos2\theta} + \frac{1}{\cos2\theta'}\right), \tag{26}$$

$$m_{\phi_0}^2 = \frac{4g_2}{\bar{g}^2\bar{C}_2\cos2\theta}, \tag{27}$$

$$m_{h^\pm}^2 = \frac{4g_2}{\bar{g}^2\bar{C}_2\sin2\theta}, \tag{28}$$

$$m_{\phi_i}^2 = 0. \tag{29}$$

Hence it follows that, of the eight spinless states of the theory, three are massless Goldstone modes that are absorbed by the gauge fields (the Higgs mechanism). As can be easily seen from (22), the other five states satisfy the sum rule as

$$m_{\chi_1}^2 + m_{\chi_2}^2 + m_{\phi_0}^2 = \frac{4g_3}{\bar{g}^2\bar{C}_2\sin2\theta}, \tag{30}$$

$$m_{h^+}^2 + m_{h^-}^2 = \frac{4g_3}{\bar{g}^2\bar{C}_2\sin2\theta}. \tag{31}$$

This result differs somewhat from the Nambu sum rule. Although the sum of the squared masses of the neutral modes and the analogous sum for the charged modes equal the same expression, its value does not coincide with $4m_t^2$, as is required by the Nambu sum rule. Furthermore, instead of two Nambu partners, the first expression contains contributions by three states, which also differentiates this result from the standard rule. What are the reasons for that? To answer this question, we write two other relations that are also a consequence of mass formulae (25)–(29) as follows:

$$m_{\chi_1}^2 + m_{\chi_2}^2 = m_{\phi_0}^2 + 8m^2, \tag{32}$$

$$m_{h^+}^2 + m_{h^-}^2 = 2m_{\phi_0}^2 + 8m^2. \tag{33}$$

We should recall that $2m^2 = m_t^2 + m_b^2$. It can be seen from this that a mass of meson that is different from zero prevents the Nambu sum rule from being satisfied.
As was mentioned above, in the absence of the interaction with the coupling constant g_2, the theory has an additional symmetry. It plays the role of the global Peccei–Quinn symmetry [22, 23] and prevents the meson, which can be interpreted as an "electroweak axion", from acquiring mass. Indeed, it can be proven that, at $g_2 = 0$, the masses of the particles assume the values

$$m_{\chi_1} = 2m_b \qquad m_{\chi_1} = 2m_t,\ m_{h^\pm} = 2m \qquad m_{\phi_0} = 0. \tag{34}$$

Expressions (34) are entirely consistent with the Nambu sum rule. This indicates that it is the interaction that breaks the symmetry responsible for their breaking in (32)–(33).

5. Numerical estimates

Let us determine to which degree the four-quark interactions responsible for the $U(1)_A$ symmetry breaking modify the Higgs state spectrum. To do this, we express the mass formulae through dimensionless parameter that relates the ratio between the constants $\mathrm{g}_3$and $\mathrm{g}_2$to the mixing angle θ as

$$\frac{\mathrm{g}_2}{\mathrm{g}_3} = a\tan 2\theta. \tag{35}$$

By virtue of the gap equations, the spectrum of the Higgs particles is expressed only through the mass of the quarks and the above parameter a. Thus, according to (15)

$$\tan 2\theta' = (3 - 2a)\tan 2\theta, \tag{36}$$

we conclude that, at $a > 2/3$, the angle $\theta' < 0$ and the spectrum has the form

$$m_{\chi_1}^2 = \frac{2\mathrm{m}^2}{a-1}(2a - 1 - \Delta), \tag{37}$$

$$m_{\chi_2}^2 = \frac{2\mathrm{m}^2}{a-1}(2a - 1 + \Delta), \tag{38}$$

$$m_{\phi_0}^2 = \frac{4\mathrm{m}^2}{a-1}, \tag{39}$$

$$m_{h^\pm}^2 = \frac{4\mathrm{m}^2 a}{a-1}, \tag{40}$$

where $\Delta = \sqrt{cos^2 2\theta + (3 - 2a)^2 sin^2 2\theta}$.
If parameter a is fixed by the known value of the mass of the standard Higgs state $m_{\chi_1} = 125\ GeV \rightarrow a = 4.84$, the above formulae yield the following numerical estimates: $m_{\chi_2} = 346$ GeV, $m_{h^\pm} = 275 GeV$, and $m_{\phi_0} = 125\ GeV$. The fact that the mass $m_{\chi_1} = m_{\phi_0}$shows that $\Delta = 2a - 3$; i.e., the angle $\theta = \pi/4$. To make the final conclusion about the reasonableness of these estimates, the renormalization group approach must be applied, the findings of which are set forth in a separate study. A numerical estimation of the effect of considering the $U(1)_A$ symmetry breaking also seems to be of interest. Since the right term of sum rule (30)–(31) can be written as

$$\frac{4g_2}{\bar{g}^2 \bar{C}_2 sin 2\theta} = 8\mathrm{m}^2 \frac{a}{a-1}, \tag{41}$$

we see that the divergence of the factor $\frac{a}{a-1} = 1.26$ from one is 26%.

6. Conclusion

The numerical values obtained here for the mass of the Higgs states are in agreement with that obtained in [14]. The difference is not large. For instance, for the mass of the $\chi_2$state, the value $m_{\chi_2} = 325$GeV was obtained there and $m_{h^{\pm}} = 245\ GeV$ was obtained for the charged particles. These values are slightly lower than our estimates here; this is explained, however, by the $U(1)_A$ anomaly, which increases the right term of the Nambu sum rule and, consequently, the mass of the particles of the second Higgs doublet. The novelty is the presence in the spectrum of the "electroweak axion" with a mass that practically coincides with the mass of the Higgs ground state. Detailed phenomenological analysis will enable us to gain insight into the future of this prediction of the model.

Acknowledgements

This Research is supported by CFisUC and Fundação para a Ciência e Tecnologia through the grant CERN/FIS-COM/0035/2019. We would like also acknowledge networking support by the COST Action CA16201.

References

[1] H Terazawa, Y Chikashige and K Akama 1977 Unified model of the Nambu-Jona-Lasinio type for all elementary- particle forces *Phys. Rev.* D **15** 480–7
[2] H Terazawa 1980 Subquark model of leptons and quarks *Phys. Rev.* D **22** 184–99
[3] H Terazawa 1980 t-quark mass predicted from a sum rule for lepton and quark masses *Phys. Rev.* D **22** 2921
[4] Y Nambu 1988 Quasisupersymmetry, bootstrap symmetry breaking and fermion masses *Proceedings of the International Workshop New Trends in Strong Coupling Gauge Theories, Nagoya, Japan, Aug., 24–27 1988,* Ed. by M. Bando, T. Muta, and K. Yamawaki (World Scientific, Singapore, 1989)
[5] Y Nambu New theories in physics 1989 *Proceedings of the 11th International Symposium on Elementary ParticlePhysics, Kazimierz, Poland* Ed. by Z. Ajduk, S. Pokorski, and A. Trautman (World Scientific, Singapore,1989) 1–10
[6] V A Miransky, M Tanabashi and K Yamawaki 1989 Dynamical electroweak symmetry breaking with large anomalous dimension and t quark condensate *Phys. Lett.* B **221** 177–83
[7] V A Miransky, M Tanabashi and K Yamawaki Is the t quark responsible for the mass of W and Z bosons? 1989 *Mod. Phys. Lett.* A **4** 1043–53
[8] W A Bardeen, C T Hill and M Lindner Minimal dynamical symmetry breaking of the standard model 1990 *Phys. Rev.* D **41** 1647–60
[9] G Cvetic Top quark condensation 1999 *Rev. Mod. Phys.* **71** 513–74
[10] C T Hill and E H Simmons 2003 Strong dynamics and electroweak symmetry breaking *Phys. Rep.* **381** 235– 402
[11] Y Nambu and G Jona-Lasinio 1961 Dynamical model of elementary particles based on an analogy with superconductivity *Phys. Rev.* **122** 345–58
[12] Y Nambu 1985 Fermion-boson relations in BCS-type theories *Phys.* D **15** 147–51
[13] G E Volovik and M A Zubkov 2013 Nambu sum rule and the relation between the masses of composite Higgs bosons *Phys. Rev. D* **87** 075016
[14] G E Volovik and M A Zubkov 2013 Nambu sum rule in the NJL models: From superfluidity to top quark condensation *JETP Lett.* **97** 301 (*Preprint* 1302.2360)
[15] J Schwinger 1951 On Gauge invariance and vacuum polarization *Phys. Rev.* **82** 664–79
[16] B S DeWitt 1965 *Dynamical Theory of Groups and Fields*
[17] B. S. DeWitt 1975 Quantum field theory in curved spacetime *Phys. Rep.* **19** 295–357

[18] R D Ball 1989 Chiral Gauge theory *Phys. Rep.* **182** 1–186
[19] G C Branco, P M Ferreira, L Lavoura, M N Rebelo, M Sher and J P Silva 2012 Theory and phenomenology of two-Higgs-doublet models *Phys. Rep.* **516** 1–102
[20] A A Osipov and M M Khalifa 2019 Catalysis of the $\langle \bar{b}b \rangle$ Condensate in the Composite Higgs Model *JETP Lett.* **110** 387
[21] A A Osipov and M M Khalifa 2020 The Nambu Sum Rule in the Composite Two Higgs Doublet Model *Phys. of Part. And Nucl. Lett.* **17** 296
[22] R D Peccei and H R Quinn 1977 CP conservation in the presence of pseudoparticles *Phys. Rev. Lett.* **38** 1440–42
[23] R D Peccei and H R Quinn 1977 Constraints imposed by CP conservation in the presence of pseudoparticles *Phys. Rev.* D **16** 1791–97