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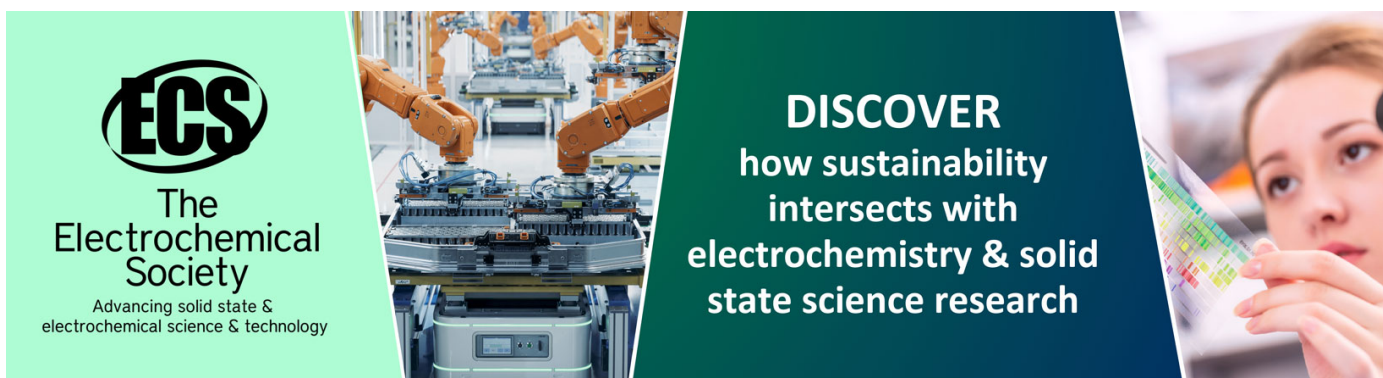
Students depend on the Pythagorean theorem: Analysis by the three parallel design of abstraction thinking problem

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Students depend on the Pythagorean theorem: Analysis by the three parallel design of abstraction thinking problem

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Abstract. Based on students' geometry knowledge, there are contrast and less relevant skills goals being prepared for the students. Students are more dominant in learning how to apply procedural knowledge so there is a need for students to use the Pythagorean theorem when facing a triangle properties problem. Therefore, this case study aims to analyze how students' conceptual knowledge depends on the Pythagorean theorem. The analysis uses a cognitive diagnostic assessment framework through the three parallel design of abstraction problem. This study was conducted for students at the senior high school. The findings are the Pythagorean theorem as a result of thinking abstraction at least two of the three design problem formations, including for the effect of claims and metacognitive knowledge them. There is a disconnected conceptual system between the products of thought and the claims elicited so that abstraction is not optimal. Development for in-depth understanding of conceptual experience is needed in the instructional intervention so that more adequate reasoning.

1. Introduction

Learning is the process of preparing self-skills for the use of knowledge acquired, both for problem solving and scientific development [1]. Students have spent a lot of time studying geometry in taking formal education. For example, geometry began to be studied in Indonesia from elementary to advanced levels [2,3]. However, recent conditions in the geometry class are more prevalent in guiding students to apply procedural knowledge [4,5], with the hope that students will also simultaneously gain conceptual knowledge. While learning geometry is very dependent on conceptual knowledge [2,6]. Notwithstanding how the implementation learning geometry, we see that there are contrast and less relevant goal between the skills that are prepared for students and the knowledge they should have.

According to the results of the study by Minarti et al. [3], there is sometimes a difference between the student's conceptions and the knowledge underlying the concepts. Several factors affect students' ability to solve geometry problems, such as abstraction [7,8], conceptual knowledge as the basis of understanding [1,5,9], and the analytical capabilities of geometry concepts relevant to the problem [3]. The result of the study by Alghadari & Herman [4] found that there is a problem with epistemological concepts and hierarchy of concepts on students' geometry knowledge. As a result, students incorrectly interpret essential concepts for the connections between known information, applied concepts, and the problem is being solved. Some of the geometry knowledge used is out of sync and not complementary to the corrected completion [2]. The rigidity of the familiar mathematical procedure emerged as a cause and led students towards these findings. The mathematical procedure means the Pythagorean theorem. The results of the study show that procedural knowledge controls the conceptual knowledge of students



and not vice versa. Even though, procedural knowledge itself must be organized based on conceptual knowledge [5,10,11].

In the benefits of helping to solve geometry problems, the Pythagorean theorem is an essential geometrical procedure on the relationship between the sides of a right triangle. Usually applied after the relationship meets the conditions of the concept. Procedural knowledge understood by students need not be doubted. However, if there is a conceptual process of knowledge then it is not enough to guarantee that they will be able to solve geometry problems [4,9,11]. Conceptual knowledge is made up of separate pieces of information, how about the functions and operations information, and when to use it [5,12]. Conceptual knowledge will reveal the relationship between information [5,11], specifically on geometry through thinking ability at abstraction level because of the product of thought is the relationship between properties of geometry element [13]. According to research findings by Ramdhani & Suryadi [14], where students have not been able to identify relevant conceptual knowledge between the Pythagorean theorem and other geometrical calculation concepts, then there are difficulties that students will experience in solving problems.

We do not question the relevance between learning content, educational levels, and students' thinking level when everything corresponds to the level that they should. Learning geometry is more than just applying the Pythagorean theorem but also there is an abstracting process before applying. Abstraction is a suggested thinking level for geometry learning content in high school students [9]. However, there is a need for students to use the Pythagorean theorem of course when they encounter geometry problems involving numerical computation or relationships between geometrical properties in triangular shapes [4], because learning geometry during their school time is more dominant in applying the routine procedure [3]. Student conceptual knowledge is a guarantee of the abstraction and resolution of problems [5]. The aim of this study is therefore to analyze how students' conceptual knowledge depends on the Pythagorean theorem in order to solve the geometry problem in abstraction level. This research is important as a standard description of students' conceptual knowledge of geometry, which is useful as a starting point for exploring and improving their skills and abilities.

2. Method

This research is a case study in the process of solving geometry problems. There is a geometry problem, the level of geometry thinking, the students who solve it, and cases in the relationship between conceptual and procedural knowledge. The problem specifications and the level of geometry thinking are explained in the instrument. Students who solve problems are explained in the student participant. Whereas the study of students' conceptual knowledge and problem-solving cases is classified in the data analysis section. Data analyzed were collected by tests and students' interpretation as verification.

2.1. Instrument

In this study, a test item with an abstraction thinking level specification is prepared. The geometry problem is designed to match the topic in the school curriculum, namely solid geometry. The solid one we chose was the pyramid. We create geometry problems in the pyramid, which is the distance between the points and the lines. The problem is that there is a three-one formation. Students will be able to determine the size of the distance between points and lines after three parallel problems are solved first. Three parallel problems are referred to for three geometry problems that can be solved without being affected by solving other problems.

Before students apply the geometrical procedure, they will be involved in abstraction thinking to find the relationship between properties. Relationship means the relevance of properties of shape and computation rules. The specifications we designed in the geometry problem presented to students are as follows.

In Figure 1, given that a pyramid $T.ABCD$. $ABCD$ is a square where AB is 4 cm. The slanted edge is $2\sqrt{6}$ cm. Q is the intersection between AC and BD . TP is half of TD . R on TC so that QR is perpendicular to TC . Find the distance between P and QR .

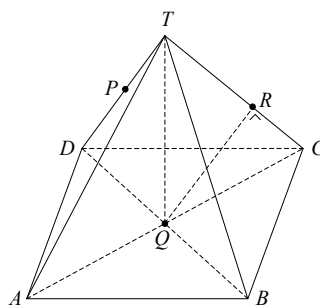


Figure 1. A pyramid $T.ABCD$

2.2. Students Participant

Students who solve the problem are from 12th grade senior high schools at Tanjungpandan in the 2019/2020 academic year. A total of 58 students were involved in this study. Students try to solve problems using paper and pencil after they learn the concept of distances between elements of geometry in math class. Capable or not they solve it and involving the Pythagorean theorem, its procedural has been one important indicator for sampling. Then, in the problem solving process, some different students have similarities in problem analysis techniques. Therefore, this study only presents data based on different conceptual analysis after we have identified the problem solving process. Eventually, we only found three male students who represent the applied sampling standard and are initialed with AL, AD, and GD. These three students interpret the problem solving and we direct them to describe the process in detail.

2.3. Data Analysis

We identify the problem-solving process of students and confront their interpretation. Conceptual knowledge of students is the way in which the results of their identification, illustration or presentation of symbols are shown [5]. Knowledge submitted after students' abstraction is an attribute in problem solving. There are attributes of content and process. Content attributes are basic concepts and how to operate, while the process attribute is cognitive activity towards knowledge [15]. When there are parts that are not explained in detail by students so it is a possible sign that there is a case for their knowledge. Knowledge, skills and processes are the elements analyzed [15–17]. The relevance of the three elements was investigated on the basis of the diagnostic model suggested by Hwang et al. [18]. That is the concept of eliciting and integrating phase, the relationship-eliciting phase, and the relationship-integrating phase. The cognitive diagnostic assessment provides an alternative for the analysis of skills mastery, processing skills, cognitive knowledge status, and cognitive structure of students [17]. Besides, five frameworks are substantive theoretical construction; design selection; test administration; response testing; and design revision. In the instrument, three initial frameworks have been explained. Response testing is included in the results and part of the revision design will be detailed on the implications for learning.

3. Result and Discussion

Based on the results of data analysis on the knowledge and skills element, there are similarities in students' points of view in the identification of the problem. But there is a difference in the process of analyzing the problem, geometry thinking and abstracting, and the impact of abstracting it when synthesizing problem solving. The other finding is that there is a problem with the product of thought after students have abstracted relevant to reasoning.

3.1. Geometry Knowledge in Problem Solving

Analysis of student knowledge in problem solving is grouped by the size of the geometry elements needed to find the distance between P and QR on the pyramid. Some of the required sizes are distance: Q to R , P to Q , and P to R . The focus on this analysis is on the eliciting concept. Radmehr & Drake [19] states that there are four types of knowledge dimensions, namely factual, conceptual, procedural, and metacognitive. The focus of this study is on conceptual knowledge because students' recognition of their

known and understood concepts will appear on how are the three other types of elicited knowledge. The results of data analysis found that students see geometry problems are the problem of elements and properties triangle. Knowledge of the geometry of the triangle concept is dominant in the solution. Here are the corresponding research data on how students parse a geometry problem based on a triangle as the class of shape.

Table 1. Analysis into problem solving based on geometry knowledge

Students' Initial Name	Geometry Problem			
	$Q \text{ ke } R$	$P \text{ ke } Q$	$P \text{ ke } R$	$P \text{ ke } QR$
AL	ΔTQC	$PQ=QR$	-	$\Delta POQ, PO \perp QR$
AD	ΔTQC	ΔTOD	ΔRPT	ΔPQR
GD	ΔTQC	ΔTPQ	-	ΔPQR

Based on Table 1, some triangle has been engaged by students in the relevant property analysis with geometry problems. There seems to be the same triangle they used as a first step and a goal to solve problems. This is the concept of the right triangle TQC for property QR and triangle PQR for the distance between P and QR . Giannakopoulos [6] states that the problem solving method is knowing the beginning and end of the work. There are similarities in the problem solving methods of students due to the analysis approach everything by the triangle concept. Other concepts may be used to solve geometry problems in this study design and have been explained as a vector approach in Alghadari & Herman [4]. However, because the problem is informed in parallel then There are also different concepts involved based on each point of view. Conceptual knowledge of each student chooses different classes of shapes to analyze PQ and PR properties. According to this fact, Hwang et al. [18] revealed that the difference in expertise and knowledge used in solving problems was due to the cases they experienced and the knowledge they built. Furthermore, strategy in problem solving shows how individuals' extent of knowledge of their cognitive strengths and weaknesses refers to metacognitive knowledge [19,20].

The first case of this results study is the tendency of students to see the problem of distance between points and lines as a problem involving the triangle concepts. In this case, if there is a geometry problem and it is solved by the triangle approach, then some geometrical concepts are possible involved, like the Pythagorean theorem, trigonometry, congruence, and area [2,4,12]. Here, students' understanding and conceptual knowledge of the triangle is the foundation of their performance [7] although many students fail to develop a deep understanding of basic geometric concepts [21]. Through the triangle TQC , students abstract the relationship between triangle properties. Conceptualization as such requires synthesis through association, abstraction and differentiation between properties before analysis [1]. After students have abstracted, two concepts are serially integrated, the first is the Pythagorean theorem and the second is the triangle area to find QR . The tendency of students to involve the Pythagorean theorem also be the point of view of the analysis of this study, when the procedure is applied, and what the purpose is. Here, the focus of our analysis is in the integration concept phase. The following is the analysis result for the triangle element which is the product of thought and apply the Pythagorean theorem.

Based on Table 2, analysis element of triangle TQC and abstraction the triangle properties relationship produce the Pythagorean theorem as a product geometry thinking to find TQ . Size of TQ is needed to abstract the relationship of triangle properties between TQC and QR . The product of abstraction thinking the properties of triangle TQC by students is similar because the object being analyzed is also the same so that the purpose of the analysis produces the same findings. However, when the object of analysis is different, then the abstraction product is also different although the analyzed size of the property is the same [12]. The example is on the size of PQ and PR of each of the triangle properties analyzed. The object analyzed may be different, but the size of the property found must be the same. However, the second case source in this study findings is a difference in the size of the property found by students. The product of abstraction thinking is not the same, there is the Pythagorean theorem product by GD, and the congruence by AL. One measure of property that students find may be correct or maybe it's all wrong.

Table 2. Application Pythagorean theorem for triangle property

Students' Initial Name	Geometry Problem				Addition Analysis
	$Q \text{ ke } R$	$P \text{ ke } Q$	$P \text{ ke } R$	$P \text{ ke } QR$	$Q \text{ ke } PR$
AL	TQ	-	-	PO	-
AD	TQ	-	PR	-	$QO, QO \perp PR$
GD	TQ	PQ	-	-	-

Based on Table 2, analysis element of triangle TQC and abstraction the triangle properties relationship produce the Pythagorean theorem as a product geometry thinking to find TQ . Size of TQ is needed to abstract the relationship of triangle properties between TQC and QR . The product of abstraction thinking the properties of triangle TQC by students is similar because the object being analyzed is also the same so that the purpose of the analysis produces the same findings. However, when the object of analysis is different, then the abstraction product is also different although the analyzed size of the property is the same. The example is on the size of PQ and PR of each of the triangle properties analyzed. The object analyzed may be different, but the size of the property found must be the same. However, the second case source in this study findings is a difference in the size of the property found by students. The product of abstraction thinking is not the same, there is the Pythagorean theorem product by GD, and the congruence by AL. One measure of property that students find may be correct or maybe it's all wrong.

After further analysis, both in the case of the Pythagorean theorem and congruency as a product of thought for PQ or PR size, there are properties of shape which are not involved in abstracting the relationship between properties. For example, the type and size of triangles and given other geometry elements. Even though, as part of the analysis, its properties will contrast the sizes of triangles for the classification type, and other geometry elements will also include in the abstraction process, but the Pythagorean theorem or congruence is not relevant as a product of thought. On the other hand, in Table 2, there is an additional analysis on the triangle property being the geometry element required in the process of problem solving. This property analysis is by AD when he calculates the distance between Q and PR . The abstraction thinking product from its analysis is the Pythagorean theorem. Here, the theorem for triangles as a procedure and relatively involved. AD noted the reason that the calculation involves is to find the distance between P and QR based on his conceptual knowledge by the principle of triangle similarity area. According to the case analysis, the Pythagorean theorem is dominant in abstracting triangle property as a product of thought, both as a single product and as part of a series. Qualitatively, we found the application of the Pythagorean theorem as a result of abstraction thinking in students' process of problem solving, at least two of the three design formations of the geometry problem.

3.2. Conceptual Knowledge for Abstracting Properties

Referring to the tendency of students for certain procedural knowledge, van de Walle et al. [13] states that analysis for their conceptual knowledge is the reason underlying the product abstraction. Alex & Mammen [7] states that concepts are elements of understanding and knowledge. They understand the concepts so the connotation and denotation are the basis on which procedural knowledge is integrated into problem solving [13,18]. This analysis focuses on the relationship-eliciting and the relationship-integrating phase. The analysis results of the concepts in the abstraction phase are as follows.

Based on Table 3, some concepts elicited as a basis for analyzing properties. For example, the triangle classification like an equilateral and isosceles. The classification and category are a subtype of conceptual knowledge [19]. The concepts are integrated into abstracting with others. The abstraction product of integration between triangle classification and the property, such as the triangles TQC and TQD , is congruent. Radmehr & Drake [19] states that conceptual knowledge refers to the knowledge of the interrelationships among the basic elements within a larger structure that enable them to function together. Involved concepts in problem solving should have been interrelationships [1,12]. Therefore, the validity of the problem solving process by students can be analyzed based on relevance between geometry problems, Table 1, Table 2, and Table 3. For example, AL finds QR involving triangle TQC (in Table 1), he found out that QC and the Pythagorean theorem were applied to TQ (in Table 2). The

analysis results of PQ find that it is the same as QR (in Table 3), but on the given information that PQ is not perpendicular to TD , or R is not in the middle TC because TQ is not the same as QC . There is an integration of relationship which is not reasonable for problem solving. This fact is no different from the study by Alghadari & Herman [4] that an issue of problem solving geometry concern the relevance between given information, the concept used, and the problem addressed. Even though the subjects and places of research are different. In addition, according to the study results by Hutapea, Suryadi & Nurlaelah [22] that there are epistemological obstacles because students' conceptual understanding is not sufficient when problem solving is relevant to the Pythagorean theorem.

Table 3. Concepts in Abstraction Thinking

Students' Initial Name	Geometry Problem			
	Q ke R	P ke Q	P ke R	P ke QR
AL	Right triangle, size of property and the area	Claim $PQ=QR$	-	Claim PR , ΔPQR is the equilateral triangle
AD	Right triangle, size of property and the area	Right triangle, ΔTQC congruent to ΔTQD , $PQ=QR$	Claim the size of RT and PT , ΔRPT is the right triangle, $PT \perp PR$	ΔPQR is the isosceles triangle
GD	Right triangle, size of property and the area	Claim ΔTPQ is the right triangle, error on $TD=4$.	-	-

According to Table 3, students claim some properties of the triangle, and its claims are included in the abstraction process. Students' interpretations of the claim are disconnected, so it is a source of indication for cases in our analysis. After an investigation, the result is the student abstract the relationship between properties is not optimal. There is a serial abstraction process, but there is a triangle property that is not part of the first abstraction process, or the product of abstraction that is not part of the next abstraction, so there is a disconnected conceptual system between the product of thought. One basic cause that students to look at the same figure of the triangle between the current and the past. The congruence concept is the abstraction product of abstraction directly. As a result, the property of shape claimed. In these cases, solving geometry problems just focus on triangle concept may contribute to the error of completion [2,4]. However, if students only know bits of information, that is the lowest level of conceptual knowledge, while those who can build relational rules and conclude pieces of information are at a higher level [5,12]. Because conceptual knowledge refers to the understanding of mathematical principles and how everything is connected [23] and can be achieved by establishing relationships between pieces of information [10].

Our analysis of the next case based on students' claims. Some of the effects lead to triangle classification, which is the right and equilateral triangle. The effect claimed is the application of the Pythagorean theorem. According to Gunhan [24], three factors have an impact on student responses, namely knowledge, visual perception, and logical argument. Besides, procedures and techniques for solving problems are information about student reasoning skills. This case headed to knowledge and logical argument, so the direction was abstraction and reasoning issues. Abstraction is one level below of deduction in van Hiele's theory of geometry thinking [1,7,9,12,13]. When there are issues at the level of abstraction, deductive reasoning is more extensive. Fabiyi [25] states that poor reasoning involves unfounded and hasty reason. Furthermore, Gunhan [24] states that students may sometimes choose the correct problem solving strategy, yet follow the wrong course of action when finding solutions becoming oriented towards familiar solution patterns due to conceptual shortcomings and they see without giving much thought to their reasons for doing so. The strategic competence have strongly connected and interrelated with metacognitive monitoring [13,20]. Thus, in general, we see that the influence of metacognitive knowledge has brought students' focus to apply the Pythagorean theorem for finding the required properties of shape to solve geometry problems. Gunhan [24] recommends that students should

develop their arguments in support of the claims made with adequate, as well as appropriate conceptual knowledge and associative skills. Furthermore, if their reasoning skills remain underdeveloped, students will come to view mathematics as an aggregate of specific rules, execute calculations and drawings thoughtlessly.

3.3. *Implication for Learning Geometry*

The focus of educational research on learning geometry is a process. Learning geometry is important because it aims at developing deductive reasoning skills and gaining spatial awareness [1,7,12,20,23,25]. The thesis we states based on this study is when students encounter geometry problems involving triangles as the class of shapes, they will tend to apply the Pythagorean theorem as a relationship between properties to determine the size of a particular property, and students' geometrical problem solving is influenced by several factors such as the process of abstraction, reasoning, and the influence of metacognitive knowledge. How to learn geometry while existing issues are part of the students' abstraction of conceptual knowledge. This issue concerns the disconnected conceptual system between the product of thought and it is the result of the tendency and influence of metacognitive knowledge. There should be an accommodation process because at that time students experience what Piaget called disequilibrium in the theory of cognitive development so that the work of solving problems becomes new stable knowledge for students [13,26]. Howse & Howse [21] cites that K-grade 12 geometry instruction should empower students with the ability to analyze properties of geometry shapes and to base sound arguments on the understanding of relationships among these properties.

According to the results of our study that there is a tendency for students to engage in Pythagorean theorem. Implementing procedures do not always make students aware of relevant concepts unless they abstract the information system [5,6], and that tendency has hampered reasoning. While, according to the cognitive-constructivist perspective, cognitive schemas is the product of constructing knowledge and the tools with which additional new knowledge can be constructed [13,26]. The reason for performance efficiency in solving geometry problems requires students to have in-depth conceptual knowledge [1]. Luneta [9] argues that conceptual knowledge of geometric concepts goes beyond the skills needed to manipulate geometry shapes. Developing a conceptual understanding of geometry in teaching is important [7] because the conceptual experience will support the transition from concrete to abstract thinking [27] by encourage the making and testing of hypotheses or conjectures, examine properties of shapes to determine necessary and sufficient conditions for a shape to be a particular shape, use the language of informal deduction, and encourage students to attempt informal proofs [13]. Relevant to this study, Sia & Lim [17] has identified that cognitive diagnostic analysis results can be used to bridge the gap between the level of competence and the learning objectives planned through instructional intervention. Because of the assertion in the van Hiele theory that students must develop masterfully at each level before they can progress to the next [9], recommended instructional intervention for in-depth conceptual knowledge is learning in accommodation process situation context, for the intended ability is to analyze properties of shapes on the language of informal deduction in geometric hypotheses or conjectures and to attempt informal proofs.

4. Conclusion

Based on the results of the study, it was concluded that students see the three parallel designs of abstraction thinking problems, or the problem of distance between point and line, as the problem of triangle properties. There are the same and different triangles in the students' analysis. In their problem solving, there is a serial abstraction process on each triangle before the parallel products are synthesized. The Pythagorean theorem is one of the students' abstraction products and its benefits for further abstraction processes in the serial or the parallel. The Pythagoras theorem appears as a result of abstraction thinking at least two of the three design formations geometry problem. But there are issues in the abstraction process, because the property of the triangle is not involved in the first of the serial processes, or the product of thought is not involved in the next abstraction so that the conceptual system between the products of thought is disconnected. When the abstraction process is not optimal, in a serial or parallel abstraction session, then the claim comes for properties. The effect claim leads to triangle classification and has implications for applying the Pythagorean theorem. Factors due to the influence

of metacognitive knowledge, the students' focus has been directed towards their tendency to apply the procedure for finding the properties of shape needed to solve geometry problems. Generally, issues in solving geometry problems based on students' conceptual knowledge are the relevance between given information, the concept used, and the problem addressed.

5. Acknowledgments

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