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Investigation of the second harmonic influence on focusing of moderate energy protons in travelling wave accelerating structure

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Investigation of the second harmonic influence on focusing of moderate energy protons in travelling wave accelerating structure.

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Abstract. For proton acceleration in the traveling wave mode with backward n=-1 spatial harmonic the faster main harmonic n=0, having a larger amplitude, influences on proton beam focusing. In the report an additional focusing by the faster main harmonic n=0 is estimated in acceleration of proton beam with energy above 15 MeV assuming the traveling wave accelerating structure in the S band frequency range.

1. Introduction

The possibility to get simultaneously bunch acceleration and focusing in the same structure is always of large interes in the linac design, especially for relatively small linacs for applications. The opportunity to provide simultaneously conditions for both longitudinal and transverse stability for charged particles bunches accelerating in the field of two spatial harmonics, was intensively investigated for low energy heavy ions, [1]. As is natural for heavy ions acceleration, the application of accelerating structure with a relatively low operating frequency f~150 MHz was assumed. As was shown in [2], for certain conditions in the field of a slower accelerating spatial harmonic and a faster focusing harmonic both longitudinal and transverse stability of accelerated bunches can be obtained simultaneously. Such a regime was proposed for proton acceleration with backward n=-1 spatial harmonic in the medium energy range by using a traveling wave accelerating structure, operating in the S band frequency range. In this case the faster main spatial harmonic n=0 serves as a focusing one. The accelerating and focusing harmonics have different signs of phase velocities, thus providing a very large relative velocity. The amplitude of the main focusing harmonic is larger than the amplitude of the accelerating one. Simultaneously, the backward n=-1 harmonic in the expansion of electric field distribution into the Fourier set is the nearest one to the main harmonic n=0 and has a sufficient field amplitude for effective proton acceleration in the range from ~15 MeV to 60 MeV. These properties are favorable for realization of conditions described in [2], making proposal [3] attractive for further investigation.

2. Beam dynamics for approximation of two harmonics.

In approximation of two spatial harmonics in fields distributions the longitudinal accelerating component $E_z(z)$ is:

$$E_z(z) = E_{-1}\cos(\Psi+\varphi_s) + E_0\cos(\Psi+\varphi_s+\frac{2\pi z}{D_{-1}}), \Psi = \omega t - k_{-1}z,$$
$$D_{-1} = \frac{\beta\lambda(2\pi-\theta)}{2\pi}, k_{-1} = \frac{2\pi}{\beta\lambda}, \lambda = \frac{c}{f} = \frac{2\pi c}{\omega}, \qquad (1)$$

where E_{-1} and E_0 are the amplitudes of accelerating (n=-1) and focusing (n=0) harmonics, respectively, φ_s is the synchronous phase, D_{-1} and θ are the length of period and operating phase advance of the accelerating structure, k_{-1} is the wave number of accelerating harmonic, β is the relative velocity of the particle.

Following the usual procedure, let us derive the difference in the strength of longitudinal field , acting on the synchronous $\Psi = 0$ and other $\Psi \neq 0$ particles in the bunch:

$$\Delta E_z(z) = E_{-1}\cos(\Psi+\varphi_s) + E_0\cos(\Psi+\varphi_s+\frac{2\pi z}{D_{-1}}) - E_{-1}\cos(\varphi_s) - E_0\cos(\varphi_s+\frac{2\pi z}{D_{-1}}). \quad (2)$$

Transferring to the equation of small longitudinal oscillations, instead of a constant force, proportional to eE_{-1}, in the single harmonic approximation, for approximations of two harmonics we get a variable, periodically oscillating force:

$$e\Delta E_z(z) = eE_{-1}(\sin\varphi_s + \chi\sin(\varphi_s+\frac{2\pi z}{D_{-1}})), \chi = \frac{E_0}{E_{-1}}. \qquad (3)$$

Instead of equation with constant coefficients in the case of a single harmonic, for small longitudinal oscillations in the case of two harmonics we get the Mathieu equation.
As opposed to dealing with low energy heavy ions, [2], for protons in moderate energy range from 10 MeV to 60 MeV we have to take into account the action of the magnetic field. In linear approximation the transverse motion of a synchronous particle is defined by the transverse components of electric and magnetic fields, assuming the operating TM0 mode for the structure:

$$e(E_r(z) - \mathrm{c}\beta \mathrm{B}_\varphi(z)) = eE_{-1}\frac{r\pi}{\beta\lambda}(-\frac{\sin\varphi_s}{\gamma^2} + \chi(\frac{\theta}{2\pi-\theta}+\beta^2)\sin(\varphi_s+\frac{2\pi z}{D_{-1}})), \qquad (4)$$

where γ is the Lorentz factor. Taking into account the second harmonic, we also obtain a periodically oscillating focusing force instead of a constant one for a single harmonic. The sign of the constant components in periodical parts of the equation is defined by synchronous phase. Considering the canonical form of the Mathieu equation:

$$\frac{d^2x}{d\tau^2} + (a + q\sin(2\pi\tau+\varphi))x = 0, \qquad (5)$$

for equations, describing small longitudinal and transverse oscillations, we have expressions for coefficients in canonical form (2):

$$a_z = \frac{2\pi eE_{-1}D_{-1}^2\sin(\varphi_s)}{(\beta\gamma)^3\lambda W_0} = \Omega^2\sin(\varphi_s), q_z = \chi\Omega,$$
$$a_r = -\frac{a_z}{2}, \mathrm{q}_r = \frac{\chi\pi eE_{-1}D_{-1}^2}{\beta^3\gamma\lambda W_0}(\frac{\theta}{2\pi-\theta}+\beta^2), \qquad (6)$$

where W_0 is the rest energy of protons. The sign of constant terms a_z and a_r in periodical parts of equations (3) is defined by synchronous phase. In equations for longitudinal and transverse motions the signs of constants terms are naturally different. Periodically oscillating terms emerge from the addition of the second harmonic and for $\chi = 0 \Rightarrow E_0 = 0$ we naturally come to the case of a single harmonic, which has been studied very well, [4].

3. Motion analysis.

Analyzing the second equation in (6), one can show that the focusing effect of the second slow harmonic depends on λ/β. As compared to the conditions considered in [2], for proton acceleration in the range $0.15 \le \beta \le 0.35$ and using the structure with S band frequency, $\lambda \approx\sim 10cm$, the focusing effect decreases by more than two orders. It makes it impossible to assure stability of transverse motion only by the field of the faster second harmonic. Some additional focusing elements are necessary.

As was mentioned in [3], RF parameters of the accelerating structure allow proton acceleration with a the high accelerating rate gradient $E_{-1} \sim (5-25)MV/m$. In this case, the defocusing effect of the accelerating gap, proportional to E_{-1}, [4], becomes of primary importance. If the faster second harmonic cannot ensure required the focusing alone, it can probably help to relax requirements to external focusing elements by defocusing effect dumping.

In order to understand how much defocusing is reduced, we need to compare the transformation matrices for the structure period in one and two harmonic approximations.

For the single accelerating harmonic this matrix of transformation is as (7a), where $|a| \sim \Omega$ - is the module of the defocusing parameter.

$$\begin{pmatrix} ch(\sqrt{a}) & \frac{sh(\sqrt{a})}{\sqrt{a}} \\ \frac{sh(\sqrt{a})}{\sqrt{a}} & ch(\sqrt{a}) \end{pmatrix} \quad (7a)\,. \qquad \begin{pmatrix} ch(\sqrt{\mu}) & \frac{sh(\sqrt{\mu})}{\sqrt{\nu}} \\ \frac{sh(\sqrt{\mu})}{\sqrt{\nu}} & ch(\sqrt{\mu}) \end{pmatrix} \quad (7b)$$

As was estimated above, in the presence of a focusing harmonic the period is unstable, the transformation matrix has the form (7b), depending on two parameters μ and ν. The ratio $a/\mu > 1$ characterizes defocusing relief.

The analysis of the solutions of the Mathieu equations (6) shows that a significant defocusing relief occurs at a ratio $q_r/a_r > 50$, i.e. with a significant excess of the focusing harmonic amplitude over the accelerating harmonic amplitude.

4. Realistic ratio for harmonics amplitudes

The practical implementation of the travelling wave accelerating structure is based on a disk loaded circular waveguide, Fig. 1a.

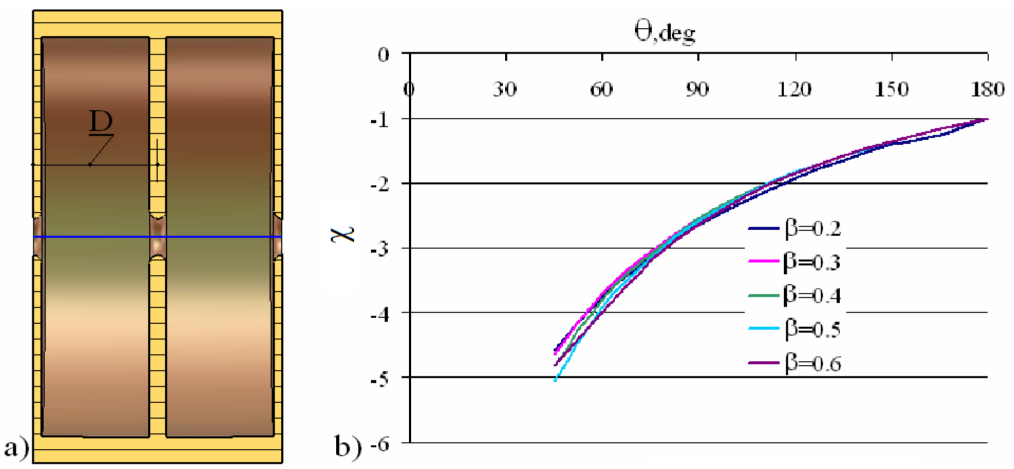

Figure 1. A typical geometry of accelerating structure cells (a) and plots of amplitudes ratio $\chi = E_0/E_{-1}$ for different particles velocities β (b)

Representing $E_z(z)$ distribution at the structure axis, calculated before, as usual:

$$E_z(\mathrm{z}) = \sum_{n=-\infty}^{\infty} E_n e^{-ik_{zn}z}, \mathrm{E}_n = \frac{1}{D}\int_0^D E_z(z) e^{ik_{zn}z} dz, k_{zn} = \frac{\theta + 2n\pi}{D}, D = \frac{\beta\lambda(2\pi - \theta)}{2\pi},$$

we can easy calculate the ratio of harmonic amplitudes. Plots of $\chi(\beta) = E_0 / E_{-1}$ dependences for different particles velocities β are illustrated in Fig. 1b. As can be seen in Fig. 1b, parameter $\chi(\beta)$ is practically independent of the particle velocity β and only can be controlled by selection of phase advance θ. But in the slow travelling wave we always have $\chi(\beta) = 1$ for $\theta = 180^0$. In acceleration with spatial harmonic n=-1 for $\theta < 90^0$ RF efficiency of acceleration drops rapidly. In the acceptable range $60^0 < \theta < 180^0$, as one can see from Fig. 1b, the ratio of harmonics amplitudes does not exceed 4. With the practically realized value $\chi < 5$, when the efficiency of acceleration is still reasonable, there is no significant decrease in defocusing. To have this, one needs a substantially, by orders of magnitude, larger value of amplitude for a faster harmonic.

5. Summary

The attractive concept of simultaneous focusing with an axial symmetric accelerating field has a natural region of application. This is the beginning of particles acceleration – the region of low particle velocities. As the particle velocity, i.e. particle energy, increases, the efficiency of such focusing decreases. This is valid for both concept of two waves, considered in [2], and more sophisticated schemes, considered in [1] and [5]. For protons acceleration in the moderate energy range from ~15 MeV to 60 MeV by using the backward n=-1 spatial harmonic in the travelling wave structure there is no noticeable for practical use focusing effect of a faster spatial harmonic.

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