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Mutualinfluence of two dipole antennas

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Abstract. Accounting for the mutual influence of emitters is one of the important problems in antenna theory. Due to the fact that the results of numerical calculations are poorly presented in the scientific literature, it is very important to develop a theory and perform calculations. In this paper, we obtain a one-dimensional integral equation with a dedicated main operator for two identical emitters. An efficient numerical method is constructed, a program is developed, and numerical calculations are performed over a wide range of input parameters.

1. Introduction

Electrodynamic analysis of dipole antennas is based on solving integral equations for surface currents. Knowledge of surface currents allows finding the main characteristics of antennas: input resistance, field in the far zone, field in the near zone, active and reactive parts of the radiation power. The kernels of integral equations have singularities; therefore, the numerical solution of integral equations presents certain difficulties. To overcome them, the problem operator is represented as a sum of two operators: the main one and the compact one. The principal operator matrix can be found analytically. The matrix elements of a compact operator are efficiently calculated on a computer using standard programs. In this work, the method of separating the main operator is supposed to be applied to calculate the input resistance of a system of two vibrators.

2. Formulation of the problem. Integral equation

Let us consider two identical tubular linear vibrators of length $2l$ and radius a. The distance between the axes of the vibrators is equal to d. It is required to determine the currents flowing through the vibrators and other characteristics through them. We compose the integral equation to determine the currents from the condition of equality to zero of the tangential component of the electric field[1–4].

$$\frac{kl}{2\pi^2}\int_0^{+\infty}\left(x^2-1\right)I_0\left(\sqrt{x^2-1}ka\right)K_0\left(\sqrt{x^2-1}ka\right)\int_{-1}^{1}I(t)\cos\left(lx(\tau-t)\right)dtdx+$$

$$\frac{kl}{2\pi^2}\int_0^{+\infty}\left(x^2-1\right)K_0\left(\sqrt{x^2-1}kd\right)\int_{-1}^{1}I(t)\cos\left(lx(\tau-t)\right)dtdx=i\sqrt{\frac{\varepsilon}{\mu}}\frac{1}{k}E^0(l\tau) \quad (1)$$

where ε, μ –are absolute dielectric and magnetic permeabilities, k –is a wave number, $E^0(l\tau)$ – is a primary field, I_0 – is a modified zero-order Bessel function, K_0 – is a zero-order Macdonald function, t, x – are variables of integration, τ – is an observation variable, $I(t)$ – is an unknown current function. Let us note that the second term on the left-hand side of (1) is due to the presence of the second vibrator, $i=\sqrt{-1}$.

3. Allocation of the main positive definite operator

Using the asymptotics of the modified Bessel functions $I_0(xka)K_0(xka)\approx\dfrac{1}{2xka}$, at

$$xka\to+\infty, \tag{2}$$

the equation (1) can be reduced to the form with a distinguished main operator [4]

$$\beta(AI)(\tau)+(KI)(\tau)+(K_1I)(\tau)=i\sqrt{\frac{\varepsilon}{\mu}}\frac{1}{k}E^0(l\tau), \tag{3}$$

where

$$(AI)(\tau)=\frac{1}{\pi}\int_0^{+\infty}x\int_{-1}^{1}I(t)\cos(x(\tau-t))\,dtdx\,,\beta=\frac{1}{4\pi(kl)(ka)}, \tag{4}$$

$$(KI)(\tau)=\frac{kl}{2\pi^2}\int_0^{+\infty}\left((x^2-1)I_0\left(\sqrt{x^2-1}ka\right)K_0\left(\sqrt{x^2-1}ka\right)-\frac{x}{2ka}\right)\times$$

$$\times\int_{-1}^{1}I(t)\cos(lx(\tau-t))\,dtdx\,, \tag{5}$$

$$(K_1I)(\tau)=\frac{kl}{2\pi^2}\int_0^{+\infty}(x^2-1)K_0\left(\sqrt{x^2-1}kd\right)\int_{-1}^{1}I(t)\cos(lx(\tau-t))\,dtdx\,. \tag{6}$$

The method for isolating the principal operator based on asymptotics (2) is widely used in scientific literature and is one of the main mathematical techniques. The main operator A in this problem is the same as in the problem of analyzing a dipole antenna in free space. As follows from formula (2), the effectiveness of this approach depends on the radius of the vibrator a, and is not very effective for very small asymptotics a (2)

The operator K_1 is conditioned by the presence of a second vibrator. The vibrators are located at a distance d from each other. The emission point is on one vibrator, and the observation point is on the other. Therefore, the kernel of the integral operator K_1 is an infinitely differentiable function. And the operator K_1 is a completely continuous operator in the space of square-summable functions $L_2[-1,1]$ and does not affect the structure of the equation (3).

4. Numerical method for solving an integral equation

An operator A is a positive definite operator in space $L_2[-1,1]$. This allows us to introduce an energy space H in which the scalar product is determined by the expression

$$[u,v]=(Au,v). \tag{7}$$

It is important to note that the space H is limited to the original space $L_2\left[-1,1\right]$, or in other words, does not go beyond the original space. Using the properties of the operator A and the specific form of equation (3), one can prove that it has a unique solution and is equivalent to the Fredholm equation of the second kind in space H. We apply the following system of functions for the numerical solution of the integral equation:

$$\varphi_n\left(\tau\right)=\sqrt{\frac{2}{\pi n}}\sqrt{1-\tau^2}U_n\left(\tau\right), n=1,2,3,... \tag{8}$$

These functions represent the product of the weight function by the Chebyshev polynomials of the second kind. Due to the weighting function $\sqrt{1-\tau^2}$, each basis function vanishes when approaching the boundaries of the segment $\left[-1,1\right]$. Chebyshevpolynomials $U_n\left(\tau\right)$ are well studied and we present the first three:

$$U_1\left(\tau\right)=1,\ U_2\left(\tau\right)=2\tau,\ U_3\left(\tau\right)=4\tau^2-1.$$

An important property of the system of functions is its orthonormality

$$\left(A\varphi_n,\varphi_m\right)=\begin{cases}1, if \ \ m=n\\ 0, if \ \ m\neq n\end{cases} \tag{9}$$

Thus, the matrix elements of the operator A are found analytically, and, in addition, the matrix of the operator A in the given basis turns out to be unit. This condition plays an important role in the construction of a stable approximate solution method. The solution to equation (3) will be sought in the form

$$u\left(\tau\right)=\sum_{n=1}^{+\infty}c_n\varphi_n\left(\tau\right). \tag{10}$$

We substitute (10) into (3), then multiply by basis functions and integrate. As a result, we obtain a system of linear algebraic equations

$$\beta c_n+\sum_{m=1}^{+\infty}c_m K_{mn} = f_n, n=1,2,3,..., \tag{11}$$

where

$$K_{mn}=\left(K\varphi_n,\varphi_m\right)+\left(K_1\varphi_n,\varphi_m\right),$$

$$f_n=\left(i\sqrt{\frac{\varepsilon}{\mu}}\frac{1}{k}E^0\left(l\tau\right),\varphi_n\right).$$

Matrix elements K_{mn} are represented in the form of one-dimensional integrals, since the integration over variables t and τ is carried out analytically. It is possible to efficiently calculate the matrix elements K_{mn} for this reason, which is the most laborious part of the computational process.

In conclusion, we describe a method for solving system (11). The first N of unknowns are found from the solution of the truncated system

$$\beta c_n + \sum_{m=1}^{N} c_m K_{mn} = f_n,\ \ n = 1, 2, 3, \ldots N, \tag{12}$$

and the rest - analytically by the formula

$$c_n = \frac{f_n}{\beta},\ \ n = N+1, N+2, N+3, \ldots. \tag{13}$$

5. Results of numerical calculations

In all calculations, the results of which are presented below, the primary field was specified as

$$E^0(z) = U_0 f(z),$$

where $z = l\tau$,

$$f(z) = \frac{1}{2\Delta}\begin{cases} 0, |z| \geq \Delta, \\ 1, |z| < \Delta \end{cases}.$$

Voltage $U_0 = 1B$. The parameter $\frac{\Delta}{l}$ is defined through T. Input impedance $Z = R + iX = \frac{U_0}{I(0)}$, $I(0)$ –is the current in the middle of the vibrator.

Figure 1 shows the dependences of the active - R - and reactive - X - part of the input resistance of a half-wave vibrator with the ratio $\frac{l}{a} = 25$ and $T = 0.01$. The value d is indicated in wavelengths λ. Figure 2 and figure 3 show similar dependences for the parameters $\frac{l}{a} = 50$ and $\frac{l}{a} = 100$, accordingly, the parameter $T = 0.01$. The calculation was carried out using 40 basis functions $(N = 40)$.

Analysis of the calculated dependencies allows us to make an unambiguous conclusion about the mutual influence of vibrators, and in an oscillating form. The closer the antennas are to each other, the stronger the effect is. As the antennas move away, the input impedance values tend to those for free space. The decrease in mutual influence occurs rather slowly, the effect is clearly visible at a distance $d = 10\lambda$.

6. Conclusion

Thus, a theory has been developed that makes it possible to calculate the parameters of antennas taking into account their mutual influence. This is necessary for practical purposes when setting up and matching antennas with feed feeders. The results can also be useful in the field of electromagnetic compatibility.

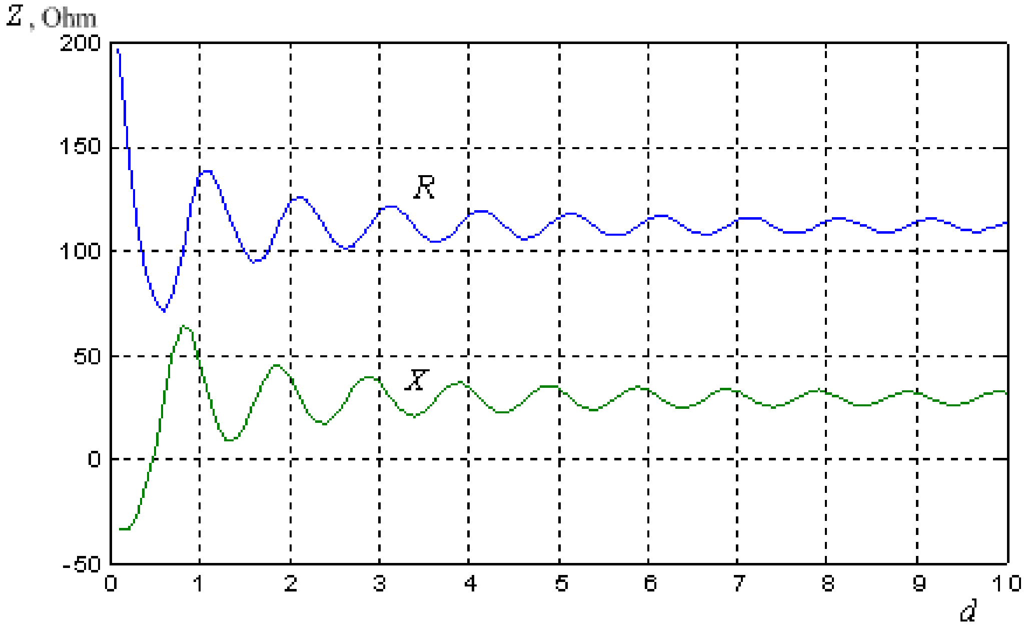

Figure 1. Dependence of the active - R and reactive - X part of the input resistance of a half-wave vibrator with the ratio $\frac{l}{a} = 25$ and $T = 0.01$. The value d is indicated in wavelengths λ.

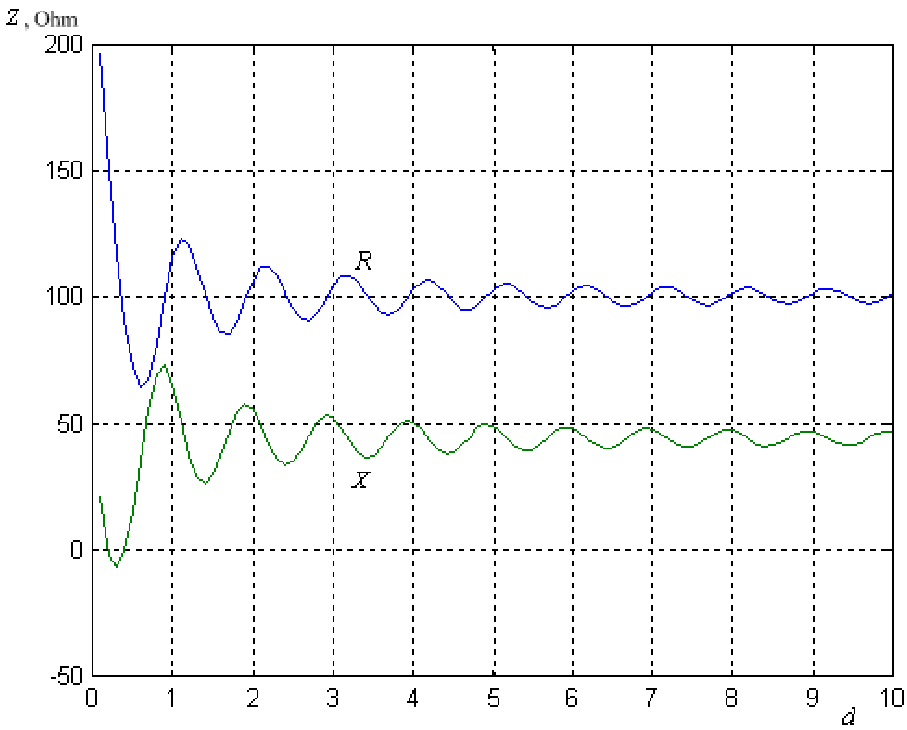

Figure 2. Dependence of the active - R and reactive - X part of the input resistance of the half-wave vibrator with the ratio $\frac{l}{a} = 50$ and $T = 0.01$. The value d is indicated in wavelengths λ.

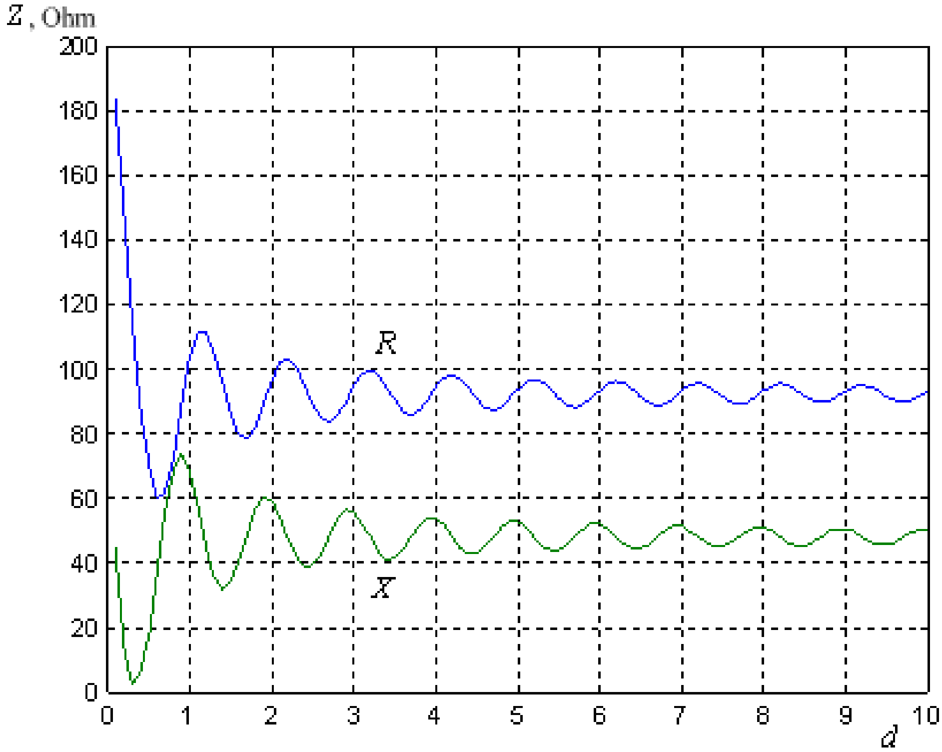

Figure 3. Dependence of the active - R and reactive - X part of the input resistance of the half-wave vibrator with the ratio $\frac{l}{a} = 100$ and $T = 0.01$. The value d is indicated in wavelengths λ.

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