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Students' understanding of the concept of function and mapping

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Abstract. Function is a concept that is very much related to daily life. Likewise with the concept of mapping. The purpose of this study is to describe students' understanding of the concept of function and mapping by students who are at a high level (ie extended-trans level). The student is the subject of this study. This research is exploring the ability of research subjects. The research subjects were taken from mathematics education students at Bengkulu University. We selected one subject of research, Mg. Student was interviewed in depth about the concept of function. Data were analyzed based on genetic decomposition. The results of this study are that students are able to present an overview of the concept of function with a broad extension level. The function from the set A to B is the set f whose members are sequential pairs of the two cross-sets of non-empty sets A and B such that (a, b) and (a, b') elements of f then $b = b'$. Also able to make a simplification in the form of mapping definition, which states that the domain of function f is the set A. The conclusion of this study is that high-level students have an in-depth understanding of the concept of function. He is able to compile extensive extensions. Also, make the exact simulations.

1. Introduction

The concept of function is very important in mathematics. Also, in school mathematics as well as in everyday life, though not always explicitly called functions [1]. At school, teachers often refer to it as mapping. Function and mapping are two different concepts, but are interrelated. Many students have misconceptions about these two things. There are students who claim that each function is a mapping and vice versa also applies. It is a concern for teachers to correct errors about the concept of function and mapping. In mathematics, terminology about functions uses words like functions, domains, range, linear maps, sets, coordinate systems and the like. Also, words that describe the role of mathematical objects, such as examples, definitions, properties, and formulas [2]. There are students who state that a function must include several algebraic formulas. Also, a function must have an explicit analytical representation [3]. In other studies [4] show that, a student believes that all functions are linear. When faced with a coordinate axis where several points are plotted and asked to draw as many functions as possible through the points, which form a line. Therefore, a student's conception influences how the concept is applied. Also, his conception is proof of his understanding of the concept.



There are three students' misconceptions in understanding the concept of function, namely students think that the relation of a mapping must be mapped back to the domain with a single value. Students misunderstand how restrictions on the domain of a function affect relations. The third misconception is that students rely on shallow or misleading indicators, such as the so-called vertical line tests, to determine whether a relation is a function [5]. It is useful to consider each source of students' misconceptions about function.

The results of the research Herawaty *et al.* [6], shows that students are able to simplify the concept of function through realistic problems based on "and dance" culture. He simplifies through the concept of mapping. For example, f is a function from set A to B , students can simplify the concept of function f with $Df = A$ (the domain function f is A). Students have been able to explain with a variety of real examples and are able to show the position of the concept of function in the deductive structure of mathematics. Through realistic mathematics learning and ethnomathematics approaches can be a vehicle for students to simplify the concept of functions to be more meaningful.

Many mathematics books, both used in schools and in tertiary education, present different definitions of functions. Different here can be in the sense of the intention of the definition and also can be in the sense of extension of the definition [1]. Bartle [7] defines a function as a set of sequential pairs. The function f from set A to set B is a set f whose members are (a, b) in $A \times B$ provided that if (a, b) and (a, b') elements of f then $b = b'$. In another article states that A function is a rule that assigns to each member of a set D , the domain, a unique member of a set R , the range [1, 8]. If you look at math textbooks at school, the function is simplified to mapping. The mapping from set A to set B is a special relation, that is, a relation where each member A is paired with exactly one member B . The numeric representation of the function gives a lower value than the other representation [9]. Therefore, teaching about numerical representation is less supportive of students developing a deep understanding of function.

The results showed that most students had a very limited understanding of the concept of function. Most of the others, students are not able to provide an exact definition, do not recognize whether the graph or rules given are a representation of a function. They cannot make correct connections between function graphs and value tables [10]. We realize that function is a very important mathematical concept. However, many students do not understand the concept. He did not understand the meaning of functions and comprehensive problems about didactic metaphors often encountered. Students often experience misconceptions about functions [11]. According to Clement [3], students often experience errors in understanding the concept of function. Like, a function must be given by a single rule. For example, a function with a divided domain is often thought of as two or more functions, depending on how the domain is split. The function graph must be continuous. For example, students generally do not consider the largest integer function graph as a representation of a function. Also, it was found students stated that the function must be one-to-one, ie the function has an additional property that for each element in the range, exactly one element exists in the domain. These are all functional concept errors that are often experienced by students.

Concept maps prove to be a useful tool for assessing conceptual understanding [12]. Other research results [9] shows that concept maps about understanding functions can be analyzed according to the main ideas related to the definition of functions, processes or objects of functions. It can produce a function representation based on the relationship between concepts. Concept maps can show different representations that can contribute to making different aspects of a function such that students can be better understood. Therefore, certain objects which show how to think

about functions are very important for a deep understanding of the concept of functions. Therefore, students' conceptions of the concept of functions are the feelings and ideas communicated about these concepts [4].

According to Victor *et al.* [13], that the theory of concept drawings shows that teaching mathematical concepts must include different approaches and representations to enable students to establish multiple and flexible connections between cognitive units. There are several main forms of representation for functions, numeric, algebra (formulas), and geometry (graphs), each of which has its own limitations. A numeric table can only have a limited number of entries that do not necessarily determine all functions. Formulas can be presented in a way that does not mention ranges or domains and physical graphs can only roughly present information needed for formal functions. Each is a description that emphasizes certain aspects of the concept, but also gives a shadow over the others.

Understanding the concept of function can be done through defining the intention and extension. The difference between the intention and extension of a definition turns out to give clues to the existence of a simplified function definition. The simplified function definition is in the form of a narrowing of the initial element's loading set of sequential pairs of sets. That is narrowing the extension [1]. But functions can have many representations, even if only one equation. Function representations can be algebraic or non-algebraic representations. Some functions, such as mapping each student to a place identified by letters along the wall, do not have algebraic similarities or representations at all. Students see algebraic representations more often than non-algebraic representations. That's so much that it can mess up the equation and function completely. In learning it is necessary to introduce non-algebraic functions [5]. That is to reach a concept that is stored in the memory of students carefully. Teachers need to understand it through their genetic decomposition. It also means a structured collection of mental activities that build blocks (categories) to describe how concepts / principles can be developed in the mind of an individual [14-16]. Genetic decomposition as a collection of mental activities carried out by someone about how mathematical concepts are connected in information processing systems. It can be analyzed through action activities, processes, objects, and schemes that someone does in solving a problem [17-19]. Thus, the teacher can have an understanding of the cognitive abilities of students in understanding the concept of function. The focus of this research is to understand the students' understanding of the concept of function and mapping.

2. Methods

This research is part of the research development of the theory of representation structure of students in understanding mathematics. That is the research master chaired by Widada [20]. The subjects of this study were mathematics education students at Bengkulu University. There is one extended trans level student [21-22] interviewed in depth about the concept of function and mapping. He was asked to explain the extension and intention of the concept of function and mapping. For example given two sets A and B, the subject is asked to define f a function from A to B. What is the domain of f and what about the other constraints. Interviewers are researchers using a semi-open interview guide sheet. That is to direct the interviewer to be more focused and in-depth in tracing the ability of students to understand concepts of function and mapping. This interview was interrupted through audio-visual, in order to capture all mental and physical activities carried out by the research subjects. Data in the form of descriptions and writing on paper based on the tasks given. That will complement the subject's genetic decomposition of understanding the

concept of function and mapping. The analysis was carried out descriptively qualitatively through appropriately classified interview footage.

3. Results and Discussion

According to Soedjadi [1], the definition of a concept in mathematics may be very different from the intention or extension with the definition of the same concept in mathematics in higher education. The definition of a concept in school mathematics can occur not equivalent to the definition of another concept in mathematics. That's because the extension is different. Also, it gives an opportunity for differences in the deductive structure of the related theorem and provides opportunities for different approaches to presentation in school mathematics. There are two different definitions of functions related to the extent of the extension. Extensions in one definition are a subset of extensions to other definitions. Therefore researchers traced to a high-level research subject. The subject is Mg, with the interviewer (Q). The following are excerpts from our interview.

Q : What is your explanation of the concept of function?

Mg : I state that the concept of function can be defined through relations, it can also be defined by a set of sequential pairs.

Q : What do you mean?

Mg : Yes ... for example A and B non-empty sets, then f is a function from A to B is a special relation that matches each member of set A with exactly one member of set B. This definition states that set A must be completely paired with members- member of set B. Also means that A is the domain of function f.

Mg : A broader definition of the extension is for example A and B two non-empty sets, for example f functions from A to B, then f is a set whose members are sequential pairs (a, b) members of $A \times B$, such that if (a, b) and (a, b') elements of f, then $b = b'$.

Mg : The final definition states that it does not require that every member A has a partner with member B. It also means that the domain f does not have to be the same as A, but the subset of A.

R : Ok ... alright.

Based on the excerpt from this interview, then the description of the second definition of function f by Mg can be represented in the text as shown in Figure 1. This illustrates the set f whose members are sequential pairs of multiplication crossings of two non-empty sets A and B such that: (a, b) and (a, b') elements of f, then $b = b'$. This is a very general definition, has a very broad extension. As Mg revealed that ... the final definition states that it does not require that every member of A have a partner with member B. It also means that the domain f does not have to be the same as A, but the subset of A.

This sample can be concluded that, Mg already has a high level of understanding of the concept of function. He is able to understand the definition that is simplified from a broader concept. This gives an illustration that extended-trans level students already have the maturity of the scheme stored in their memory.

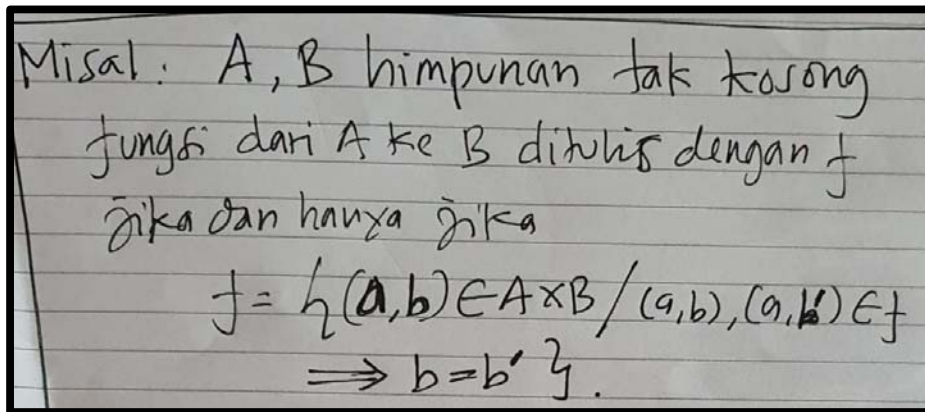


Figure 1. Definition of Functions from A to B by Mg

(See: <https://www.youtube.com/watch?v=UFWA0wBH0OE>)

Based on Figure 1, that set A is a superset of the domain f, and B is a superset of the range f. This is a picture of Mg's complete understanding of the concept of function. Consider the following interview excerpt.

- Q : How can you understand the concept of functions like Figure 1.*
Mg : well ... I understand that for each a member of the set A so that a is the first element of member f then a is a member of the domain f (... ie Df)
Mg : the same thing happens that for each b member of the set B such that b is the second element of member f, then b is a member of range f (written Rf).
Q : Ok ... What next?
Mg : That means that Df is a subset of A and Rf is a subset of B.
Mg : Df members do not have to be completely paired with members of group B ... The representation can be seen in Figure 2.

Interview footage and Figure 2 provide information that Mg is able to understand the concept of function completely. He said that the domain of the function f (Df) is a subset of A and the range of the function f (Rf) is a subset of B.

Mg has a good mental abstraction, because the concept is a mental abstraction from the general characteristics of a series of experiences. The elements of a collection can involve objects as collections of concepts, actions as operational of a concept. Based on interview excerpts, Mg formulated the definition explicitly. The background is non-empty sets A and B. The term defined is function f, with the proximal genus sequential pairs (a, b) members of $A \times B$. The special differentiator in the definition made by Mg is if (a, b) and (a, b') elements of f then $b = b'$. Verbal definitions understood by Mg have a very broad extension. This description is supported by a representation of the Mg shown in Figure 2.

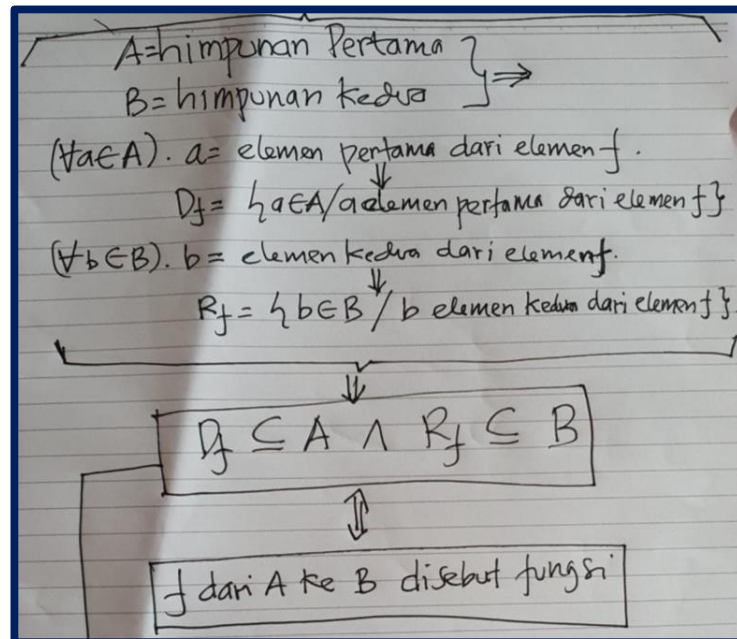


Figure 2. Explanation of Function Definition from A to B by Mg

Based on Figure 2, Mg understands the concept of functions that are different from concepts that are commonly understood in schools. It is as an abstract idea to classify pairs of sequences which fulfill the limits of function and which do not fulfill. Mg's conception of the concept of function can influence his thought patterns in learning the concept further in the mathematical deductive structure and also its application. The rules are understood by Mg as one of the mathematical objects that apply in the concept of functions. Mg understand well the mathematical notation for a function, Mg try to understand it based on past experience with mathematics [4].

Simplification of the concept of function is carried out by Mg through a special differentiator in the definition. Simplification that influences the intention or extension of a definition, it can be assumed that it will have a certain deductive structure or structure of school mathematics material. [1]. Consider the following excerpt from our interview with Mg.

Q : What is your explanation of the simplification of the concept of function?

Mg : I mentioned that the concept of function can be narrowed down in extension ...

Q : What do you mean?

Mg : Yes ... the limit that the D_f subset of A is simplified to D_f is equal to A ... this has the effect that the members of set A have been paired with the members of set B.

Q : Ok ...

Mg : ... through these limitations, then f is managed by mapping ... in my opinion is a simplicity of the concept of function.

Mg : ... thus, if D_f is equal to A then f from A to B is a mapping and written $f: A \rightarrow B$. You can see the representation in Figure 3.

Based on interview excerpts and Figure 3, it can be interpreted that Mg understands the concept of mapping as a simplification of the concept of function. That will happen if D_f is equal to A. Mg states that if D_f is equal to A then f from A to B is a mapping and written $f: A \rightarrow B$.

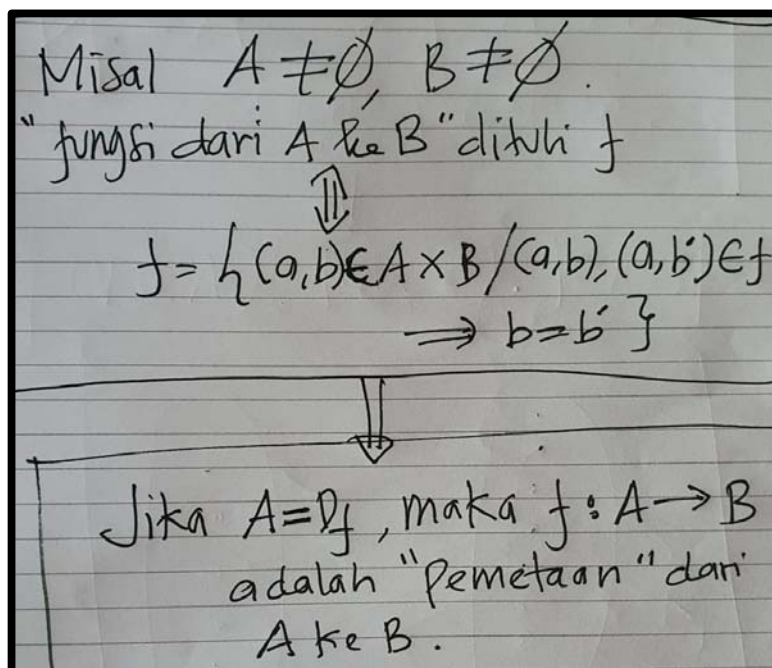


Figure 3. Explanation of the Definition of Mapping from A to B by Mg

Figure 3 is a representation of a simulation of the concept of function. This extension is part of the previous definition. According to Soedjadi [1], that the simplification of a concept gives the meaning that the concept is released as related to another concept; concept extension is a subset of the concept extension presented with the same term (in this case the term function); and one concept only prioritizes certain characteristics of the concepts presented with the same terms. Note that Mg understands the characteristics of $D_f \subseteq A$ on one side and $D_f = A$ on the other side. The limitation of $D_f = A$ is to understand the function term which is simplified into a new term that is mapping. This will affect the schemes stored in the memory of students for further learning. The student's conception of the concept of function can influence his future efforts to learn more about or apply the concept [4]. As a result of such a simplification it can be expected that there will be an impact on the deductive structure of the related theorem as well as on the presentation structure of the related school mathematics [1]. According to Evangelidou *et al.* [11], there are three strong trends in students' ideas about the concept of function. First, identify functions as more specific concepts of one-to-one functions. It is an understanding of uniqueness that leads to identification of functions as one-to-one functions. Second, it functions as an analytic relationship between two variables. Finally, it functions as a diagram, either a Cartesian graph or a mapping diagram. Students make understanding as an algebraic expression. Thus, it can completely be understood about Mg's ability to understand the concept of function and its simplification. He interpreted the concept of function as an extension of the concept of mapping.

4. Conclusion

The concept of function becomes one of the concepts in mathematics which has a very broad influence in understanding mathematics further. The extension and intention of these concepts have an impact on students' cognitive development, and the deductive structure of mathematics. A concept of function taught to students does not always, or has not, reached all the traits or aspects possessed by more general concepts. That is meant as simplification. This study concludes that

students are able to understand the concept of functions with extension levels that have a wide range, ie the background is non-empty sets A and B . The terms defined are functions f , with the proximal genus sequential pairs (a, b) members of $A \times B$. The special differentiator in the definition made by Mg is if (a, b) and (a, b') elements of f then $b = b'$. He understood the characteristics of $Df \subseteq A$ for the concept of function and $Df = A$ for mapping. Mg is a subject with extended trans-level thinking. The subject understands that $Df = A$ is for the function term which is simplified into a new term called mapping.

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