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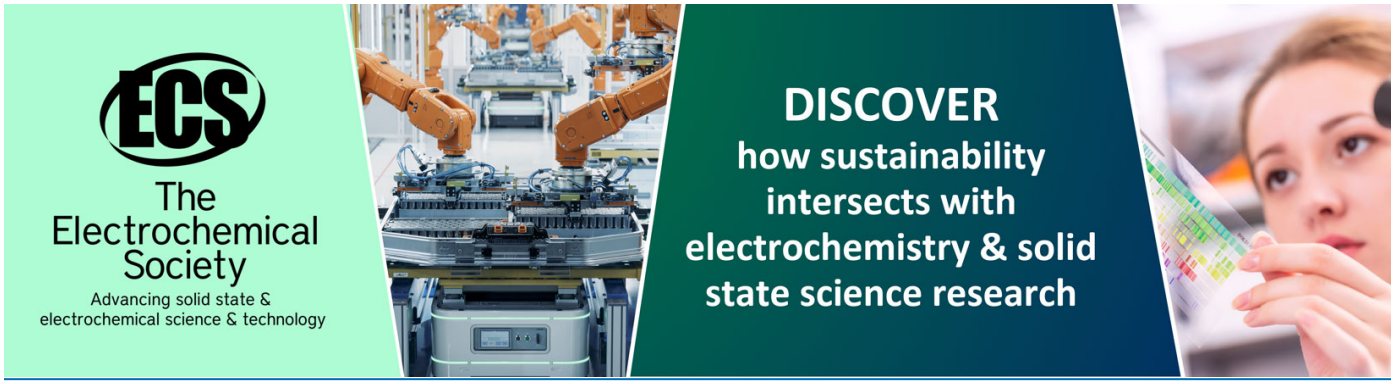

A Time Dependent Process Zone Model for Slow Crack Growth of Polyethylene Pipe Material

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Abstract. Polyethylene (PE) pipe is widely used in water supply and gas transportation engineering. Its fracture behavior is very important for the safe operation of the pipeline. The objective of this work is to simulate the slow crack growth (SCG) process of PE pipe material via a time dependent crack-tip process zone model. To achieve this end, the crazing damage caused by stress concentration at crack tip of PE material is taken into account, the stress on the bulk/craze interface layer is supposed to be uniformly distributed due to the micro necking mechanism of the layer, and a linear crack growth kinetics is used. In order to describe the delay characteristics of the fracture process of PE materials, the fracture surface energy is considered as an exponential decay function of time, which is attributed to the creep damage of the craze fibrils. A simulation algorithm of the proposed process zone model is suggested using iterative method. The model is verified by comparing the simulation with the discontinuous SCG behavior reported in the literature.

1. Introduction

Polyethylene (PE) pipes have been widely used in gas transportation and water supply. The service lifetime of the pipe is related to its internal pressure: higher stress causes shorter service lifetime. It is well recognized that the pipe fails with various modes when operated under different stress levels [1]. As depicted in Figure 1, there exist three different failure modes corresponding to the circumferential tensile stress in the pipe wall. In the case of high circumferential tensile stress, the ductile failure occurs due to the local yielding and bulging at stress concentration sites; when the circumferential tensile stress is reduced to moderate level of magnitude, the service lifetime of the pipe increases. In this case, the failure model of the pipe changes from ductile to quasi-brittle manner, and the underlying mechanism is slow crack growth (SCG) [2], which is also called subcritical crack growth (SCG); however, when the circumferential tensile stress further decreases, the reducing stress has little effect on the prolonging service life of the pipe. Because in this case, the brittle failure of the pipe is mainly due to the material corrosion by the inner fluid/gas medium and the aging induced deterioration of the mechanical properties of the pipe material, so that the stress corrosion cracking (SCC) or environmental stress cracking (ESC) is the dominant failure mechanism [3]. In practical

engineering projects, such as gas pipeline engineering, the PE pressure pipes majorly are subjected to intermediate stresses and fail due to SCG.

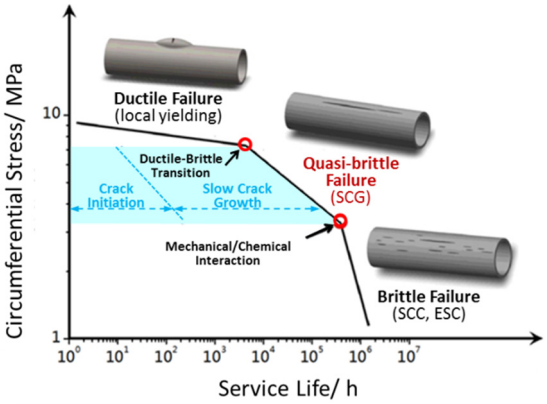

Figure 1 Schematic figure of polymer pressured pipe failure modes and stress-service life curve.

At experiment aspect, several standard test methods have been issued to evaluate the SCG resistance of PE pipe, most of the methods are based on the constant load tests with specimen containing notch, including Notch Pipe Test (NPT, ISO 13479), Pennsylvania Edge Notch Test (PENT, ISO 16241), Notched Ring Test (NRT, ASTM D2290) and Full Notch Creep Test (FNCT, ISO 16770). Although these methods accelerate the measurements by using a combination of high stress, elevated temperature and/or chemical agent, the measured failure times are typically thousands of hours, which are excessive for material ranking and new resin development. Therefore, it is necessary to develop alternative rapid test methods. Recently the Strain Hardening (SH) test with standard tensile specimen at elevated temperature (ISO 18488) and the fatigue test of Cracked Round Bar (CRB, ISO 18489) are available to rank the SCG resistance. The main advantage of both methods is that they shorten the test time to a few hours.

In addition, there are few reliable models to predict the delayed failure behavior and the creep lifetime of polymer pressure pipes under various stress levels. In particular, the research on theory and simulation of discontinuous SCG is still in its infancy [4, 5]. Most of the current researches are based on linear elastic fracture mechanics or elastioplastic fracture mechanics, which hold that when the stress intensity factor (SIF) or energy release rate at a crack tip reaches the critical value, the crack begins to expand. To avoid stress singularity at the crack tip, the classic elastic-plastic fracture analysis uses the Dugdale-Barenblatt model to describe the crack-tip fracture process zone. According to elastic-plastic fracture mechanics, the crack will never propagate in material subjected to a constant load, as long as the crack-tip SIF or energy release rate at the moment of loading is less than the critical value. This is obviously inconsistent with the fact that PE pressure pipes present delayed failures. Therefore, it is real need to develop corresponding delayed failure theory. At present, there are two research strategies. One is to adopt theory of viscoelastic fracture mechanics. Since Knauss [6] established the framework of linear viscoelastic fracture mechanics for polymers in the mid-1970s, viscoelastic fracture mechanics has been in slow development [7-9]. Based on the process zone model and the work of fracture criterion, Schapery discussed the crack initiation of a viscoelastic material whose second derivative of logarithmic creep compliance respect to time is very small, and obtained the relationship between crack propagation rate and SIF [10]. Frassine et.al verified the Schapery's theory through experimental tests and pointed out that the time dependent cohesive stress in the crack-tip process zone should be considered [11]. Yang and Luo et.al considered the linear viscoelastic polymer bulk, and proposed a modified crack-tip craze zone model by introducing a time-dependent damage variable. In such modified model, the cohesive stress is inhomogeneous in space and unsteady in time, and the numerical calculation of craze zone growth is in good agreement with the experimental measurements [12]. The premise of numerical calculation is to define the craze damage evolution law, which is difficult to be experimentally determined. The other method is to modify the Dugdale-Barenblatt

model taking into account the deterioration characteristics or time-dependent properties of the material in the crack-tip process zone. In this paper, we adopt the second method to establish a process zone model, which can describe the discontinuous SCG behavior, based on the reasonable assumption of stress distribution in crack-tip craze zone and time-dependent crack growth criterion.

2. Process Zone Model

2.1. *Crazing at crack tip*

Crazing is a unique damage phenomenon in polymer materials, which is a process containing material cavitation and fibril formation [13]. The experimental researches by Kramer and Berger suggested that there is a strain softening layer (also named active layer) between craze zone and bulk material [14, 15], as shown in Figure 2 and the thickness of active layer depends on local strain rate and temperature, which is approximately of the order of the diameter of the craze fibril [15].The general mechanism of craze tip advance has been well known to be the Taylor meniscus instability process; whereas there are two possible mechanisms for craze thickening, i.e. interface drawing thickening and creep thickening [15]. Unlike crack, craze has bearing capacity due to its interconnected fibrils. The stress at the craze-bulk interface is called cohesive stress or craze stress referred in some literature [12]. Due to the stretching under craze stress, the molecular chains in the active layer undergo disentanglement and/or breakage, and are continuously drawn into the craze fibrils, so that the length of the fibrils increase and the craze thickens [15]. In addition, the creep of fibrils under the action of craze stress is also one of the causes of fibril lengthening and craze zone thickening [16]. Berger and Yang et.al suggested that craze fibrils are formed by micro-necking deformation of the bulk material in the active layer [14, 17], such interface drawing process is similar to the necking development of the material after macroscopically yielding and softening under tension. Therefore, the craze stress in the interfacial active layer is believed to be consistent with the cold drawing stress of macro-necking development [18]. As the craze zone grows by thickening, the cavitation damage increases, and the fibrils at the crack tip break and then the crack grows when the length and thickness of the craze reach to their critical value.

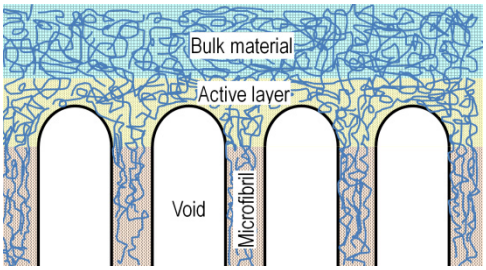

Figure 2 Schematic view of the formation of the craze fibrils by micro-necking of the active layer.

2.2. *Crack-tip process zone model*

Consider a notched elastic-plastic body subjected to remote stress of σ_∞, a craze zone of length l_{cz} is formed at its crack tip. The crack length is denoted by l_{ck}. Such mechanical problem is modeled in Figure 3(a). It is equivalent to the superposition of the two subsystems depicted by Figure 3(b) and Figure 3(c). In Figure 3(b) the bulk material containing a notch of virtual length $l_{\mathrm{cz}} + l_{\mathrm{ck}}$ by removing the process zone from the original problem, it is subjected to a combination of the remote stress σ_∞ and the virtual closure stress σ_{dr} , that is the cohesive normal stress transferred through the craze fibrils to the surrounding bulk material. Figure 3(c) presents the craze zone material cut out from the crack-tip damage region, it is stretched by the craze stress σ_{dr}. Experimental data [14, 19] have shown that the craze fibrils almost undergo the same stretch ratio λ, indicating the uniform distribution of σ_{dr} along the craze length.

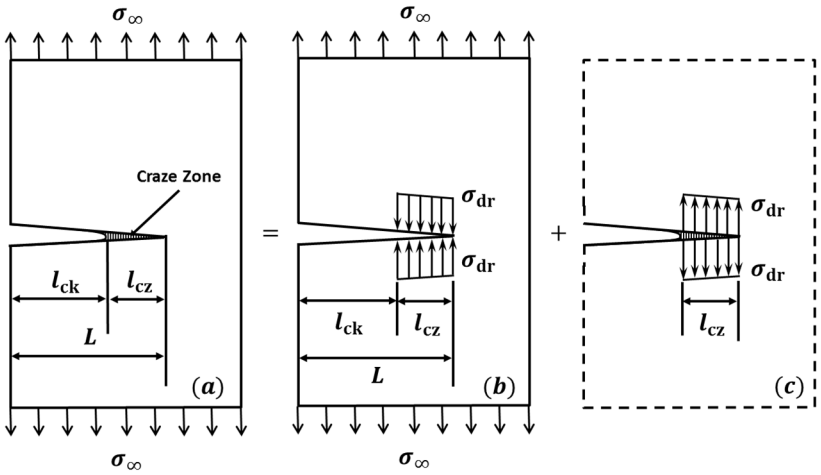

Figure 3 Schematics of the crack-tip process-zone model.

2.3. *Crazing at crack tip*

The discontinuous SCG behavior will be modeled with the above mentioned process zone model. However, unlike most of the literatures, the time-dependent mechanical property of the craze fibrils is taken into account, in other words, the fracture surface energy of the craze fibrils decays with load-bearing time.

The crack propagation is physically the consequence of the fact that crack driving force exceeds crack resistance. So the crack propagation criterion is expressed by $X^{\mathrm{ck}} \geq 0$, while X^{ck} reads:

$$X^{\mathrm{ck}} = -\frac{\partial \mathcal{G}}{\partial l_{\mathrm{ck}}}\Big|_{L=\mathrm{const}} = J_1 - 2\gamma = \frac{K^2}{E'} - 2\gamma \tag{1}$$

where $\mathcal{G}$ is the Gibbs potential energy of the notched material under loading(see Figure 3(b)), J_1 is the energy release rate due to crack propagation into the process zone, i.e. the crack drive force; K is the SIF at the crack tip due to remote stress; E' is the elastic modulus in plane strain state; γ is time-dependent Griffith fracture surface energy due to crazing, i.e. the crack resistance. As stated before, the crack would grow into craze zone when the craze fibrils near the crack tip break, thus the crack resistance is actually dominated by the mechanical properties of craze fibrils, especially the fracture surface energy. According to the growth behavior of craze zone, crazing damage becomes severer upon craze thickening, the intuitive and simplest criterion for craze fibril breakdown, which has been proved to be effective for glassy polymers, is that craze fibrils break at a critical length, which is exactly the critical thickness of the craze zone. The growing crazing damage results in sustainable decrease in resistance force of craze fibril breakdown. Therefore, the crack resistance also thereupon decreases, and the corresponding specific fracture surface energy decays with the time of craze growth. Chudnovsky et.al [18] expressed the specific fracture surface energy as a decreasing function of time by Equation (2), connecting the degradation of the material in craze zone with its creep behavior.

$$\gamma = \gamma_0 \cdot \left(1 + \frac{t_c}{t^*}\right)^{-1} \tag{2}$$

where γ_0 stands for the specific surface energy of the undamaged bulk material and it is taken as a constant, t_{c} is the elapsed time of craze fibrils degradation, which is considered to be the time of craze growth; and *t** is the characteristic time and controls the degradation speed, depending on temperature. Smaller *t** causes more serious degradation. We suggest, in this paper, the exponential decay function:

$$\gamma = \gamma_0 \exp\left[-\left(\frac{t_{\mathrm{c}}}{t^*}\right)^{\mathrm{n}}\right] \tag{3}$$

where n is the material constant depending on temperature. Since the crazing damage is caused by the stress concentration in crack tip zone, the damage degree is directly related to the crack-tip stress field which is controlled by the SIF, thus we here assume the characteristic time *t** is linearly proportional to the SIF (K):

$$t^* = A \cdot K + B \tag{4}$$

in which A and B are temperature dependent material constants.

As the craze zone thickening, its tip advances simultaneously so as to increase its length. The craze tip advance criterion is expressed by $X^{\mathrm{cz}} \geq 0$, and X^{cz} denotes the excessive force for craze tip advance:

$$X^{cz} = -\frac{\partial \mathcal{G}}{\partial l_{cz}}\Big|_{l_{ck}=const} = \frac{K_{tot}^2}{E'} - \psi^{tr} \cdot \omega_0(x)|_{x=l_{ck}} \tag{5}$$

As seen in Figure 3(b), the total SIF (K_{tot}) at the craze tip consists of two parts: the positive SIF (K_{∞}) arising from the remote stress σ_{∞} and the negative SIF (K_{dr}) from closure stress σ_{dr}. The first term of right hand side of Equation (5) is the energy release rate representing the drive force of craze tip advance, while ψ^{tr} and ω_0 in the second term are the energy density and the thickness of the original bulk material that is drawn into fibrils respectively. So the second term is the energy required for the bulk material to be drawn to form a unit area of craze zone, representing the resistance of craze tip advance. Although the thickness ω_0 is unknown in advance, the thickness of craze zone after craze thickening is $\omega = \omega_0 \cdot \lambda$ (λ is the natural draw ratio of the drawn fibril)according to the thickness of craze zone in equilibrium cold drawing. So the thickness change due to craze thickening is $\omega - \omega_0 = (\lambda - 1)\omega_0$.Since the wedge typed craze zone has a small thickness-length aspect ratio, its thickness change can be approximately expressed by crack opening displacement (COD) δ [3]. Accordingly the original thickness ω_0 can be determined through Equation (6).

$$\omega_0 = \delta \cdot (\lambda - 1)^{-1} \tag{6}$$

To determine the COD δ in Equation (6)and the SIFs in Equation (1) and Equation (5), the Green function method can used for the case of single edge notched (SEN) specimen[20, 21]. Besides the Green function method, finite element method is the alternative way to obtain the total SIF and total COD.

The following sequence events is assumed during the discontinuous SCG process: (1) the initiated process zone ahead of the crack tip grows by increasing its length and thickness in each incremental time step till the moment of local equilibrium ($X^{\mathrm{cz}} = 0$) or crack onset ($X^{\mathrm{ck}} \geq 0$), while the length of the crack remains unchanged; (2) as long as the crack onset criterion is met, the crack begins to extend till the strain energy released at the crack tip is not much enough to produce new more crack surfaces. Simultaneously the craze tip advances to a certain distance because of the break of its own equilibrium; (3) repetitive cycles between the craze growth to its next equilibrium size and the subsequent crack growth occur till to unstable fracture. SCG kinetics formulate the relationship between the crack/craze zone growth rate ($\dot{l}_{\mathrm{ck}}$ and $\dot{l}_{\mathrm{cz}}$) and their excessive drive forces (X^{ck} and X^{cz}).

Equilibrium size of craze length can be calculated from $X^{\mathrm{cz}} = 0$. Any positive deviation from the equilibrium would cause the change of craze length. Chudnovsky and Wee et al [5, 18] assumed a linear law of growth kinetics for craze and crack as shown in Equation (9) and Equation (10) respectively:

$$\dot{l}_{ck} = k\langle X^{ck} \rangle \tag{7}$$

$$\dot{l}_{cz} = k'\langle X^{cz} \rangle \tag{8}$$

where $\langle x \rangle = (|x| + x)/2$, k and k' are the kinetic coefficients corresponding to propagation of crack and craze zone respectively.

The governing equations for SCG consist of Equations (1), (5), (7)and (8) Although these equations are nonlinear due to the change of X^{ck} and X^{cz} with the crack length and craze length, they can be solved under the given initial boundary conditions through numerical iteration, as will be illustrated in the next section.

3. Application to SCG of PENT specimen of PE material

Below we use the proposed process zone model and its numerical algorithm to simulate the uniaxial tensile creep fracture process of a notched middle density polyethylene (MDPE) specimen at elevated temperature, and compare the simulation with the experimental results from Lu et al [22]. The Pennsylvania Edge Notch Tensile (PENT) specimen, as shown in Figure 4, was machine cut from the MDPE pipe and used for creep test, the main notch depth was 3.5 mm. Such SEN specimen was designed to ensure the creep crack growth under plane strain condition. The creep test was conducted

under constant tensile stress of 2.4 MPa and at 80 ℃. The experimental measured crack length vs. loading time is replotted in Figure 5 by digitization from the literature [22]. It can be seen that after the action of constant remote stress, the artificially prefabricated crack of 3.5mm long almost did not expand in a long period of about 310 minutes, and it also grew very slowly in three other time segments which were about 360 to 480, 530 to 640, and 680 to 830 min. As previously described for the discontinuous SCG process, the craze initiated and grew during these crack arrest periods. Furthermore, there was a jump in crack length after each crack arrest. The jump increment was about 0.5~0.7 mm and the time duration for crack jump was about 40~50 minutes.

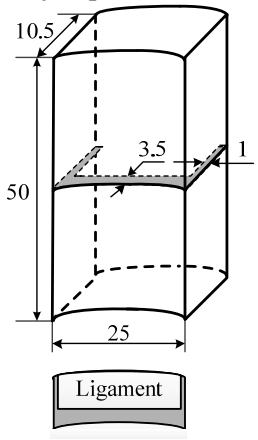

Figure 4 PENT specimen used in creep test.

The above described discontinuous SCG process is simulated using the proposed process zone model and the aforementioned numerical algorithm. The model parameters are listed in Table 1, and the numerical results of crack length and craze zone size are plotted as a function of time in Figure 5. From the comparison of simulation results and experimental data, the present model can well reproduce the SCG process of the material.

Table 1 Parameters of the process zone model.

σ_∞	σ_{dr}	E'	λ	ψ^{tr}	K_{IC}	k	k'	n	A	B
MPa	MPa	MPa	-	$\mathrm{mJ/mm^3}$	$\mathrm{MPa\sqrt{mm}}$	$(\mathrm{MPa\cdot s})^{-1}$	$(\mathrm{MPa\cdot s})^{-1}$	-	-	-
2.4	13.5	600	6	11	95	0.007	0.06	0.3	20	-178

From the simulation results in Figure 5, it can be seen that the crack is stationary at the beginning, while the craze zone grows to its equilibrium length almost at the very moment of remote loading. And then their lengths keep unchanged until the degradation of the craze fibrils triggers the crack's initial propagation into the craze zone at 155 min, as depicted by the green dash dot line and the red solid line for $0 \le t \le 155$ min. The crack passes through the craze zone and then stops at t =190 min when it reaches to the tip of the previous mature craze zone, because the microfibrils in the newly formed craze zone ahead of the growing crack tip is still strong enough to arrest the crack. Such events are then repeated to produce the whole discontinuous SCG until the complete fracture of the specimen. The time of five crack jumps in simulation occur at t =333 min, 487 min, 629 min and 723 min respectively. As a comparison, the corresponding crack jumps time observed in experiment takes place at t =319 min, 483 min, 643 min and 830 min respectively.

It should be noted that the first jump observed in the experiment is simulated as two jumps by the process zone model. This may be attributed to the artificial notch. Generally, the first craze initiation and growth ahead of the artificial notch tip precedes the first crack jump. The craze initiation and growth are influenced or even dominated by the stress field at the notch tip. The tip curvature of the artificial notch is quite different from that of the initial sharp crack assumed in simulation or the mature craze tip, which is assumed as the subsequent equivalent crack tip. The blunt notch tip restrains the tip stress intensity, prolongs the craze initiation and growth duration, delays the time of initial

crack propagation and accordingly causes the greater crack increment in the first jump. As seen in Figure 5, two times of sharp crack jump propagation in simulation is equivalent to one time of the first jump propagation observed in the experiment.

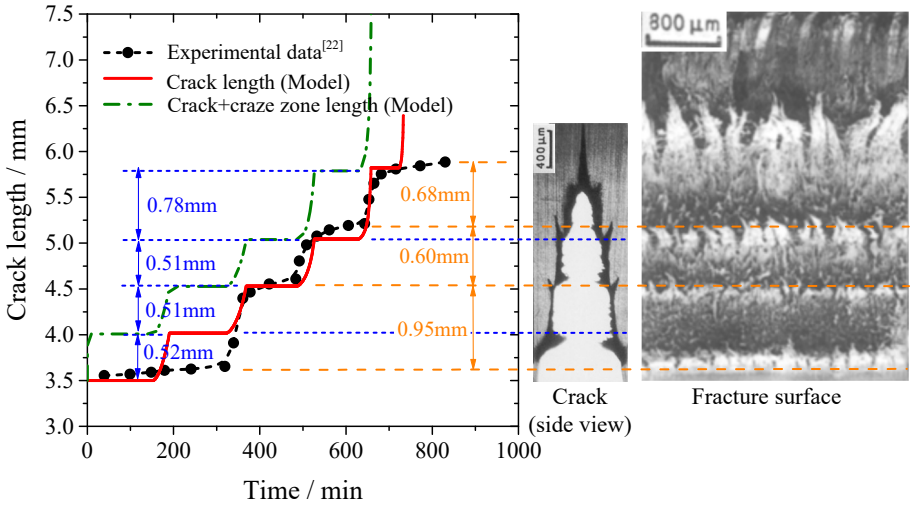

Figure 5 SCG of PENT specimen of PE material: model simulation vs. experiment by Lu et al [22].

From the crack side view in the inset of Figure 5 the average distance between two adjacent ligament bridges is about 0.6 mm; by contrast, the craze zone length calculated by the process zone model varies from 0.51 mm to 0.75 mm depending on the time. The calculations generally coincide with the experimental observations. Since the SIF increases with the growing crack, the equilibrium length of the craze zone increases and the crack arrest duration decreases with jumps number. However, according to the estimation by Dugdale-Barenblatt model, the equilibrium craze zone length changes from 0.6 mm to1.95 mm with the increasing crack length, two to three times larger than the calculations and observations. It is expected, since the Dugdale-Barenblatt model ignores cavitation and fibrillation in crazing which assume large amount of energy.

4. Conclusions

SCG has been recognized to be the failure mechanism of PE material under intermediate stress. In this paper, the modified process zone model has been proposed to simulate the discontinuous SCG behavior of PE material. The delayed fracture can be characterized by the process zone model with time-varying fracture surface energy, which is attributed to the creep damage of the craze fibrils. The simulation is in good agreement with the experimental results. It is demonstrated that the modified process zone model can be used to describe the discontinuous SCG behavior of PE material. And it is reasonable and effective that the fracture surface energy is considered as an exponential decay function of time, and the characteristic time for craze fibril deterioration is dominated by the applied stress intensity factor.

Acknowledgments

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