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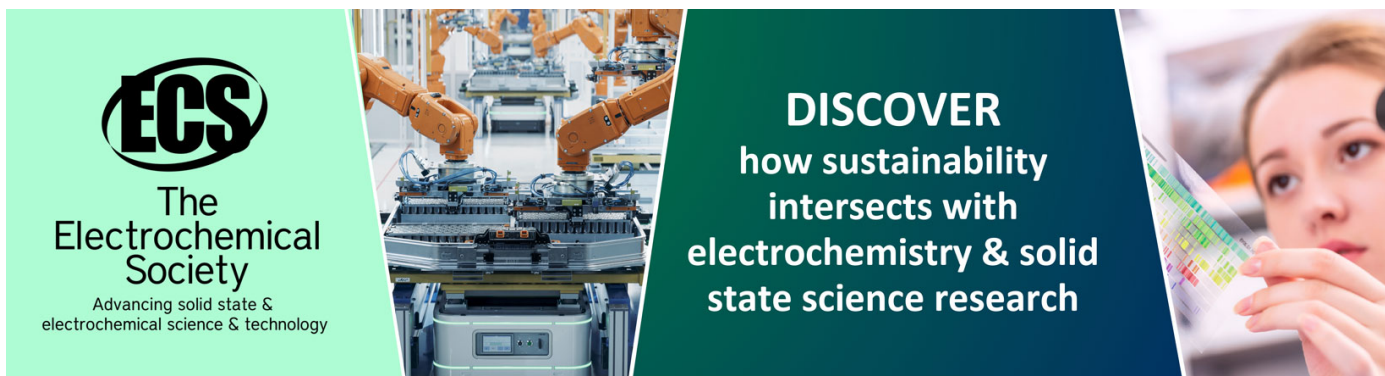
Magic covering and edge magic labelling and its application

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Magic covering and edge magic labelling and its application

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Abstract. Graph theory is developing very rapidly, especially in labelling graphs. This is proven by the number of researches examining graph labelling. In graphs labelling, research focuses on developing theories. On the other hand, graph labelling (magic covering and edge super magic labelling) has a useful application. This paper will discuss the magic covering of a simple graph (Domino graph) and edge magic labelling on a simple graph (Domino graph). The conclusion is that the magic covering can be applied to a secret sharing scheme, and also edge magic labelling can be applied to ruler models.

1. Introduction

Graph theory was first introduced by Leonhard Euler in 1735 to solve the problem of the Konigsberg bridge on the river Pregel, Russia. The proposed solution to the problem is in the form of points and edges which are then known as Eulerian graphs, a graph with connected characteristics and each point having an even degree. Graph labeling is an interesting topic in graph theory so that various types of labeling are researched and developed [1].

Graph labeling can be applied to various fields including transportation systems, communication systems, geographical navigation, radar, and also security systems [2,3]. For example, the design of the code for radar signals and missiles is equivalent to labeling a complete graph, where each point is connected to one side that has a label that is always different. This side label describes the distance between points, while the point label is the position at the time the signal is sent.

Graph labeling is a value (with integers) at the point or side of the graph or both so that it meets certain conditions. The labels used are positive integers or natural numbers. Graph labeling was first introduced in the late 1960s by Rosa. Furthermore, labeling whose domain is a set of points, a set of sides, or both is usually referred to as point labeling, side labeling, and total labeling. Until now there are several types of graph labeling discussed, including graceful labeling, harmony labeling, irregular total labeling, magic labeling, radiolabeling, anti-magic labeling, and many other graph labelings [4].

One type of graph labeling that is widely studied and developed is magic labeling. The idea of labeling was first introduced by Sedlacek, then formulated by Kotzig and Rosa [5]. Wallis [6] defines magic labeling as the one-to-one mapping of graph elements, that is, a set of points and sets of sides to a set of positive integers, where there is a magic sum. Then Stewart explains that a magic labeling is called super magic labeling if the smallest label is placed at the dot [4].

In the development of magic labeling, it is also known that the labeling of the total magic point, the total labeling of the super magic point, total labeling of the magic side, and total labeling of the super magic side. Total labeling of the magic side is a magic labeling that has been studied extensively, in more than 50 classes of graphs it has been proven to have total labeling of the magic side. From the development of total magic side labeling, it has been found a magic blanket labeling as follows.



Let $G = (V, E)$ be a finite simple graph. Gutierrez and Llado defined edge-covering of G as a family of different subgraph H_1, \dots, H_k such that any edge of E belongs to at least one subgraphs $H_i, 1 \leq i \leq k$. Then, it is said that G admits an $(H_1, \dots, H_k) - (\text{edge})$ covering. If every H_i is isomorphic to a given graph H , then we say that G admits an H -covering [7]. This paper will discuss the magic covering of a simple graph and edge magic labelling on a simple graph. The purpose of this research is a secret sharing scheme with magic covering on domino graphs.

2. Method

The research method used is a literature review that is by collecting references in the form of books about graph theory, journals, and writings that are published on the website. In the first step, we will look for magic covering from a simple graph. In the second step, we will look for edge magic labelling in a simple graph. The third step, from the results the first and second steps will be applied to a case.

3. Result and Discussion

3.1. Magic Covering

Let T be the sum of labels of all vertices, the sum of labels of all edges is s , and the magic sum of star magic covering is $m(f)$. In Figure 1, the vertices are labelled by a, b, \dots, f and the edges are labeled by g, h, \dots, m . The labels of vertices and edges are a positive integer between 1 to $|V(G) + E(G)|$. In this case, we have a domino graph.

The basic idea is to determine the label of each vertex, and then the labels on each edge are interchanged to find cycle magic covering. The label of the edges is obtained from the remainder of the vertices labels. To get cycle magic covering, first, we search all possible T to label the vertices. There is no guarantee every combination of T has cycle magic covering.

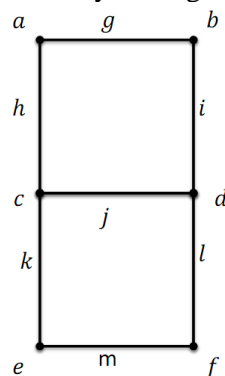


Figure 1. Domino graph

G is a domino graph if every vertex of G is contained in at most two maximal cliques of G [8]. Here is an illustration to find T , s , and $m(f)$. There are 2 C_4 -cycle magic covering on a Domino graph in Figure 2 The constant $m(f)$ can be obtained by adding all labels on star magic covering as follows.

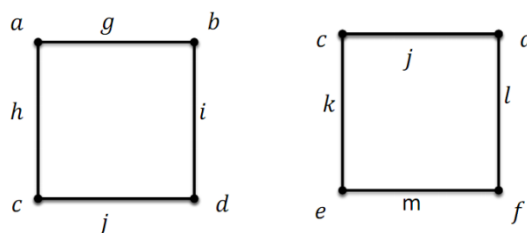


Figure 2. C_4 -cycle magic covering

$$2m(f) = (a + b + \dots + m) + (c + d + j)$$

$$m(f) = \frac{a + b + \dots + m}{2} + \frac{T}{2}$$

$$m(f) = \frac{91}{2} + \frac{T}{2}$$

From Equation, T is a vertex label sum mod 2. To obtain a $m(f)$, first, we combined all of possible T . However, not all combination labels on T has cycle magic covering. To find all possible labelling, we also use a duality [6]. Given cycle magic covering is λ , its dual labeling λ' is defined by $\lambda' = v + e + 1 - \lambda(x)$ for every vertex x and $\lambda'(x, y) = v + e + 1 - \lambda(x, y)$ for every edge $\{x, y\}$.

Proposition 3.1. If the sum of labels of vertices T is odd, then there is no cycle magic covering on domino graphs.

3.2. Illustration Secret Sharing

For each graph, we number all vertices and all edge, then we call these number positions. Thus, a graph labeling can be represented as a set of ordered pairs of positions and its label. According to Baskoro [9], critical set of a graph G with a labeling λ is a set $Q_\lambda(G) = \{(a, b) | a, b \in \{1, 2, \dots, |V(G) \cup E(G)|\}\}$ with ordered pair (a, b) represents label b in position a , which satisfy:

1. λ is the only labeling of G which has label b in position a .
2. No proper subset of $Q_\lambda(G)$ satisfies (1).

If a critical set has c members, thus it has size c .

Once a critical set of graphs is found, it will be applied to the secret sharing scheme [9]. Independently, a secret sharing scheme was introduced by Shamir in 1979 and Blakley in 1979. Many scientists has been studied this scheme by until today. Some important application to several areas of information security has a secret sharing scheme now. From October 2010, In Japan, NRI (Nomura Research Institute) Secure Technologies which is one of the private sectors in the area of information security. It has provided clients with some cloud computing products named Secure Cube. This cloud is utilized by a secret sharing scheme that computing product. It is one good example of the application to an external storage unit [10].

The secret sharing scheme is a method to distribute shares of a secret value K among a set of participants P such a way that only the qualified subsets of P can reconstruct the value of K from their shares. Since the security of a system depends on the amount of information that must be kept secret, the size of the shares given to the participants is a key point in the design of secret sharing schemes [11]. Hence, the information rate is an important criterion for a measure of a secret sharing scheme.

In this case, we search a C_4 -cycle magic covering on domino graph with $T = 21$, $m(f) = 56$, and $s = 70$, and construct its dual labelling Figure 3. Here are the steps to get a secret sharing.

1. First, we determine the label of each vertex of the combination of label T , for example, the set of the label of the vertices obtained is $\{1, 2, 3, 4, 5, 6\}$.
2. Second, we label each edge by labels not used by T , for example, the label of an edge 7, 8, 9, 10, 11, 12 and 13.
3. Third, we interchange each label of edge, until we get cycle magic covering. In the process of interchange the labels of edges, we can obtain more than one labeling.
4. Next, we will search for the dual-labeling.
5. Once a critical set is found, it will be used to devise a secret sharing scheme [9].

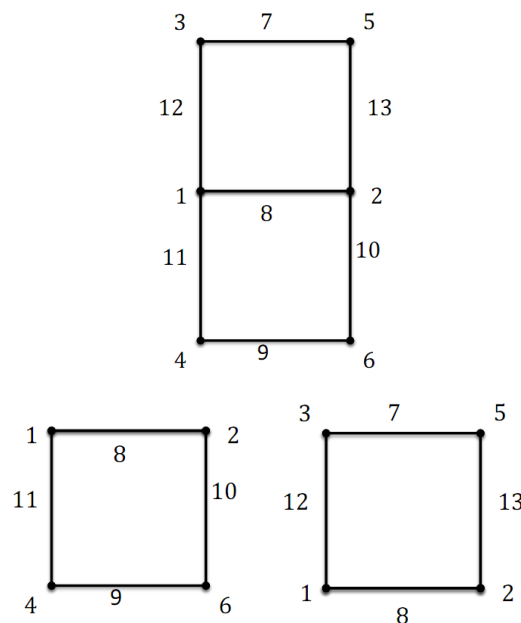


Figure 3. C_4 -cycle magic covering on domino graph with $T = 21$, $m(f) = 56$, and $s = 70$

A method of distributing secret data among a set of participants are a secret sharing scheme. So that only specified qualified subsets of participants can recover the secret [12]. Also, if the unqualified subsets collectively yield no extra information, i.e., the scheme called perfect if the joint shares are statistically independent of the secret. The access structure is the subset if qualified subsets among all possible subsets of participants. In this paper, we propose a secret sharing scheme with magic covering on domino graphs.

3.3. Ruler Models and Application of Edge Magic Total Labelling

In 1992, Bača [13] have found on magic labelling of grid graphs. Domino graph is a special case of a grid graph. The following papers only discuss development theory. This sub-section will discuss the application.

Most of the notations and terminologies in this paper follow that of Wallis [6]. Suppose λ is an edge-magic injection of Domino graph. If λ is any labeling of Domino graph, a ruler model of λ is constructed as follows. For each vertex of Domino graph, place a mark distance $\lambda(x)$ from the start of the ruler. The ruler can be used to measure all distances corresponding to the distance between two marks.

The ruler models derived from edge-magic injection have the following special property. No two of the measurable difference are equal, except possibly for a difference of the form $\lambda(x) - \lambda(y)$ and $\lambda(y) - \lambda(z)$. This is very nearly the semi-graceful property, so magic sublabeling can be used in many of the situations where semi-graceful labelings have been applied [4].

One important application of semi-graceful labelling is to radar pulse codes. And the most efficient way to proceed is to time the pulse in accordance with the marks on a ruler derived from semi-graceful labelling. If the ruler from a magic sub labelling is used, there may be two detectors excited at once (if $\lambda(x) - \lambda(y) = \lambda(y) - \lambda(z)$ occurs) but it is still very unlikely that the reflected pulse will be misidentified even if part of the signal is lost through dispersion. In this application, the duration of the train will be minimized if the size of the largest label is minimized, so again sub labeling with small excess is better [6].

4. Conclusion

The magic covering can be applied to secret sharing schemes. Also, Edge magic labelling can be applied to ruler models.

5. Acknowledgments

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