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## Unsteady conjugate convective heat transfer in a vertical channel at a sudden heating of the bottom

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**Abstract.** The processes of unsteady heat transfer in the mode of thermogravitational convection in a vertical channel with solid walls of finite thermal conductivity after a sudden heating from bottom was studied numerically by the finite element method in three-dimensional problem statement. The system of thermogravitational convection equations was solved in the Boussinesq approximation in the variables of vortex–vector potential of the velocity field and temperature. The calculations were performed with the Prandtl number of 10, the Grashof number of 5000, and the ratio of the thermal conductivity of solid walls to the thermal conductivity of the liquid equal to 6.29. Distributions of the unsteady temperature field into the liquid and solid walls, temperature gradient fields, and velocity fields in the liquid were obtained. The process of formation, loss of axial symmetry and subsequent development of the rising stream was shown.

### 1. Introduction

Knowledge of common and local characteristics of non-stationary conjugate natural convective heat transfer is necessary not only in the design of technical systems such as the passive cooling system of a nuclear reactor, but also in the analysis of various-scale geophysical phenomena. An example is a system of temperature observations in a water-filled well, which serves as a tool for geothermal research. The well inevitably develops an unsteady process of conjugate natural convective heat transfer with fluid flows of complex spatial shape (screw flows), which significantly distorts the results of the necessary high-precision temperature measurements. Understanding the patterns of non-stationary processes of conjugate natural convective heat transfer occurring in liquid-filled vertical channels with walls of finite thermal conductivity is important and relevant from the point of view of analyzing the processes occurring in geophysical and technological systems. The analysis of available information on the results of research in natural and laboratory conditions, as well as the results of numerical studies, has shown that data on common and local characteristics of non-stationary processes of conjugated natural convective heat transfer is insufficient. Similarly, there is insufficient information about the influence of thermal parameters of materials and the geometry of the region on these patterns.

Unsteady conjugated natural convective heat transfer in a high vertical channel with massive walls of finite thermal conductivity after sudden heating of the bottom can be the simplest model of natural and man-made systems, such as faults and cracks in the earth's crust, deep depressions on the bottom of the oceans, deep quarries and workings [1]. Mantle plumes, volcanoes, and kimberlite pipes occupy an important place among geodynamic systems where flows occur due to heating from below [1]. The



energy source of the formation of mantle plumes and kimberlite pipes is the heat flux from the upper mantle to the earth's surface.

Experimental and numerical studies of the processes of plume formation and moving out to the day surface, developing over linear heat sources were performed at the IT SB RAS [2, 3]. The evolution of the spatial flow shape, temperature and velocity fields depending on the input power was studied. The development of a non-stationary convective flow in a rectangular cavity, after sudden heating from below, in a conjugate two-dimensional setting was studied in [4].

It is extremely difficult to obtain data on the distribution of the non-stationary temperature field inside solid walls during physical modeling. The application of mathematical modeling is relevant. The paper presents the results of numerical simulation of three-dimensional unsteady conjugate heat transfer in a regime of thermogravitational convection in a vertical cylindrical channel with solid walls of finite conductivity fluid-filled after a sudden heating from below. By the finite element method [5] the equations of thermogravitational convection are solved in the Boussinesq approximation in terms of temperature, vortex, and vector potential of the velocity field. The results obtained numerically will be used for planning and optimizing experimental studies in the closest possible problem statement.

## 2. Model

Calculations are made in a three-dimensional computational domain in Cartesian coordinates. The computational domain consists of a liquid-filled vertical cylinder with a radius to height ratio of 1:5. The liquid-filled domain is surrounded by massive walls with finite thermal conductivity with a thickness of 0.3 radius of the cylinder. Thus, the ratio of the radius of the calculated domain to its height is 13: 50. An ideal thermal contact condition is set on the inner side of the walls. The outer side of the walls is heat-insulated. The upper end of the calculated domain is isothermic cold, and the bottom of the area is suddenly heated and maintained at a constant temperature. The temperature of the liquid and solid walls at the initial time corresponds to the temperature of the upper end.

The problem is solved in dimensionless form, the radius of the cylinder filled with liquid  $R$  is chosen as the geometric scale. The speed scale is used for  $v/R$ , where  $v$  is the kinematic viscosity of the liquid. The temperature scale is  $\Delta T = T_{\max} - T_{\min}$ , where  $T_{\max}$  and  $T_{\min}$  are the temperatures on the hot and cold walls, respectively. The time scale is  $v/R^2$ .

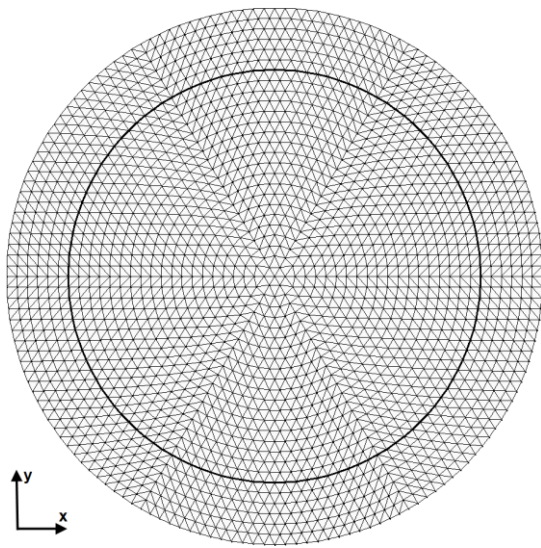
The process is unsteady conjugate convective heat transfer in a regime of thermogravitational convection described by the dimensionless Navier-Stokes equations in the Boussinesq approximation, which is written in terms of temperature, vector potential of the velocity field and vortex:

$$\begin{cases} \frac{\partial T}{\partial t} + \bar{V} \cdot \nabla T = \frac{1}{Pr} \Delta T \\ \frac{\partial \omega}{\partial t} - \nabla \times [\bar{V} \times \bar{\omega}] = \nabla \times [\nabla \times \bar{\omega}] + Gr [\nabla \times (T \cdot \bar{g})] \\ -\Delta \psi_x = \omega_x \quad -\Delta \psi_y = \omega_y \quad -\Delta \psi_z = \omega_z \\ \bar{V} = \nabla \times \bar{\psi} \end{cases}$$

where  $T$ ,  $\omega$ ,  $\psi$ ,  $V$  are the temperature, the vortex, the vector potential of the velocity field, and the velocity field, respectively.

In dimensionless equations,  $Gr = (\beta \cdot g / v^2) \cdot \Delta T \cdot R^3$  is the Grashof number. Here  $\beta$  is the volume expansion coefficient of the liquid,  $g$  is the acceleration of gravity,  $v$  is the kinematic viscosity of liquid, and  $\Delta T$  is the temperature difference. Prandtl number  $Pr = v/a$ , where  $a = \lambda_F / \rho \cdot C_P$  is the coefficient of thermal diffusivity of the liquid,  $\lambda_F$  is coefficient of thermal conductivity of the liquid,  $\rho$  is the density, and  $C_P$  is the heat capacity at constant pressure.

The problem is solved under the following boundary conditions. The maximum temperature in the system is set at the bottom of the area at the initial time:  $T|_{\Gamma_1} = 1$ . At the upper end of the area, the minimum temperature in the system is maintained  $T|_{\Gamma_2} = 0$ .



**Figure 1.** A fragment of the grid at height  $z = 0$

The outer surface of the vertical walls is heat-insulated  $\left. \frac{\partial T}{\partial n} \right|_{\Gamma_3} = 0$ . On the inner surface of the

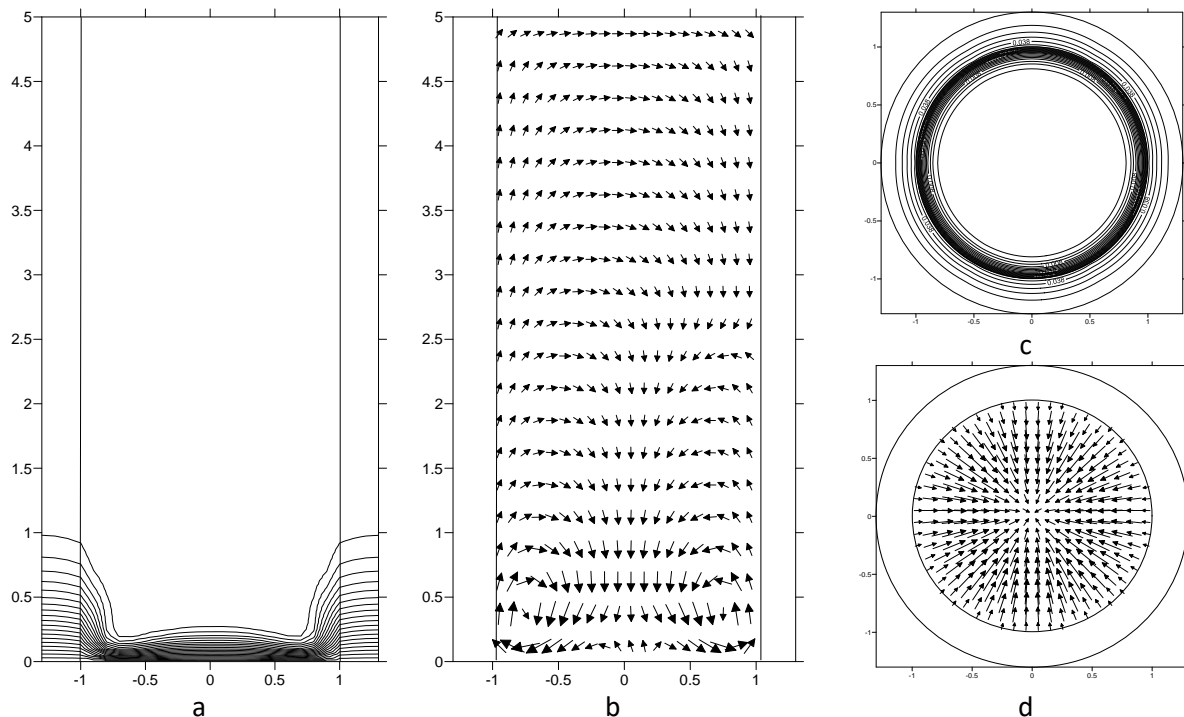
walls, the conditions for non-flow, adhesion, and ideal thermal contact are set:  $\psi|_{\Gamma_4} = 0$ ,

$$\omega|_{\Gamma_4} = -\Delta\psi|_{\Gamma_4}, \quad -\lambda_s \left. \frac{\partial T}{\partial n} \right|_{\Gamma_{4-}} = -\lambda_f \left. \frac{\partial T}{\partial n} \right|_{\Gamma_{4+}}.$$

Numerical simulation was performed using the finite element method on a tetrahedral grid with 212 thousand nodes. Linear basis functions are used in the calculations. The grid fragment in the height section is shown in figure 1. Calculations are performed for the Prandtl number  $Pr = 10$ , the Grashof number  $Gr = 5000$ , and the ratio of the thermal conductivity of solid walls to the thermal conductivity of the liquid  $\lambda_s/\lambda_f = 6.29$ .

### 3. Result and discussion

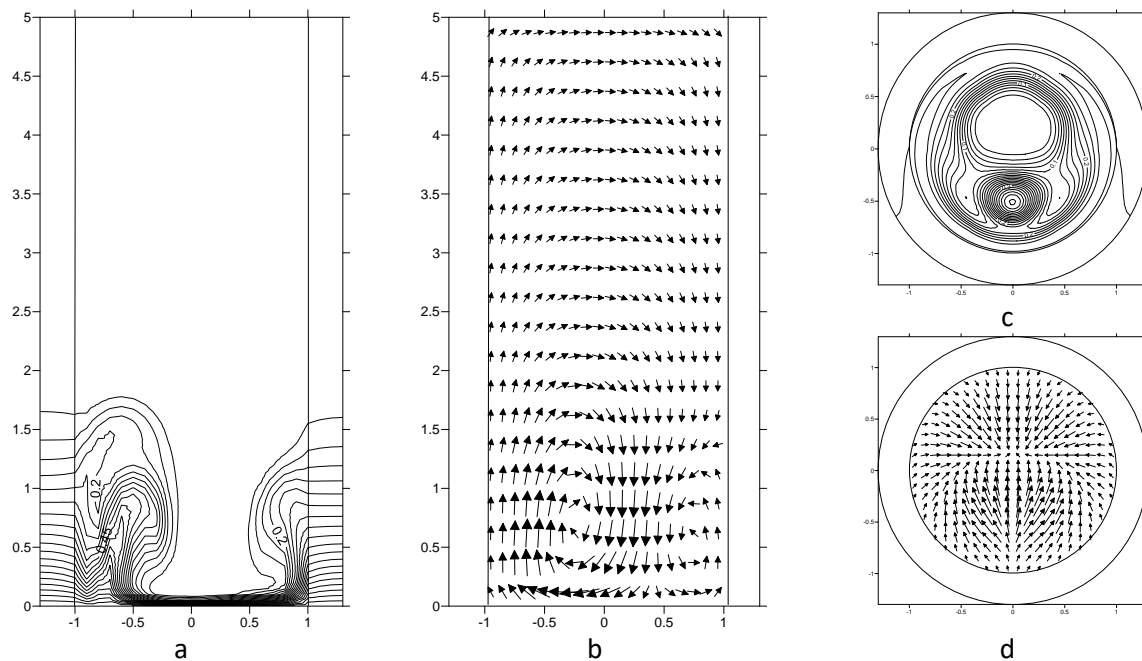
Calculations of non-stationary conjugate heat transfer are performed in the regime of thermogravitational convection with the ratio of the radius of the area filled with liquid to its height of 1:5.



**Figure 2.** Temperature field on the time layer  $t = 20$  in sections: a –  $x = 0$ ; c –  $z = 1$ , velocity field on the time layer  $t = 20$  in sections: b –  $x = 0$ ; d –  $z = 1$ .

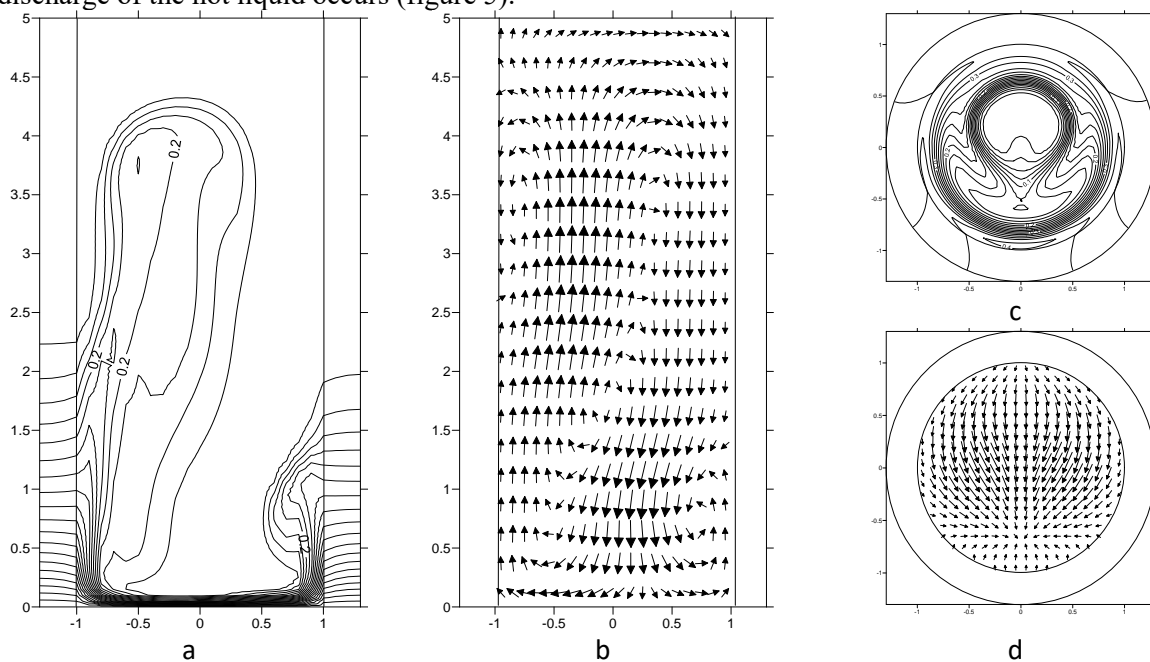
Figure 2 shows the temperature and velocity fields on the time layer  $t = 20$ . It is noticeable that due to higher thermal conductivity, solid walls warm up faster than the liquid layer in the bottom area. As a result, an annular upward flow of hot liquid is formed on the inner surface of the walls. A downward

flow of liquid is formed in the center of the cylindrical area. Figure 3 shows that the flow and temperature field lose their axial symmetry and the initial moment of formation of the rising plume is noticeable.

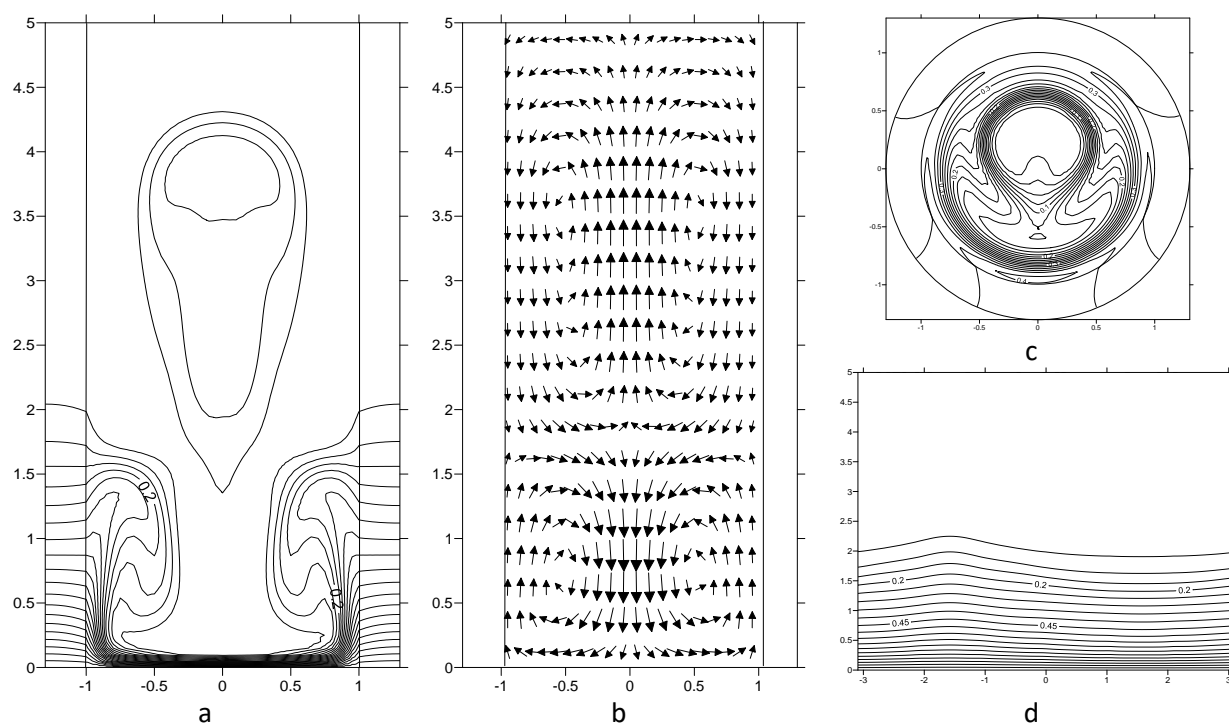


**Figure 3.** Temperature field on the time layer  $t = 60$  in sections: a –  $x = 0$ ; c –  $z = 1$ , velocity field on the time layer  $t = 60$  in sections: b –  $x = 0$ ; d –  $z = 1$ .

Under the action of the descending central flow of cold liquid, the upward flow of the plume is pushed to the wall (figures 3-4). After the critical mass of the heated liquid accumulates, a local discharge of the hot liquid occurs (figure 5).

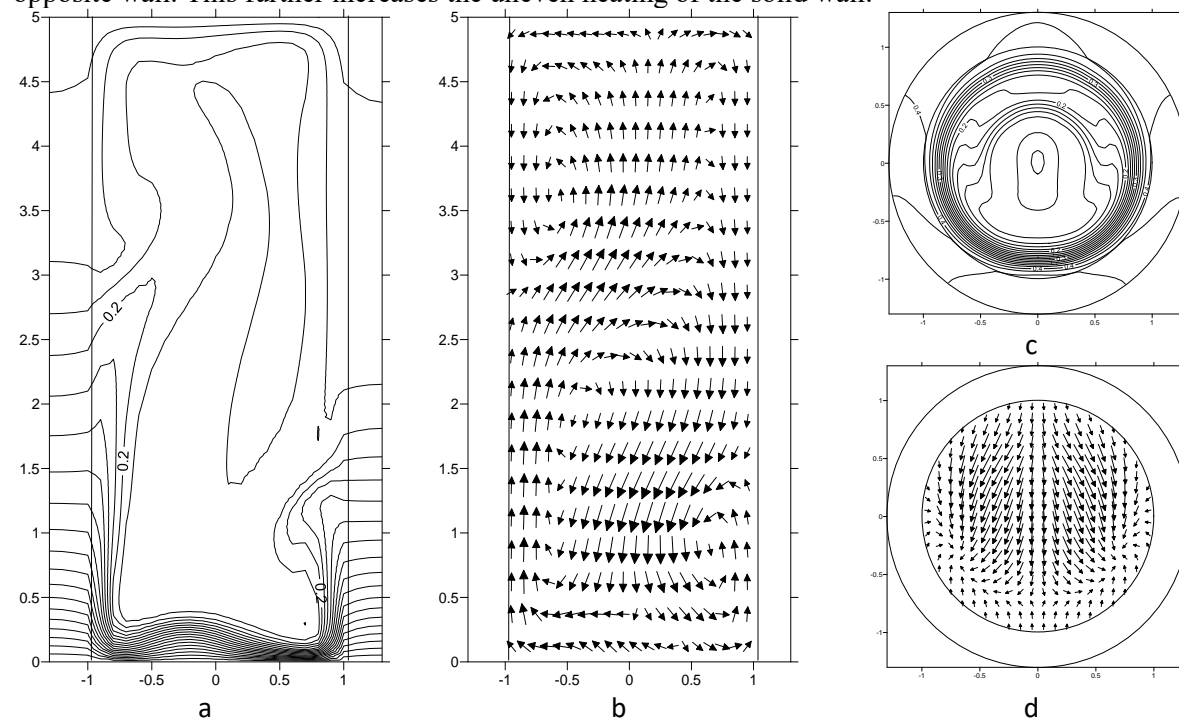


**Figure 4.** Temperature field on the time layer  $t = 100$  in sections: a –  $x = 0$ ; c –  $z = 1$ , velocity field on the time layer  $t = 100$  in sections: b –  $x = 0$ ; d –  $z = 1$ .



**Figure 5.** Temperature field on the time layer  $t = 100$  in sections: a –  $y = 0$ ; c –  $z = 1$ ; d –  $r = 1$ , velocity field on the time layer  $t = 100$  in sections: b –  $y = 0$ .

Figure 5d shows the temperature distribution on the inner wall surface. Local overheating is clearly visible in the place where the plume was pushed, which led to the release of hot liquid. In figure 4b, it can be seen that a local increase in the lift flow led to an increase in the downward flow on the opposite wall. This further increases the uneven heating of the solid wall.

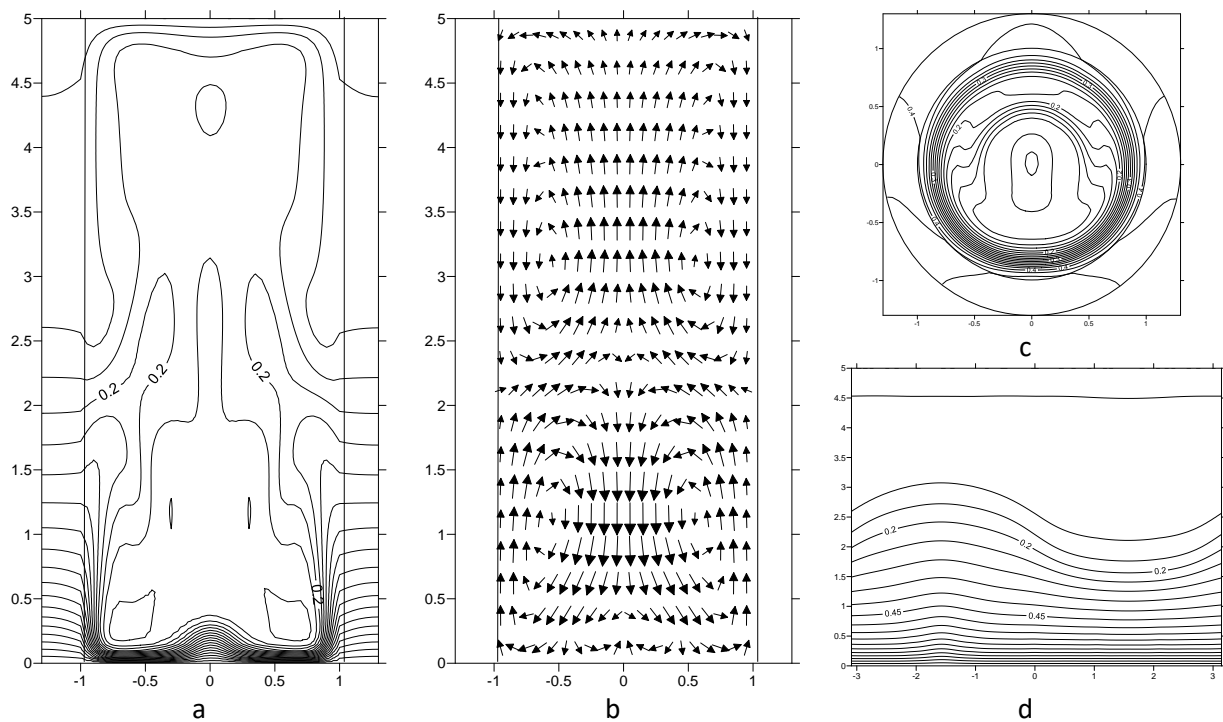


**Figure 6.** Temperature field on the time layer  $t = 200$  in sections: a –  $x = 0$ ; c –  $z = 1$ , velocity field on the time layer  $t = 200$  in sections: b –  $x = 0$ ; d –  $z = 1$ .

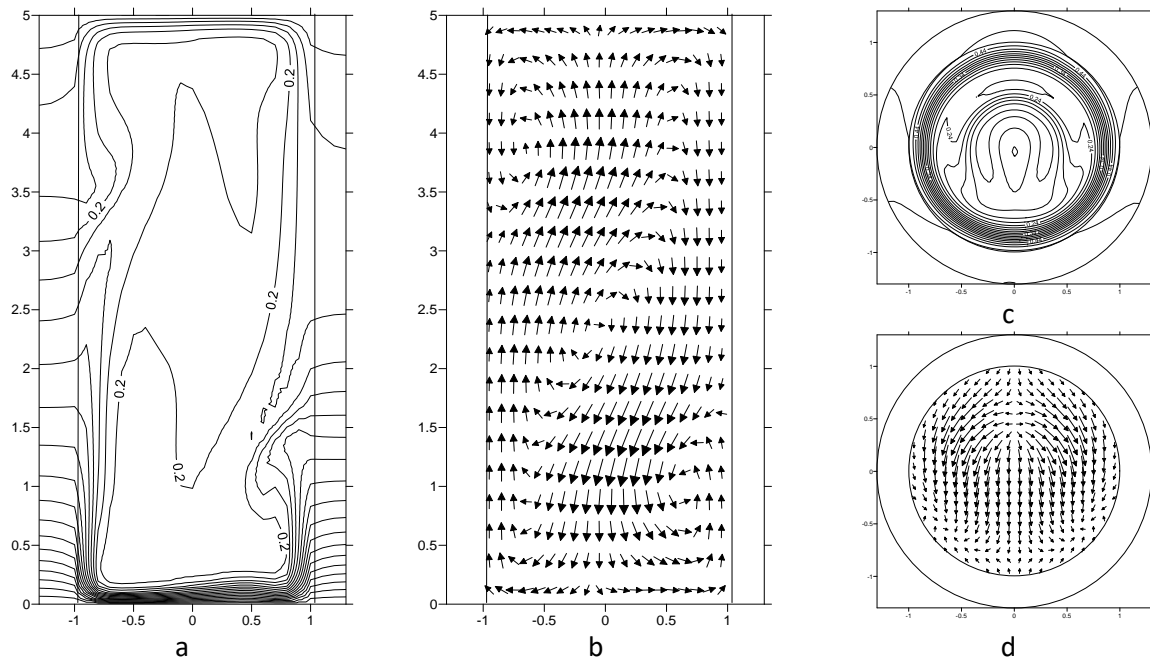
An interesting feature can be noted in figures 4a, 4b, 5a, 5b, which represent the temperature field and the velocity field at the same time, but in different sections. It is noticeable that the temperature field and velocity field lost their symmetry in the x-section, but this did not happen in the y-section. The distribution of the temperature and velocity fields in the y-section reflects the process of detaching the head part of the plume and lifting it to the cold upper end. Whereas the distribution of the temperature field and the velocity field in the cross section over x shows the process of local ejection of hot liquid masses over the surface of the hot wall.

Another noteworthy feature is the phenomenon of separation of the boundary layer, which is noticeable in figures 4a, b. It can be seen that at a height of 2.5, the updraft breaks away from the wall and moves to the center of the area. The downward flow behaves in a similar way, separating at a height of 1.5 from the wall and mixing to the center of the area. Due to this, the growth of the upstream flow along the opposite wall is blocked.

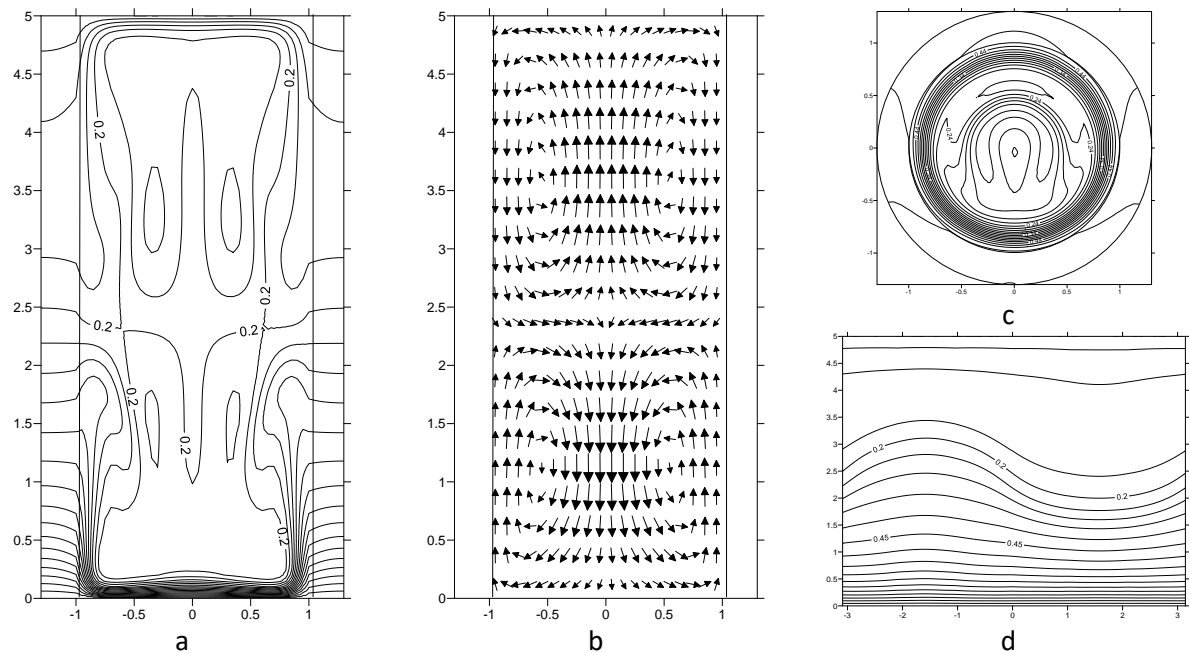
Figures 6-7 show the further evolution of the process of local discharge of hot liquid along the wall. It can be seen that under the action of the increasing hot ascending flow on one side of the wall and the increasing cold descending flow on the opposite side of the wall, the inhomogeneity of wall heating significantly increases. As a result, when the next plume is born, which is visible in figure 7a, the plume will also be pushed in the same direction as the previous plume. That is, there will be another local release of hot liquid in the same place. The initial stage of this process is visible in figure 6a, by the displacement of the ascending central flow to the wall.



**Figure 7.** Temperature field on the time layer  $t = 200$  in sections: a –  $y = 0$ ; c –  $z = 1$ ; d –  $r = 1$ , velocity field on the time layer  $t = 200$  in sections: b –  $y = 0$ .



**Figure 8.** Temperature field on the time layer  $t = 320$  in sections: a –  $x = 0$ ; c –  $z = 1$ , velocity field on the time layer  $t = 320$  in sections: b –  $x = 0$ ; d –  $z = 1$ .



**Figure 9.** Temperature field on the time layer  $t = 320$  in sections: a –  $y = 0$ ; c –  $z = 1$ ; d –  $r = 1$ , velocity field on the time layer  $t = 320$  in sections: b –  $y = 0$ .

At the same time, figures 8-9 show that with further heating of solid walls, there is a tendency to change the situation. Figure 8a shows that the next central rising flow that has begun to form starts shifting to the opposite wall. In other words, it is possible, even if temporarily, to implement the oscillatory process of the emergence and development of a large-scale two-vortex flow observed in the two-dimensional case [4]. In which the intensity of the flow alternately changes in the right and left vortex. As a result, a wave-like pulsating rising stream is formed.



## Conclusions

The process of unsteady conjugate heat transfer in the regime of thermogravitational convection in a vertical cylindrical channel with massive walls of finite thermal conductivity, after sudden heating from below, has been studied numerically in the conjugate three-dimensional formulation using the finite element method. The system of thermogravitational convection equations has been solved in the Boussinesq approximation in terms of vortex, vector potential of the velocity field and temperature. The calculations have been performed with the Prandtl of 10, the Grashof number of 5000, and the ratio of the thermal conductivity of solid walls to the thermal conductivity of the liquid equal to 6.29. Distributions of the non-stationary temperature field in the liquid and solid walls, temperature gradient fields, and velocity fields in the liquid have been obtained.

Due to the higher thermal conductivity of solid walls, they warm up faster, resulting in a more dynamic growth of an upward flow at their surface. An ascending flow of hot liquid along the heated wall generates a descending flow of cold liquid, which interacts with the periphery of the ascending flow. Loss of stability in the azimuthal direction on the leading edge of the hot liquid ascending flow at the vertical wall leads to a break of the axial symmetry of the convective flows. This leads to local overheating of the wall section and local discharge of hot liquid along the wall. The temperature field of the solid wall becomes inhomogeneous.

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