

PAPER • OPEN ACCESS

Short-range correlations for $0\nu\beta\beta$ decay and low-momentum NN potentials

To cite this article: L Coraggio *et al* 2020 *J. Phys.: Conf. Ser.* **1643** 012124

View the [article online](#) for updates and enhancements.

You may also like

- [Status and future of nuclear matrix elements for neutrinoless double-beta decay: a review](#)
Jonathan Engel and Javier Menéndez
- [Symmetry restoration in mean-field approaches](#)
J A Sheikh, J Dobaczewski, P Ring et al.
- [New applications of renormalization group methods in nuclear physics](#)
R J Furnstahl and K Hebeler



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

Short-range correlations for $0\nu\beta\beta$ decay and low-momentum NN potentials

L Coraggio¹, N Itaco^{1,2}, and R Mancino^{1,2}

¹Istituto Nazionale di Fisica Nucleare,

Complesso Universitario di Monte S. Angelo, Via Cintia - I-80126 Napoli, Italy

²Dipartimento di Matematica e Fisica, Università degli Studi della Campania “Luigi Vanvitelli”, viale Abramo Lincoln 5 - I-81100 Caserta, Italy

E-mail: luigi.coraggio@na.infn.it

Abstract. We approach the calculation of the nuclear matrix element of the neutrinoless double- β decay process, considering the light-neutrino-exchange channel, by way of the realistic shell-model. In particular the focus of our work is spotted on the role of the short-range correlations, which should be taken into account because of the short-range repulsion of the realistic potentials. Our shell-model wave functions are calculated using an effective Hamiltonian derived from the high-precision CD-Bonn nucleon-nucleon potential, the latter renormalized by way of the so-called $V_{\text{low-k}}$ approach. The renormalization procedure decouples the repulsive high-momentum component of the potential from the low-momentum ones by the introduction of a cutoff Λ , and is employed to renormalize consistently the two-body neutrino potentials to calculate the nuclear matrix elements of candidates to this decay process in mass interval ranging from $A = 76$ up to $A = 136$. We study the dependence of the decay operator on the choice of the cutoff, and compare our results with other approaches that can be found in present literature.

1. Introduction

The neutrinoless double- β decay is currently one of the main targets to explore the limits of Standard Model and to understand the intrinsic nature of the neutrino (see Ref.[1] for a brief but upgraded review of current and future experiments). As is well known the detection of such a rare decay would assess the neutrino as a Majorana particle, namely that neutrinos are their own anti-particles, and correspond to a lepton number violation, which will introduce us to “new physics” beyond the Standard Model.

On the other side, the measurement of the decay half life may provide an estimation of its effective mass via the relationship

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} |M^{0\nu}|^2 \langle m_\nu \rangle^2, \quad (1)$$

where $G^{0\nu}$ is the so-called phase-space factor (or kinematic factor), $\langle m_\nu \rangle$ is the effective neutrino mass that takes into account the neutrino parameters associated with the mechanisms of light- and heavy-neutrino exchange, and $M^{0\nu}$ is the nuclear matrix element (NME) directly related to the wave functions of the parent and grand-daughter nuclei.



The expression (1) evidences that a reliable estimate of the NME is a key point both to understand which are the most favorable nuclides to detect $0\nu\beta\beta$ decay, and to link the experimental results to the value of neutrino effective mass.

Currently, the nuclear structure models which are largely employed to study the $0\nu\beta\beta$ decay of nuclei of experimental interest are the Interacting Boson Model (IBM) [2–4], the Quasiparticle Random-Phase Approximation (QRPA) [5–7], Energy Density Functional methods [8], and the Shell Model (SM) [9–13].

One of the issues to be tackled in the calculation of the $0\nu\beta\beta$ NME is the evaluation of the short-range correlations (SRC), which account the fact that the action of a two-body decay operator on an unperturbed (uncorrelated) wave function is not equal to the action of the same operator on the real (correlated) nuclear wave function [14, 15].

This follows from the highly repulsive nature of the nuclear interaction in its short range, which requires - for nuclear structure calculations - a consistent regularization of the nucleon-nucleon (NN) potential V^{NN} and of any two-body transition operators [16].

The most common way to soften the matrix elements of the $0\nu\beta\beta$ decay operator and include SRC is by way of Jastrow type functions [17, 18], and in recent years SRC have been modeled by the so-called Unitary Correlation Operator Method (UCOM) [10, 15], this approach allowing to provide a unitary operator which prevents the overlap between the wave functions of a pair of nucleons [19].

In this work we present an original approach to the evaluation of SRC that is consistently linked to the derivation of the effective shell-model Hamiltonian H_{eff} , the latter being calculated starting from a realistic NN potential. More precisely, our first step is to consider the high-precision CD-Bonn NN potential [20], whose repulsive high-momentum components are renormalized by way of the $V_{\text{low-}k}$ approach in order to make it suitable for the derivation of H_{eff} by way of the many-body perturbation theory [21, 22].

The renormalization of V^{NN} by way of the $V_{\text{low-}k}$ procedure [23, 24] occurs through a unitary transformation $\Omega_{\text{low-}k}$ in the momentum space of the two-nucleon Hamiltonian H^{NN} , by truncating the full Hilbert space to a subspace where only relative momenta below a cutoff Λ are allowed. Obviously, the unitary transformation preserves the physics of H^{NN} , namely the calculated values of all observables are the same as those reproduced by the original realistic potential.

This renormalization procedure needs to be applied to any two-body operator, for consistency reason, before employing the same operator in nuclear structure calculations which employ wave functions obtained starting from the same $V_{\text{low-}k}$. Consequently, we have renormalized $0\nu\beta\beta$ decay operator by way of $\Omega_{\text{low-}k}$ in order to consider effectively the high-momentum (short range) components of the NN potential, in a framework where their direct contribution is dumped by the introduction of a cutoff Λ .

In the following section we will sketch out a few details of our theoretical framework, more precisely how the renormalization procedure of the $0\nu\beta\beta$ decay operator is carried out. In Section 3 the results of the calculation of $M^{0\nu}$ for the $0\nu\beta\beta$ decay, within the light-neutrino exchange, of ^{76}Ge , ^{82}Se , ^{130}Te , and ^{136}Xe are reported, comparing those obtained with the bare operator with the results provided by the renormalization of the decay operator using two different values of the cutoff Λ . The wave functions of the parent and grand-daughter nuclei have been calculated using the SM H_{eff} reported in Refs. [25, 26], where the abilities of these H_{eff} to reproduce the spectroscopic properties of the nuclei involved in the decay have been extensively reported, as well as the results of the calculations of two-neutrino double-beta decay NME $M^{2\nu}$. Conclusions and perspectives of our work will be reported in Section 4.

2. Theoretical framework

As is well known, in nuclear structure calculations with realistic potentials, practitioners have to face the problem that the basis states, which constitute the Slater determinants of unperturbed non-correlated wave functions Φ , are non-zero in the region of short-range interaction. This contrasts with the need that, because of the short-range repulsion of V^{NN} (repulsive high-momentum components in the momentum space), the “real” correlated wave function Ψ has to approach to zero as the internucleon distance diminishes, as fast as the core repulsion increases (see Fig. (1)).

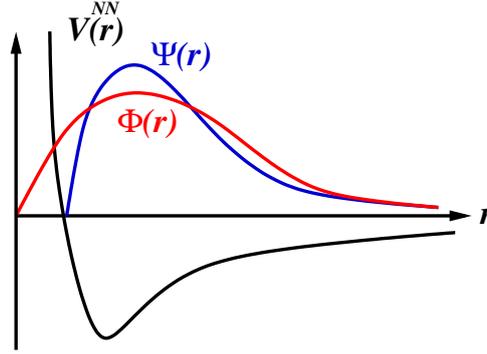


Figure 1. Representation of a realistic potential V^{NN} , as a function of the internucleon distance r , and of the correlated and non-correlated wave functions Ψ (blue line) and Φ (red line), respectively (see text for details).

This leads to the need to renormalize the short-range (high-momentum) components of the NN potential, when a perturbative approach to the many-body problem is pursued.

Here, we briefly introduce the so-called $V_{\text{low-}k}$ approach, whose details may be found in Refs. [23, 24]

The eigenvalue problem of the two-nucleon Hamiltonian $H^{NN}(k, k') = H_0(k, k') + V^{NN}(k, k')$ - $H_0(k, k')$ being the kinetic-energy term - may be written in the full momentum space of the plane-waves basis $\langle k | \Psi_\nu \rangle$ in the following form:

$$\int_0^\infty [H_0(k, k') + V^{NN}(k, k')] \langle k | \Psi_\nu \rangle k^2 dk = E_\nu \langle k' | \Psi_\nu \rangle \quad . \quad (2)$$

We look for a Hamiltonian $H_{\text{low-}k}(k, k') = H_0(k, k') + V_{\text{low-}k}(k, k')$ that is defined in a reduced subspace $P = \int_0^\Lambda |k\rangle \langle k| k^2 dk$, whose subset of eigenvalues $\{\tilde{E}_\mu\}_{\mu \in P}$ belongs to the set of eigenvalues $\{E_\nu\}$ of the Hamiltonian $H^{NN}(k, k')$ defined in full Hilbert space:

$$\int_0^\Lambda [H_0(k, k') + V_{\text{low-}k}(k, k')] \langle k | \Phi_\mu \rangle k^2 dk = \tilde{E}_\mu \langle k' | \Phi_\mu \rangle \quad . \quad (3)$$

This goal may be achieved through a similarity transformation $\Omega_{\text{low-}k}$, that leads to the identity $\mathcal{H} = \Omega_{\text{low-}k}^{-1} H^{NN} \Omega_{\text{low-}k}$. $\Omega_{\text{low-}k}$ needs to satisfy the decoupling condition which decouples the low-momentum subspace P from its complement $Q = 1 - P$:

$$Q \mathcal{H} P = Q \Omega_{\text{low-}k}^{-1} H^{NN} \Omega_{\text{low-}k} P = 0 \quad (4)$$

A very convenient expression of the operator $\Omega_{\text{low-}k}$ may be obtained according to the Lee-Suzuki formulation [27], which is:

$$\begin{aligned} \Omega_{\text{low-}k}P &= I_P & P\Omega_{\text{low-}k}Q &= 0 \\ Q\Omega_{\text{low-}k}P &= \omega & Q\Omega_{\text{low-}k}Q &= I_Q \end{aligned} \quad (5)$$

where I_P, I_Q represents the identity operator in the P and Q spaces, respectively. This form leads to a non-linear matrix equation for the ω operator, which can be solved using iterative techniques [28]:

$$QH^{NN}P + QH^{NN}Q\omega - \omega PH^{NN}P - \omega PH^{NN}Q\omega = 0 \quad (6)$$

Once Eq. (6) is solved and the operator ω is obtained, an hermitization procedure, based on the Cholesky decomposition of the operator $\Omega_{\text{low-}k}$ [28], evolves the Lee-Suzuki similarity transformation to a unitary transformation.

The $V_{\text{low-}k}$, which is explicitly zero for momenta above the cutoff Λ , may now be suitable as an input for the derivation of H_{eff} by way of the many-body perturbation theory [22]. In Refs. [25, 26] we have reported the results for the calculation of $M^{2\nu}$ for ^{76}Ge , ^{82}Se , ^{130}Te , and ^{136}Xe , and in the present work we employ the same wave functions obtained from the diagonalization of H_{eff} obtained renormalizing the CD-Bonn potential with a cutoff $\Lambda = 2.6 \text{ fm}^{-1}$.

As regards the calculation of $M_\alpha^{0\nu}$ - α denoting the Fermi (F), Gamow-Teller (GT), or tensor (T) decay channels - we recall that, within the closure approximation [29], it can be written in terms of the two-body transition-density matrix elements $\langle f | a_p^\dagger a_n a_{p'}^\dagger a_{n'} | i \rangle$, the indices i, f denoting the parent and grand-daughter nuclei:

$$M_\alpha^{0\nu} = \sum_{j_n j_{n'} j_p j_{p'} J_\pi} \langle f | a_p^\dagger a_n a_{p'}^\dagger a_{n'} | i \rangle \langle j_p j_{p'}; J^\pi | \tau_1^- \tau_2^- O_{12}^\alpha | j_n j_{n'}; J^\pi \rangle \quad (7)$$

The operators O_{12}^α are expressed in terms of the neutrino potentials H^α and form functions $h^\alpha(q)$:

$$\begin{aligned} O_{12}^{GT} &= \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r) \\ O_{12}^F &= H_F(r) \\ O_{12}^T &= [3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_1 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2] H_T(r) \end{aligned} \quad (8)$$

$$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty \frac{j_{n_\alpha}(qr) h_\alpha(q^2) q dq}{q + \langle E \rangle} \quad (9)$$

The average energies $\langle E \rangle$ have been evaluated as in Ref. [30], the parameter R is $R = 1.2A^{1/3} \text{ fm}$, and $j_{n_\alpha}(qr)$ are the spherical Bessel functions, $n_\alpha = 0$ for Fermi and Gamow-Teller components, $n_\alpha = 2$ for the tensor one. The explicit expression of neutrino form functions $h_\alpha(q)$ for light-neutrino exchange may be found in Ref. [31].

We transform the operators in Eq. (9), that are expressed in a local configuration-space form, to a momentum-space representation [32], in order to construct a low-momentum decay operator $O_{\text{low-}k}^\alpha = P\Omega_{\text{low-}k}^{-1}O^\alpha\Omega_{\text{low-}k}P$. The latter unitary transformation allows to take into account effectively the high-momentum (short-range) correlations on the two-nucleon wave function. In the following Section, the results of the calculation of $M_\alpha^{0\nu}$ for ^{76}Ge , ^{82}Se , ^{130}Te , and ^{136}Xe decays will be presented, using both bare and renormalized $0\nu\beta\beta$ decay operators.

3. Results

Our starting point is the calculation of nuclear wave functions of parent and grand-daughter nuclei for ^{76}Ge , ^{82}Se , ^{130}Te , and ^{136}Xe decays, using the SM effective Hamiltonians derived from CD-Bonn potential, the latter being renormalized via the $V_{\text{low-}k}$ procedure employing a cutoff $\Lambda = 2.6 \text{ fm}^{-1}$ [25, 26]. In those works, where tables with the theoretical single-particle energies and two-body matrix elements of the residual interaction have been also reported, it has been carried out an extensive study of the doubly-beta decay of such nuclei, with the perspective to check the theoretical framework for future studies of their $0\nu\beta\beta$ decay.

In Table (3) the results of the calculations of $M^{0\nu}$ for the ^{76}Ge , ^{82}Se , ^{130}Te , and ^{136}Xe decays are reported. We have neglected the tensor component in the expression (7), since its contribution is about 2-3 order of magnitude smaller of the Fermi and Gamow-Teller components.

The calculations have been performed both with bare $0\nu\beta\beta$ operators and those renormalized via the $V_{\text{low-}k}$ approach, as reported in Section 2. We have employed for the renormalization of the high-momentum components of the decay operator two cutoffs, $\Lambda = 2.6, 2.1 \text{ fm}^{-1}$, in order to evaluate the dependence of the results on this choice.

Table 1. Results for the calculation of $M^{0\nu}$ relative to ^{76}Ge , ^{82}Se , ^{130}Te , and ^{136}Xe decays. Calculations with bare operator are compared with results obtained taking into account the SRC by way of the $V_{\text{low-}k}$ approach and cutoffs $\Lambda = 2.6, 2.1 \text{ fm}^{-1}$. In parentheses they are reported the variations in percentage of SRC results with respect to the bare ones.

Decay	bare operator	$\Lambda = 2.6 \text{ fm}^{-1}$	$\Lambda = 2.1 \text{ fm}^{-1}$
$^{76}\text{Ge} \rightarrow ^{76}\text{Se}$	3.35	3.29 (1.8%)	3.27 (2.4%)
$^{82}\text{Se} \rightarrow ^{82}\text{Kr}$	3.30	3.25 (1.5%)	3.23 (2.1%)
$^{130}\text{Te} \rightarrow ^{130}\text{Xe}$	3.27	3.22 (1.6%)	3.20 (2.1%)
$^{136}\text{Xe} \rightarrow ^{136}\text{Ba}$	2.47	2.43 (1.6%)	2.41 (2.4%)

As can be seen, the variation of the calculated $M^{0\nu}$, with respect to the ones obtained with the bare operator, is about 2%, the softening being mildly larger with the smaller cutoff $\Lambda = 2.1 \text{ fm}^{-1}$ since it corresponds to a larger renormalization effect. The result that the $\Omega_{\text{low-}k}$ transformation leads to a tiny renormalization effect can be ascribed to the behavior of neutrino form functions $h^\alpha(q)$, which approach rapidly to zero for momenta $q \rightarrow \infty$ [31]. Consequently, they are scarcely sensitive to the renormalization of high-momentum components by the $\Omega_{\text{low-}k}$ operator.

It should be pointed out that the effect in magnitude of this renormalization is very close to the one obtained by way of UCOM SRC by Menendez and coworkers (see Table 8 in Ref. [10]) for the same nuclear decays, leading to a lighter softening of NME with respect to the one provided by Jastrow type SRC. As a matter of fact, the authors experienced a reduction of the calculated NMEs, with respect to the bare decay operator, about 20-25% employing standard Jastrow type correlations, and 5-6% the UCOM ones.

It is worth to stress again that our calculations manage the correlations induced by the renormalization of the high-momentum components of V^{NN} on an equal footing, both for the NN potential and the two-body matrix elements of the $0\nu\beta\beta$ decay operator. It should be also mentioned that a similar approach was pursued in works by Kuo and coworkers [16, 33], where the renormalization of realistic potentials by way of the reaction matrix G was employed to calculate SRC in terms of the defect wave functions [14].

4. Conclusions and perspectives

In this work we have introduced an original approach to consider the effects of short-range correlations in the calculation of the nuclear matrix element for the $0\nu\beta\beta$ decay within the realistic shell model.

This has been done by renormalizing the two-nucleon Hamiltonian for a realistic NN potential and the $0\nu\beta\beta$ decay operator, consistently, by way of the so-called $V_{\text{low-}k}$ approach [23]. Then, we have calculated the $M^{0\nu}$ for $0\nu\beta\beta$ -decay candidates ^{76}Ge , ^{82}Se , ^{130}Te , and ^{136}Xe , using both the bare decay operator and the renormalized one. The wave functions of parent and grand-daughter nuclei employed for these calculations are the same as in Refs. [25, 26], where the SM effective Hamiltonians have been derived by way of the many-body perturbation theory from the CD-Bonn NN potential, the latter being renormalized employing the $V_{\text{low-}k}$ method.

Our results show that this novel approach to the evaluation of SRC reveals a tiny effect, when compared to the inclusion of standard Jastrow type correlations.

The next step will be to build up SM effective operators for the two-body $0\nu\beta\beta$ -decay operator by way of the many-body perturbation theory, as in Refs. [16, 34], consistently with the derivation of the SM Hamiltonian from realistic NN potentials.

Our goal is to perform fully-consistent calculations of $M^{0\nu}$ which avoids to resort to parameters fitted to experiment, providing an improvement of the reliability and predictivity of nuclear-structure calculations for the $0\nu\beta\beta$ decay.

References

- [1] Henning R 2016 *Reviews in Physics* **1** 29 – 35
- [2] Barea J and Iachello F 2009 *Phys. Rev. C* **79**(4) 044301
- [3] Barea J, Kotila J and Iachello F 2012 *Phys. Rev. Lett.* **109**(4) 042501
- [4] Barea J, Kotila J and Iachello F 2013 *Phys. Rev. C* **87**(1) 014315
- [5] Šimkovic F, Faessler A, Mütther H, Rodin V and Stauf M 2009 *Phys. Rev. C* **79**(5) 055501
- [6] Fang D L, Faessler A, Rodin V and Šimkovic F 2011 *Phys. Rev. C* **83**(3) 034320
- [7] Faessler A, Rodin V and Simkovic F 2012 *J. Phys. G* **39** 124006
- [8] Rodríguez T R and Martínez-Pinedo G 2010 *Phys. Rev. Lett.* **105**(25) 252503
- [9] Menéndez J, Poves A, Caurier E and Nowacki F 2009 *Phys. Rev. C* **80**(4) 048501
- [10] Menéndez J, Poves A, Caurier E and Nowacki F 2009 *Nucl. Phys. A* **818** 139
- [11] Horoi M and Brown B A 2013 *Phys. Rev. Lett.* **110**(22) 222502
- [12] Neacsu A and Horoi M 2015 *Phys. Rev. C* **91**(2) 024309
- [13] Brown B A, Fang D L and Horoi M 2015 *Phys. Rev. C* **92**(4) 041301
- [14] Bethe H A 1971 *Annu. Rev. Nucl. Sci.* **21** 93
- [15] Kortelainen M, Civitarese O, Suhonen J and Toivanen J 2007 *Phys. Lett. B* **647** 128
- [16] Wu H F, Song H Q, Kuo T T S, Cheng W K and Strottman D 1985 *Phys. Lett. B* **162** 227
- [17] Miller G A and Spencer J E 1976 *Ann. Phys. (NY)* **100** 562
- [18] Neacsu A, Stoica S and Horoi M 2012 *Phys. Rev. C* **86**(6) 067304
- [19] Feldmeier H, Neff T, Roth R and Schnack J 1998 *Nucl. Phys. A* **632** 61
- [20] Machleidt R 2001 *Phys. Rev. C* **63** 024001
- [21] Kuo T T S, Lee S Y and Ratcliff K F 1971 *Nucl. Phys. A* **176** 65
- [22] Coraggio L, Covello A, Gargano A, Itaco N and Kuo T T S 2012 *Ann. Phys. (NY)* **327**(9) 2125
- [23] Bogner S, Kuo T T S and Coraggio L 2001 *Nucl. Phys. A* **684** 432c
- [24] Coraggio L, Covello A, Gargano A, Itaco N and Kuo T T S 2009 *Prog. Part. Nucl. Phys.* **62** 135
- [25] Coraggio L, De Angelis L, Fukui T, Gargano A and Itaco N 2017 *Phys. Rev. C* **95**(6) 064324
- [26] Coraggio L, De Angelis L, Fukui T, Gargano A, Itaco N and Nowacki F 2019 *Phys. Rev. C* **100**(1) 014316
- [27] Suzuki K and Lee S Y 1980 *Prog. Theor. Phys.* **64** 2091
- [28] Andreozzi F 1996 *Phys. Rev. C* **54** 684
- [29] Sen'kov R A, Horoi M and Brown B A 2014 *Phys. Rev. C* **89**(5) 054304
- [30] Haxton W C and Stephenson Jr G J 1984 *Prog. Part. Nucl. Phys.* **12** 409
- [31] Horoi M and Stoica S 2010 *Phys. Rev. C* **81**(2) 024321
- [32] Veerasamy S and Polyzou W N 2011 *Phys. Rev. C* **84**(3) 034003
- [33] Song H Q, Wu H F and Kuo T T S 1991 *Phys. Lett. B* **259** 229
- [34] Holt J D and Engel J 2013 *Phys. Rev. C* **87**(6) 064315