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# The removing of multiple reflections from a curved interface at the common-source seismogram 

E V Rabinovich, A V Demianenko and A S Turkin<br>Faculty of Automation and Computer Engineering, Novosibirsk State Technical University NETI, 20 Karla Marksa ave., Novosibirsk, 630073, Russia<br>E-mail: rabinovich@corp.nstu.ru


#### Abstract

The proposed simplified version of the original solution for the actual problem of removing multiple reflections from a curved interface at common-source seismogram is suggested. The solution to this problem is complicated in the framework of the approach based on using a reflected wave's hodograph. The proposed solution is based on the analytical expression for describing a reflecting interface structural form. The expression is obtained using the location algorithm based on solving only direct kinematic seismic problems. There is an analytical expression; one can calculate the propagation paths of the multiple reflections and its arrival times at the receivers of the observation base. If a seismic wavelet is detected in a limited range, including a multiple reflected wave arrival time, it is possible to remove the detected wavelet for eliminating its distorting effect to the direct reflected signal.


## 1. Introduction

Multiple reflection waves are called waves which have more than one reflection event before they have been registered at the surface of observation. There is pure multiple reflection when the wave reflects from the interface and the ground-to-air interface or the bottom of the low-velocity layer repeatedly. A complex multiple reflections are called the wave which reflects from different interfaces during a travel path besides from the ground-to-air interface or the bottom of the low-velocity layer.

Multiple waves are considered as noise that interferes with the primary reflection waves and obstructs a distinguish detection lineups in seismograms. Therefore, it is necessary to take measures to eliminate their distorting effect on reflection registration.

The hodographs of primary and multiple reflections are the hyperbolas. That is why multiple waves can create a wrong representation of a deep structure, being accepted as typical reflections from the interfaces [1].

Various attempts are made to remove lineups of multiples on the preprocessing stage. The exact formulas of the hodographs of multiple reflected waves are derived, which can ensure lineups removing in the seismogram for plane interfaces and a homogeneous medium. A multiples hodographs research is difficult for the curved interfaces. In the case of the curved interface line, the hodograph broke up into separate branches with the formation of cusps and closed loops [2, 3].

This work shows the possibility of eliminating multiple reflection lineups from the common-source seismogram from a curved interface. This opportunity arises due to the processing of seismograms based on location technology for constructing seismic images [4].

## 2. Problem definition

Mention above original location technology uses smoothing the sequence of small linear segments of the interface for layered approximation interfaces of the depth profile model. The sequence is obtained due to the coordinated location of several closely spaced seismic sources of reflection pulse (SSRP). Location is carried out with seismic digital antenna array which uses the superresolution SSRP location algorithm [5-8]. An algorithm uses only unambiguous and structures sustainable solutions of direct kinematic seismic problems (DKSP) [9].

The layered sub-homogeneous medium is taken as the base velocity-depth 2D model. Monotypic plane reflected P-waves propagate from the point of seismic source along the straight-line ray paths in this model. It means that propagate seismic wavelet velocities along with incident and reflected ray paths are constant. However, for different seismic pairs: point source (PS) - point receiver (PR), seismic wavelets velocity in a layer may vary depending on the reflecting interface basic form [10].

The processing which can give away to remove lineups for multiple reflections from a curved interface is considered on synthetic seismogram containing lineups of double reflection. For more complex multiple reflections, the approach is the same in the case, then it is possible; only the calculations and modelling complexity increases.

The depth profile model is shown in figure 1. It contains horizontal observation site, first curved interface and second horizontal interface. The primary reflection ray paths are depicted with dotted lines and multiple reflection ray paths with solid lines. That simplified model was selected to facilitate the perception of image details. However, the model does not limit the generality of the proposed solution.
Notice that the lineup of double reflection has a concave shape, which corresponds to a synclinal disruption of the interface structure form.


Figure 1. The fragment of the depth profile model
The fragments of two common-source seismogram models are shown in Figure 2 for the part of depth profile model from Figure 1. The left model contains the double lineup reflection from the first interface. The right model does not contain this lineup.
The processing problem is searching for the proper conversion from the left model into the right model.

## 3. The construction of the reflection interface line

The problem of SSRI location which is settled on interface reduces to the problem of finding the tangent point coordinates of the line that imitate lower interface of a layer, and the isochrone line of reflection (further - the isochrone). Suppose that the interface line is such that it has a single point of contact with one isochrone.


Figure 2. The fragments of common-source seismograms before and after processing
The isochrone is a curve even order, symmetrical about the middle of the interval that connects its focuses. The isochrone focuses are determined by pair coordinates SP - RP in the first interface. In the more buried interfaces, the isochrone focuses are the interception points already known line of the previous interface with the incident and reflected rays.

Within the confines of the general deep-velocity model of the medium, each isochron connects with one specific RP (receiver) and represents ellipse, whose parameters are determined by the kinematic parameters of the layer.

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The DKSP solve-based SSRP location algorithm is used for estimation the main kinematic parameters each layer of a medium. It estimates the reflection P-wave velocity in the current layer, the path from source to receiver and coordinates of all reflection and refraction points on it. Reflection point coordinates of current layer lower interface is the desired coordinates of the isochrone tangent point.

Suppose that interface line segment, which is located between the tangent points of two adjacent isochrone lines is a small size ( $\sim 25 \mathrm{~m}$ with a distance between receivers 50 m ) straight segment. The adjacent isochrones have the one common focus (wave source) and different focuses, which are represented by neighbor receivers (the first and the second receivers on Figure 1). Reflection point coordinates are calculated based on the following relationships and reasoning for the major horizontal axis of the ellipse. Canonical equations describe two adjacent ellipses

$$
\begin{gather*}
\frac{x^{2}}{a_{1}^{2}}+\frac{z^{2}}{b_{1}^{2}}=1, \frac{(x-d)^{2}}{a_{3}^{2}}+\frac{z^{2}}{b_{3}^{2}}=1,  \tag{1}\\
a_{i}=\frac{v t_{i}}{2}, c_{i}=\frac{\sqrt{\left(x_{2 c_{i}}-x_{1 c_{i}}\right)^{2}+\left(z_{2 c_{i}}-z_{1 c_{i}}\right)^{2}}}{2}=\frac{L_{i}}{2}, b_{i}^{2}=\frac{v^{2} t_{i}^{2}-L_{i}^{2}}{4}, d=\frac{L_{3}-L_{1}}{2}>0, z_{2 c_{i}}=z_{1 c_{i}}=0, i=1,3,
\end{gather*}
$$

where $a_{i}$ and $b_{i}$ are major and minor axes respectively, $c_{i}$ is focal distance, $\left(x_{2 c_{i}}, z_{2 c_{i}}\right),\left(x_{1 c_{i}}, z_{c_{i}}\right)$ are focus coordinates, $v$ is the wave velocity in the medium, $t_{i}$ is wave arrival time on receiver, $L_{i}$ is distance between source and receiver.

The equations give the corresponding tangents

$$
\begin{equation*}
\frac{l_{1} x}{a_{1}^{2}}+\frac{h_{1} z}{b_{1}^{2}}=1, \frac{\left(l_{3}-d\right)(x-d)}{a_{3}^{2}}+\frac{h_{3} z}{b_{3}^{2}}=1, \tag{2}
\end{equation*}
$$

where $\left(l_{1}, h_{1}\right),\left(l_{3}, h_{3}\right)$ denote the coordinates of touch points.
Substituting the equation (2) into (1) and demanding coincidence of tangents, we obtain

$$
\begin{gather*}
a_{1}^{2}\left(1-\alpha^{2}\right)+\alpha^{2} \beta^{2} a_{3}^{2}=\left(a_{3}+d \sqrt{1-\alpha^{2}}\right)^{2}, \alpha^{2}=\frac{h_{3}^{2}}{b_{3}^{2}} \leq 1, \beta^{2}=\frac{b_{1}^{2}}{b_{3}^{2}} \leq 1 ; \\
\alpha^{2}=-\frac{\left(v^{2} t_{3}^{2}-L_{3}^{2}\right)\left\{\left[\left(L_{3}-L_{1}\right)^{2}+\left(t_{3}^{2}-t_{1}^{2}\right) v^{2}\right]^{2}-4\left(L_{3}-L_{1}\right)^{2} v^{2} t_{3}^{2}\right\}}{\left\{L_{3}\left(L_{3}-L_{1}\right)^{2}-v^{2}\left[L_{3} t_{1}^{2}+\left(L_{3}-2 L_{1}\right) t_{3}^{2}\right]\right\}^{2}} ;  \tag{3}\\
h_{1}=\frac{a_{3} b_{1}^{2} \alpha}{b_{3}\left(a_{3}+d \sqrt{1-\alpha^{2}}\right)} ; l_{1}=\frac{a_{1}^{2} \sqrt{1-\alpha^{2}}}{a_{3}+d \sqrt{1-\alpha^{2}}} ; h_{3}=b_{3} \alpha ; l_{3}=d+a_{3} \sqrt{1-\alpha^{2}} ; \operatorname{tg}(\varphi)=-\frac{l_{1} b_{1}^{2}}{h_{1} a_{1}^{2}} ;  \tag{4}\\
\cos ^{2}(\varphi)=\frac{1}{1+\operatorname{tg}^{2}(\varphi)}=\frac{4\left(L_{3}-L_{1}\right)^{2} v^{2} t_{3}^{2}-\left[\left(t_{3}^{2}-t_{1}^{2}\right) v^{2}+\left(L_{3}-L_{1}\right)^{2}\right]^{2}}{4\left(L_{3}-L_{1}\right)^{2} v^{2} t_{3}^{2}-\left[\left(t_{3}^{2}-t_{1}^{2}\right) v^{2}-\left(L_{3}-L_{1}\right)^{2}\right]^{2}+\left[\left(t_{3}^{2}-t_{1}^{2}\right) v^{2}-\left(L_{3}^{2}-L_{1}^{2}\right)\right]^{2}}, \tag{5}
\end{gather*}
$$

where $\varphi$ denotes the angle of incidence for a tangent to the horizontal axis.
The equations (1) - (5) contain unknown parameter $v=$ const. The second wave source and receivers 2 and 3 are used for its estimation. Here ellipses are denoted with equations

$$
\frac{x^{2}}{a_{i}^{2}}+\frac{z^{2}}{b_{i}^{2}}=1, \frac{x_{i} x}{a_{i}^{2}}+\frac{z_{i} z}{b_{i}^{2}}=1, d=0, i=2,3 .
$$

The estimation of $v$ for a linear interface is calculating with formulas

$$
\begin{gather*}
v^{2}=\frac{L_{3}^{2}-L_{2}^{2}}{t_{3}^{2}-t_{2}^{2}} \cos ^{2}(\varphi)=\frac{L_{3}^{2}-L_{2}^{2}}{\left(t_{3}^{2}-t_{2}^{2}\right)\left(1+\operatorname{tg}^{2}(\varphi)\right)}=\frac{\left(L_{3}^{2}-L_{2}^{2}\right) \alpha^{2} v^{2} t_{3}^{2}}{\left(t_{3}^{2}-t_{2}^{2}\right)\left[v^{2} t_{3}^{2}-\left(1-\alpha^{2}\right) L_{3}^{2}\right]} \\
{\left[\left(L_{3}^{2}-L_{2}^{2}\right)-\left(t_{3}^{2}-t_{2}^{2}\right) v^{2}\right] t_{3}^{2}+\left(L_{2}^{2} t_{3}^{2}-L_{3}^{2} t_{2}^{2}\right)\left(1-\alpha^{2}\right)=0} \tag{6}
\end{gather*}
$$

After substituting (3) into (6) and bicubic simplification, the equation is obtained:

$$
\begin{equation*}
v^{6}+r v^{4}+s v^{2}+t=0 \tag{7}
\end{equation*}
$$

The real solutions of this equation following v2 can be found by applying the Cardano formula.
The depth profile model fragment with two isochrones is shown in Figure 1.
In the case, when the major ellipse axis is inclined to the horizontal surface of land with angle $\theta$, the reflection point coordinates are calculated with the rotation of coordinate axes. In the coordinate system $z 0 x$, which is shown in Figure 3, the tangency point coordinates are $\left(l_{1}, h_{1}\right)$ and $\left(l_{3}, h_{3}\right)$. In the coordinate system $Z 0^{\prime} X$, where axis $0^{\prime} X$ is parallel to the horizon, we have

$$
\begin{gathered}
X=x \cos (\theta)+z \sin (\theta)+x_{0} \\
Z=-x \sin (\theta)+z \cos (\theta)+z_{0},
\end{gathered}
$$

where ( $x_{0}, z_{0}$ ) isapoint 0 coordinates. Substituting $\left(l_{1}, h_{1}\right)$ and $\left(l_{3}, h_{3}\right)$ into the equations for $X$ and $Z$, we obtain the tangency points coordinates in $X 0^{\prime} Y$ coordinate system

$$
\begin{aligned}
& \Lambda_{1}=l_{1} \cos (\theta)+h_{1} \sin (\theta)+x_{0} ; H_{1}=-l_{1} \sin (\theta)+h_{1} \cos (\theta)+z_{0}, \\
& \Lambda_{3}=l_{3} \cos (\theta)+h_{3} \sin (\theta)+x_{0} ; H_{3}=-l_{3} \sin (\theta)+h_{3} \cos (\theta)+z_{0} .
\end{aligned}
$$

Thus, after calculating the tangency point coordinates of the interface line by all isochrones, we have a sequence of small linear segments of the interface. Smoothing these linear segments, we can approximate the interface line with good accuracy and obtain its analytical expression in the form of the corresponding polynomial.


Figure3. The coordinate system rotation scheme.

## 4. The construction of the double reflected wave propagation path

The propagation path double reflected wave can be made with knowing the equation of the first interface. Seismic wave path must correspond with the Fermat principle (in this case for minimizing the travel time) and enforce the Snellius law (the incidence angle is equal to the reflected angle). Such paths are shown in Figure 1.

The wave propagation ray tracing is the most straightforward task of parametric optimization. It is necessary to minimize the wave propagation time when varying the position of reflection points on the interface line given by the corresponding equation. In this case, it is necessary to monitor the compliance with Snellius law, which requires calculation and ensuring the correctness of the gradients of all reflecting interfaces, including the ground-to-air interface or the bottom of the low-velocity layer.

One can calculate the arrival times at all geophones of the observation base and construct the propagation paths of a double reflected wave. In the case of seismic wavelet detection in a limited range in the trace, which includes the arrival time of double reflected wave, the removing of wavelet eliminates its distorting effect on the registration of primary reflections.

Thus, the propagation paths construction and the arrival time's calculation of double reflected wave to all geophones of the observation base makes it possible to eliminate the lineup related to this wave.

## 5. Conclusion

The proposed simplified version of the original solution for the actual problem of removing multiple reflections from a curved interface at common-source seismogram is suggested. The solution to this problem is complicated in the framework of the approach based on the use of reflected wave hodographs.

In full, the problem is solved with processing data of the seismic digital antenna array which uses super-resolution SSRP location algorithm. The algorithm based on finding the solutions of DKSP, it allows approximating interfaces of the depth profile model by the polynomial. To do this, it constructs and smoothes layer-by-layer the sequence of small linear sections of the recurrent interface line of the geological medium. The result of approximation is the analytical expression (polynomial) which describes interface lines of depth profile.

With this information, it is possible to calculate the reflected wave propagation paths of different multiplicity and their arrival times at all geophones of the observation base.

If a seismic wavelet is detected in a limited range, including a multiple reflected wave arrival time, it is possible to remove a detected wavelet for eliminating its distorting effect to primary reflected signal.

## References

[1] Puzyrev N N 1959 Interpretation of seismic data by reflection waves method (Moscow: Gostoptekhizdat) 452 p. (in Russian)
[2] Voskresenskiy Y N 2006 The construction of seismic images (Moscow: RSU NG) 117 p. (in Russian)
[3] Glogovskiy V M, Langman S L 2009 Properties of the solution of the inverse kinematic seismic problem Technologies of seismic prospecting $1 \mathrm{p} 10-17$
[4] Rabinovich E V, Shefel G S, Jukov A V 2018 Location technology for construction of seismic images In Proceedings of 14th International Scientific-Technical Conference on Actual Problems of Electronic Instrument Engineering Proceedings APEIE-2018(Novosibirsk) Vol. 1. Part 4. pp. 519-523.
[5] Capon J 1969 High-resolution frequency-wavenumber spectrum analysis In Proc. IEEE 57(8) 1408-1418.
[6] Tikhonov V I 1983 Optimum signal reception (Moscow: Radio and communication) 320 p. (in Russian).
[7] Ratynskiy M V 2003 Adaptation and superresolution in antenna arrays (Moscow: Radio and communication) 200 p . (in Russian)
[8] Malyshkin G S et al. 2014 Optimal and adaptive methods of processing hydroacoustic signals (review) Acoustical Physics 60(5) $570-587$.
[9] Obolentseva I R 1974 Numerical methods of solution of direct three-dimensional problems of geometrical seismics for multi-layered media with arbitry shape boundaries Geology and Geophysics 9 113-128. (in Russian)
[10] Rabinovich E V, Filipenko N Y, Shefel G S 2019 The location estimation of seismic wave propagation layer velocities In Proceedings of the $14^{\text {th }}$ International Forum on Strategic Technology (IFOST 2019 )(Tomsk: TPU Publishing House), pp. 118-121.

