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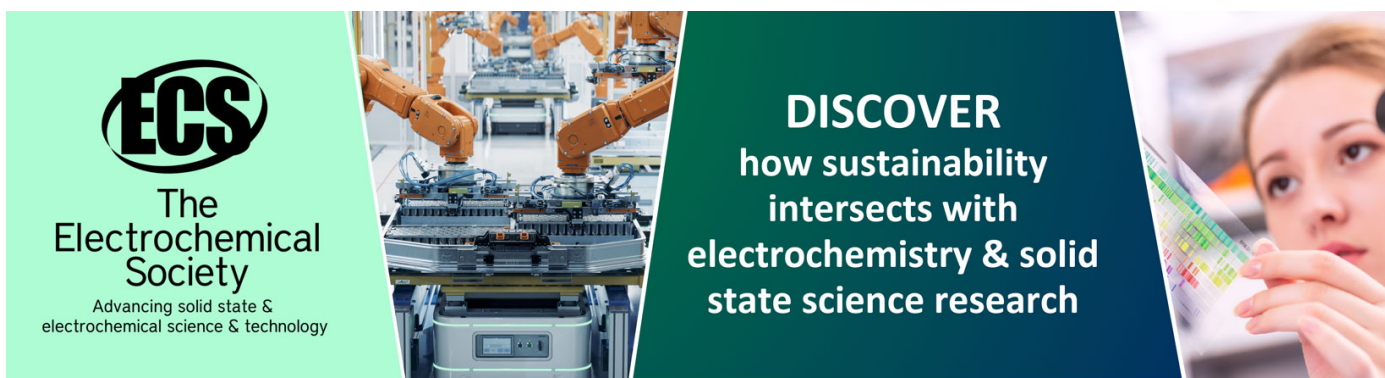
Fixed-time Adaptive Control for A New Three-dimensional Nonlinear System with Circular Equilibrium

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Fixed-time Adaptive Control for A New Three-dimensional Nonlinear System with Circular Equilibrium

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Abstract. In this paper, we proposed a new nonlinear chaotic oscillator. The model of the system is given through a predefined form. Its unique property lies in the circle-shaped equilibrium. Both analysis and simulations are presented. In particular, the stability of the neighbourhood of the equilibrium point is analysed in terms of theoretical derivation and dynamic behaviour. Moreover than that, a robust control method of a category of three-dimensional chaotic systems is designed. And the final simulation results proved the effectiveness of the controllers.

Keywords. Hidden attractor; Adaptive back-stepping technology; Fixed-time control.

1. Introduction

In the past few decades, chaos has established itself as an important branch of electrical and mathematics and applications becoming even wider, such as communication, image encryption and electronic system measurement. Especially after the first hidden chaotic attractor has been discovered, hidden attractor received widespread attention. Since without homoclinic or heteroclinic orbits [1], hidden attractors cannot be verified by the Shilnikov method. So the location, analysis and control are more complicated which may produce unexpected and potentially disastrous response to the disturbance. Recently, much more attention from researchers concentrated on the fixed-time control, for its value in practical application. In this work, a new nonlinear chaotic oscillator is proposed in Section 2 and dynamic analysis follows. The proposed control method (BASF) is presented in the next section. The final section concludes the paper.

2. The Model of the Chaotic Oscillator

Firstly, the mathematic model of the chaotic oscillator is introduced as follows:

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= a(x^2 + z^2 - r^2) \\ \dot{z} &= y \cdot f(x, y, z)\end{aligned}\tag{1}$$

in which a is a free constant, and r marks the radius of circle. Obviously the predefined form (1) contains equilibrium points which form a circle located on the plane $y = 0$. Then the nonlinear function f can contain many kinds of terms. Eventually, it reveals as:

$$f(x, y, z) = bx^2 + c|z|\tag{2}$$

in which b, c are free constants. For $a=-0.5, b=4, c=-2, r^2=2$ with initial conditions (1,0.5,-1.6), a chaotic attractor evolves with final time 1000 and a time step of 0.001 in figure 1. The grey plots are



projections of the attractor onto different axes, the black circle represents the equilibrium. The 0-1 test is shown in figure 2 with the asymptotic growth rate $K=0.966$ which close to 1 proving the chaos [2].

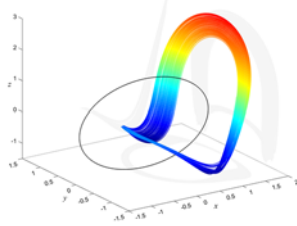


Figure 1. Hidden chaotic attractor.

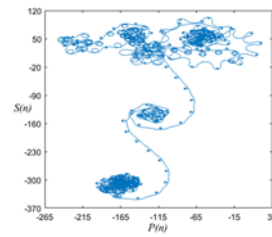


Figure 2. 0-1 test of the system.

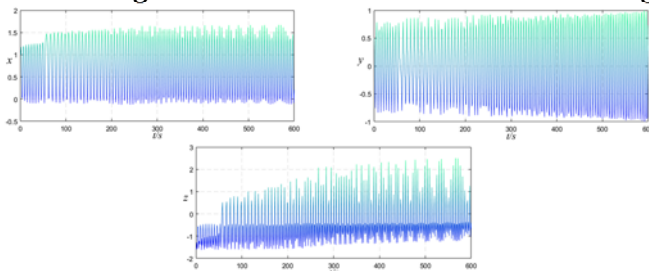


Figure 3. Time series of the states.

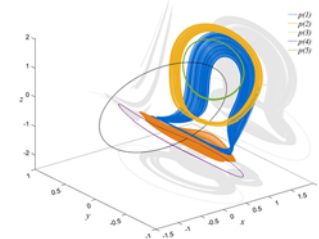


Figure 4. Coexisting hidden attractors.

And figure 3 shows the set of time domain waveforms of each states of the system. Multistability can also be observed under the same parameters, as shown in figure 4, corresponding to: p(1) (1, 0.4, -1.4), p(2) (1, 0.45, 1.5), p(3) (1, 0.4, -0.5), p(4) (-0.1, 0.4, -0.5), p(5) (1, 0.4, 0.5). It should be noticed that the attractors surrounded the circler-shaped equilibrium.

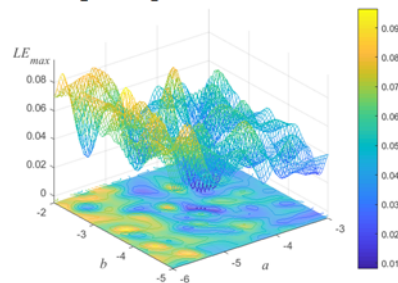


Figure 5. The largest Lyapunov exponents.

The largest Lyapunov exponent (LE) for two parameters is plotted in figure 5 with $c=2$, $r^2=2$, in which the positive value of LE indicates the chaotic solution.

Solving $\dot{x} = 0$, $\dot{y} = 0$, $\dot{z} = 0$, the circular equilibrium can be obtained:

$$y = 0, x^2 + z^2 = r^2 \quad (3)$$

There is one zero eigenvalue and two non-zero eigenvalues related to the position of the equilibrium on the circle. Thus,

$$\lambda_1 = 0, \lambda_{2,3}^2 = 2ax_0 + 2az_0(bx_0^2 + c|z_0|) \quad (4)$$

For improving the intuitiveness and clarity of analysis, the second equation in equation (4) will be rewritten. With $a=-0.5$, $b=-4$, $c=2$, $r^2=2$ we can get a function $g(x, z)$ of x, z as equation (5), and its function diagram is shown in figure 6.

$$g(x, z) = -x + 4x^2z - 2z|z| \quad (5)$$

According to the different values of the remaining two eigenvalues, the circular equilibrium on the plane $y=0$ can be divided into two types. The red part of the equilibrium corresponds to a pair of real number eigenvalues with opposite sign, and the unstable saddle can be inferred. And the blue part of the equilibrium, corresponding to a pair of pure imaginary eigenvalues, is called the centre. The dynamic motion near the centre is periodic, and the trajectory is a closed loop revolve round the equilibrium point which can also be confirmed in figure 7 and figure 8.

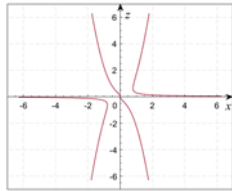


Figure 6. Diagram of $g(x, z)$.

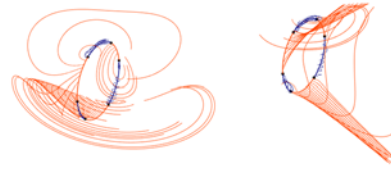
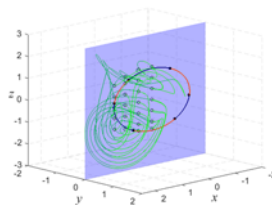
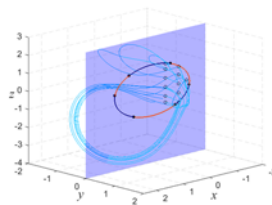


Figure 7. Dynamical behaviours with initial conditions near the equilibrium (above: inside, below: outside).

Figure 9 represents the bifurcation diagram with changing the parameter c , for $a=-0.5$, $b=-4$, $r^2=2$. From figure 9 the chaos can be observed in a wide range of parameters. So the suppression of chaos is very necessary.



(a)



(b)

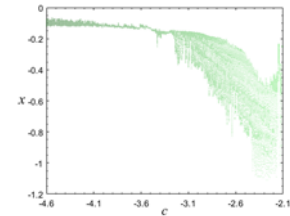


Figure 8. A local behaviour of the system in $y = 0$ plane: (a) $x > 0$, (b) $x < 0$.

Figure 9. Bifurcation diagram.

3. The Fixed-time Control

3.1. Preliminary Definitions and Lemmas

Lemma 1 [3]. Consider a dynamic system as follows:

$$\dot{x}(t) = f(x) \quad (6)$$

in which $x(t) \in R^n$ represents the state of the system. If there exists a constant $T > 0$ (T may be contingent on the initial conditions $x_0(t)$), such that $\lim_{t \rightarrow T} \|x(t)\| = 0$ and $\|x(t)\| \equiv 0$, if $t > T$, then the system (6) is finite-time stable.

Lemma 2 [4]. If system (8) is finite-time stable globally in the bounded time T , then it is fixed-time stable. That is, there exists a bounded positive constant $T_{\max}$, and $T < T_{\max}$ satisfies.

Lemma 3 [4]. Suppose a continuous, positive-definite Lyapunov function $V(x(t))$ satisfies:

(1) $V(x) = 0 \Leftrightarrow x = 0$, (2) $V(x)$ is radially unbounded, (3) with $\alpha, \beta, p, q, k > 0$, and $pk < 1$

$$D^*V(x(t)) \leq -[\alpha V^p(x(t)) + \beta V^q(x(t))]^k$$

in which $D^*V(x(t))$ is the derivative of $V(x(t))$, system (1) is globally fixed-time stable and the following estimate is valid:

$$T(x_0) \leq \frac{1}{\alpha^k(1-pk)} + \frac{1}{\beta^k(qk-1)}, \quad \forall x_0 \in R^n$$

Lemma 4 [5]. For any non-negative real numbers, that is, $x_1, x_2, \dots, x_N \geq 0$ the following inequality holds:

$$\sum_{i=1}^N \geq (\sum_{i=1}^N x_i)^\eta, \quad 0 < \eta \leq 1, \quad \sum_{i=1}^N x_i^\varsigma \geq N^{1-\varsigma} (\sum_{i=1}^N x_i)^\varsigma, \quad \varsigma > 1$$

Lemma 5. For a nonlinear function F , we assume $\hat{F}$ and F^* represent the approximate and the optimum of it, which can be respectively described by:

$$\hat{F} = \hat{A}_F^T \xi_F, \quad F^* = A_F^{*T} \xi_F$$

in which ξ_F is a nonlinear mapping of inputs, $\hat{A}_F^T$ and A_F^{*T} are vectors of the actual parameters and the ‘optimal’ parameters. Then, the difference between them will be donated by: $\tilde{A} = A_F^* - \hat{A}_F$

We assume the upper bound for the molding error exists:

$$\forall x_F \in D_F \mid F(x_F) - A_F^* \xi_F(x_F) \mid < \varepsilon_F.$$

3.2. Fixed-time Controller Design

Including the chaos system proposed in this paper, a category of third-order autonomous system can be described by:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f + pu_1 \\ \dot{x}_3 &= g + qu_2 \end{aligned} \quad (7)$$

in which, f, g are arbitrary nonlinear function with unknown parameters, p, q represent the control gain, u_1, u_2 denote the control inputs. With the desired trajectories x_{1d}, x_{2d} , for the state x_1 and x_2 we introduce an error:

$$e_1 = x_{1d} - x_1, \quad e_2 = x_{2d} - x_2 \quad (8)$$

Error dynamics is described by:

$$\dot{e}_1 = \dot{x}_{1d} - x_1 = x_{1d} - x_2 \quad (9)$$

So the desired value for x_2 can be chosen as:

$$x_{2d} = \dot{x}_{1d} + k_1 |e_1|^\alpha \operatorname{sgn}(e_1) + k_2 |e_1|^\beta \operatorname{sgn}(e_1) \quad (10)$$

where k_1, k_2 are positive design parameters and $\alpha > 1, 0 < \beta < 1$. So

$$\dot{e}_1 = e_2 - k_1 |e_1|^\alpha \operatorname{sgn}(e_1) - k_2 |e_1|^\beta \operatorname{sgn}(e_1) \quad (11)$$

The dynamics of e_2 is given by

$$\begin{aligned} \dot{e}_2 &= \ddot{x}_{1d} + \alpha k_1 e_2 + \beta k_2 e_2 - \alpha k_1^2 |e_1|^\alpha \operatorname{sgn}(e_1) - \beta k_2^2 |e_1|^\beta \operatorname{sgn}(e_1) - (f - f^*) \\ &\quad - f^* - (p - p^*)u_1 - (p^* - \hat{p})u_1 - \hat{p}u_1 \end{aligned} \quad (12)$$

So the control law is designed as:

$$\begin{aligned} u_1 &= \hat{p}^{-1} (-\hat{f} + \alpha k_1 e_2 + \beta k_2 e_2 - \alpha k_1^2 |e_1|^\alpha \operatorname{sgn}(e_1) - \beta k_2^2 |e_1|^\beta \operatorname{sgn}(e_1) \\ &\quad + k_1 |e_2|^\alpha \operatorname{sgn}(e_2) + k_2 |e_2|^\beta \operatorname{sgn}(e_2) + e_1 + \ddot{x}_{1d}) \end{aligned} \quad (13)$$

Under the control law, the error dynamics may be modified by:

$$\dot{e}_2 = -k_1 |e_2|^\alpha \operatorname{sgn}(e_2) - k_2 |e_2|^\beta \operatorname{sgn}(e_2) - \tilde{A}_f \xi_f - \tilde{A}_p \xi_p u_1 - \eta - \varepsilon \quad (14)$$

where $\varepsilon = -(f - f^*) - (p - p^*)u_1$. With Lemma 5, ε satisfies

$$\varepsilon \geq D_F \mid F(x) - A_F^* \xi_F(x) \mid = e_2 \quad (15)$$

Consider the following candidate Lyapunov function:

$$V_1 = (e_1^2 + e_2^2 + \tilde{A}_f^T \tilde{A}_f + \tilde{A}_p^T \tilde{A}_p) / 2 \quad (16)$$

Then the adaptive laws are given as:

$$\begin{aligned} \dot{\tilde{A}}_f &= \xi_f e_2 - k_1 \|\tilde{A}_f\|^\alpha - k_2 \|\tilde{A}_f\|^\beta, \quad \dot{\tilde{A}}_p = \xi_p e_2 u_2 - k_1 \|\tilde{A}_p\|^\alpha - k_2 \|\tilde{A}_p\|^\beta \\ \dot{\hat{A}}_f &= -(\xi_f e_2 - k_1 \|\tilde{A}_f\|^\alpha - k_2 \|\tilde{A}_f\|^\beta), \quad \dot{\hat{A}}_p = -(\xi_p e_2 u_1 - k_1 \|\tilde{A}_p\|^\alpha - k_2 \|\tilde{A}_p\|^\beta) \end{aligned} \quad (17)$$

Thus, we can get

$$\begin{aligned} \dot{V}_1 &\leq -m[(e_1^2 / 2)^{(\alpha+1)/2} + (e_2^2 / 2)^{(\alpha+1)/2} + (\|\tilde{A}_f\|^2 / 2)^{(\alpha+1)/2} + (\|\tilde{A}_p\|^2 / 2)^{(\alpha+1)/2}] \\ &\quad -n[(e_1^2 / 2)^{(\beta+1)/2} + (e_2^2 / 2)^{(\beta+1)/2} + (\|\tilde{A}_f\|^2 / 2)^{(\beta+1)/2} + (\|\tilde{A}_p\|^2 / 2)^{(\beta+1)/2}] \end{aligned} \quad (18)$$

where $m = 2^{(1+\alpha)/2} k_1$, $n = 2^{(1+\beta)/2} k_2$. Hence, it can be concluded from lemma 4 that:

$$\dot{V}_1 \leq -4^{(1-\alpha)/2} m \cdot V_1^{(1+\alpha)/2} - n V_2^{(1+\beta)/2} \quad (19)$$

According to Lemma 3, the state x_1, x_2 will convergence to zero in a bounded time which can be estimated by:

$$T_1 \leq \frac{2}{4^{(1-\alpha)/2} m \cdot (\alpha - 1)} + \frac{2}{n \cdot (1 - \beta)} \quad (20)$$

In addition, for the nonlinear function

$$\dot{x}_3 = g + q u_2 \quad (21)$$

where $g(x)$ denote an arbitrary nonlinear function, and q represent the control gain, u_2 denote the sliding mode control input. We define a sliding surface as

$$s = C x_3 \quad (22)$$

The control input is given by:

$$u_2 = -(Cq)^{-1} [Cg + \lambda s + k_3 |s|^\mu \text{sgn}(s) + k_4 |s|^\gamma \text{sgn}(s)] \quad (23)$$

where $\mu > 1, 0 < \gamma < 1$ and $\lambda, k_3, k_4 > 0$. So the Lyapunov function is formed as:

$$V_2 = s^T \cdot s / 2 = \|s\|^2 / 2 \quad (24)$$

The system derivative of

$$\dot{V}_2 = s^T \dot{s} < -w \cdot V_2^{(\mu+1)/2} - v \cdot V_2^{(\gamma+1)/2} \quad (25)$$

where $\lambda > 0$, $w = 2^{(\mu+1)/2} k_3$ and $v = 2^{(\gamma+1)/2} k_4$. According to Lemma 3, the state x_3 will toward to the sliding surface $s = 0$ within a bounded time T_2 , which can be estimated by

$$T_2 \leq 2 / (w(\mu - 1)) + 2 / (v(1 - \gamma)) \quad (26)$$

Hence, the time bound of the system convergence is $T = \max\{T_1, T_2\}$

3.3. Simulation Results

We assume a, r^2 are unknown parameters denoted by a', r' , and regrading z as the sliding surface, so

$$f = a'(x^2 + z^2 - r'), \quad g = y(bx^2 + c |z|) \quad (27)$$

The controller parameters are $k_1=4$, $k_2=1$, $k_3=2$, $k_4=1$, $x_{1d}=0$, $\alpha=1.2$, $\beta=0.8$, $\mu=1.5$, $\gamma=0.5$, $\lambda=2$. Hence, based on equation (21) and equation (27) the bounded time can be calculated by

$$T = \max\{T_1 = 7.08s, T_2 = 3.22s\} = 7.08s \quad (28)$$

The ODE-45 algorithm has been employed to integrate with initial conditions $X_0=(0.8,0.5,-1.6)$ and time step equals to 0.001. The curves of states are plotted in figure 10, which shows that the designed controllers can robustly stabilize the system in the bounded time (28) as expected. The time responses of the adaptive parameters a' , r' are illustrated in figure 11. It can be seen that the BASF control had a good performance no matter in the estimating speed or in accuracy.

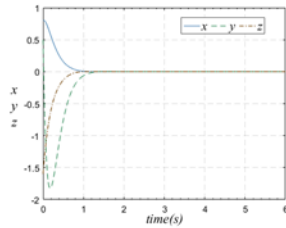


Figure 10. Time responses of the states.

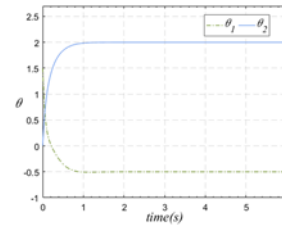


Figure 11. Estimation of the parameters ($\theta_1 : a, \theta_2 : r^2$).

3.4. Analysis of the BASF Control Effect

To further prove the validity of the control method, the control effect comparison between the BASF control and the FTSM (the finite-time sliding mode) control, which is proposed in [6], has been done. The system under FTSM control can be described by:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f + pu_1 \\ \dot{x}_3 &= g + qu_2 \end{aligned} \quad (29)$$

in which p , q represent the control gain, u_1, u_2 denote the control inputs, and f , g are the same as mentioned above. If two sliding surfaces are selected as:

$$\begin{aligned} \dot{s}_1 &= C_1 \dot{x}_1 + C'_1 \dot{x}_2 = C_1 x_2 + C'_1 (f + pu_1) \\ \dot{s}_2 &= C_2 \dot{x}_3 = C_2 (g + qu_2) \end{aligned} \quad (30)$$

So the two controllers can be designed as below:

$$\begin{aligned} u_1 &= -(Cp)^{-1} [C_1 x_2 + C'_1 f + (\mu + \gamma \|s_1\|^{\beta-1}) s_1] \\ u_2 &= -(C_2 q)^{-1} [C_2 g + (\mu + \gamma \|s_2\|^{\beta-1}) s_2] \end{aligned} \quad (31)$$

With the initial conditions (1, 0.5, -1.6), controller parameters are $p=q=1$, $\mu=2$, $\gamma=2$, $\beta=0.5$, the simulation results are shown in figure 12.

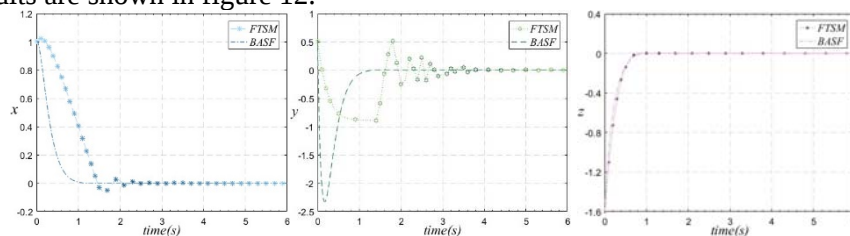


Figure 12. Comparison of control effect between method BASF and method FTSM.

Clearly, when the controllers (31) are used, the certain system can also be stabilized in finite-time. Although a set of control parameters with the best control effect has been selected after observing the control effect under different values of the parameters, the convergence time is much longer than the BASF control. And its convergence process is accompanied by obvious chattering. Distinctly, the FTSM control performs poorly in convergence speed, accuracy and stability efficiency. Thus, the BASF control is much better for its fast convergence speed, high convergence accuracy and high

efficiency stability. Besides, when the BASF controllers are applied, the unknown parameters in the system can be accurately estimated at a fast speed as shown in figure 11. Therefore, the validity and effectiveness of the BASF control method proposed in this paper are proved.

4. Conclusion

In this work, a new chaotic oscillator is proposed. And the system is analysed through mathematical analysis and numerical simulation. Then, improved adaptive laws are proposed to realize estimation of the unknown parameters. Using the Lyapunov analysis, the fixed-time stability of error signals and adaptive parameters for the controller proposed is proved. The bounded time is also given. The simulation results revealed the proposed controllers worked well for the fixed-time chaos control and accurate parameter estimation. Furthermore, compared with the FTSM control, the method proposed in this work has better control capability. In the future study, the proposed chaotic oscillator can be applied to cryptography such as secure communication, electrical and electronic system measurement.

Acknowledgments

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