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# Flat Trajectory Optimization of Tilt-rotor Aircraft Based on Gauss Pseudospectral Method

# Flat Trajectory Optimization of Tilt-rotor Aircraft Based on Gauss Pseudospectral Method

**Jiayi Zhuang[1,3], Qian Zhang[2,3,*], Jingjuan Zhang[1,3] and Jiyang Chen[1]**

1School of Instrumentation and Optoelectronic Engineering, Beihang University, Beijing, China
2School of Aeronautic Science and Engineering, Beihang University, Beijing, China
3Hefei Innovation Research Institute, Beihang University, Anhui, China

*Corresponding author email: zhangqian528@buaa.edu.cn

**Abstract.** The tilt-rotor aircraft has the advantages of high flexibility, low energy consumption and high speed, however, it faces the problems of complex operation and control difficulty. To solve the optimization problem of autonomous landing track of mobile platform, a Gauss pseudospectral method is used to design the landing trajectory of the tilt-rotor with optimal energy and time consumption. Firstly, an autonomous landing trajectory optimization problem model is developed for the tilt-rotor mobile platform. Then, the non-linear planning problem is solved, and finally, the optimal trajectory is put into the control program for control simulation. The simulation results show that the trajectory optimization based on Gauss pseudospectral method can meet the constraints well, which is in accordance with the time and energy consumption of the control program. Meanwhile, the control program simulation results can also keep up with the designed trajectory. It is proved that the Gauss pseudospectral method could design the autonomous landing trajectory with high accuracy and high performance.

## 1. Introduction

Tilt-rotor combines the advantages of helicopters and fixed-wing aircrafts, which is a kind of dual-use high-tech product, and its appearance has made helicopter technology have further developed [1].The tilt-rotor aircraft can take off and land like helicopters and fly forward like the fixed-wing airplane [2].In the flat phase, it has the characteristics of a fixed-wing aircraft with high flight speed, low energy consumption and long cruise time [3-6]. Meanwhile, with a structure which combines the design of rotorcraft and fixed-wing aircraft as well as an additional tilt mechanism, the tilt-rotor is more difficult to control for pilots [7-9]. Energy consumption and the safety of the tilt-rotor landing process are the key issues to be considered when tilting the rotorcraft for marine missions.

It will be a distance between the tilt-rotor and the landing platform when the tilt-rotor receives the landing command. At this time, online trajectory planning for the position and speed of the both tilt-rotor and landing platform is required to guide the tilt-rotor to reach the landing point. While trajectory planning suffice for the beginning and end state requirements and constraints, it also needs to consider relevant performance indicators such as flight time and flight energy consumption, and optimize the planned route on the basis of the successful completion of the task to achieve the optimal trajectory. In recent years, some studies have been made on such terminal time as the variables of the optimal control problem, mainly by direct methods [10-11], in which the pseudospectral method of discrete control variables and state variables, due to the advantage in computational efficiency, it has gradually became the hotspot of optimal control problem solving methods [12]. The common pseudospectral methods include Radau pseudospectral merhod, Legendre pseudospectral method, and Gauss pseudospectral method [13]. Compared with Legendre pseudospectral method, Radau pseudospectraland Gauss pseudospectral methods are superior in the approximation accuracy and convergence speed of state variables, control variables and coordination variables. In contrast to the Radau pseudospectral method, the Gauss pseudospectral method has higher accuracy in estimating the boundary values of coordination variables, and has advantages in dealing with problems involving initial and terminal constraints. The principle of Gaussian pseudospectral method to solve the optimal control problem is to convert it into a nonlinear programming problem through discretization, and then find the optimal solution to the performance index through the nonlinear programming algorithm [14-17]. In this paper, trajectory optimization of tilt-rotor aircraft in level flight is designed based on Gauss pseudospectral method, with minimum flight time and energy consumption as the optimization indicators.

## 2. Trajectory Optimization of Tilt-rotor Aircraft in Level Flight

### *2.1. Coordinate Definition of the Tilt-rotor Aircraft*

The schematic diagram of tilt-rotor aircraft is shown in Fig.1, the definition of the coordinate system involved in this study is as follows: $O-X_nY_nZ_n$ is the navigation coordinate system, $X_n$-axis points east, $Y_n$-axis points north, $Z_n$-axis points upward. $O-X_bY_bZ_b$ is the body coordinate system, $X_b$-axis points to transversal direction of the tilt-rotor, $Y_n$-axis points to forward direction of the tilt-rotor, $Z_n$-axis points to vertical direction of the tilt-rotor. $O-X_{m1}Y_{m1}Z_{m1}$, $O-X_{m2}Y_{m2}Z_{m2}$, $O-X_{m3}Y_{m3}Z_{m3}$ are the motor systems which are fixed with three motors, with the same direction as the body coordinate system. $O-X_wY_wZ_w$ is the wind axis coordinate system, $X_w$-axis coincident with the aircraft airspeed, $Z_w$-axis points to the front, and $Y_w$-axis is defined according to the right-hand rule.

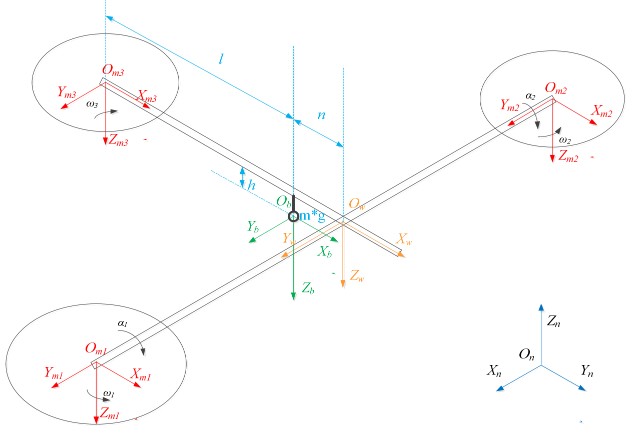


**Figure 1.** The schematic diagram of tilt-rotor aircraft

### *2.2. Mathematical Model of the Tilt-rotor Aircraft*

The dynamic equation of the tilt-rotor under the external force $\vec{F}$ is expressed as follows:

$$\begin{cases} \vec{F} = \dfrac{d}{dt}(m\vec{V}) = m(\dfrac{\delta\vec{V}}{\delta t} + \vec{\Omega}\times\vec{V}) \\ \vec{\Omega} = p\vec{i} + q\vec{j} + r\vec{k} \\ \dfrac{\delta\vec{V}}{\delta t} = \dot{u}\vec{i} + \dot{v}\vec{j} + \dot{w}\vec{k} \end{cases} \tag{1}$$

Where $\vec{V}$ is the velocity of the tilt-rotor aircraft. $p$, $q$, $r$ are the projections of the angular velocity of the body on the machine system. $u$, $v$, $w$ are the projections of the speed of the body on the machine system. $m$ is the mass of the tilt-rotor aircraft.

### *2.3. Constraints and Optimization Objective Function*

During flight, the constraints of tilt-rotor include initial state constraints, end state constraints, flight state constraints, actuator constraints and dynamical constraints. Where the dynamics constraint is the dynamics model of the tilt-rotor aircraft.

The initial state includes the position, speed and attitude angles of the tilt-rotor when it receives landing instructions, which can be represented as:

$$X(0)=\left[x_0,\ y_0,\ z_0,\ \dot{x}_0,\ \dot{y}_0,\ \dot{z}_0,\ \theta_0,\ \phi_0,\ \psi_0\right]^T \tag{2}$$

End-state constraints include reaching the predetermined destination, with arriving at the same speed as the target landing platform and reaching the same attitude angles that suffice the requirements of the inclination transition phase:

$$X(t_f)=\left[x_f,\ y_f,\ z_f,\ \dot{x}_f,\ \dot{y}_f,\ \dot{z}_f,\ \theta_f,\ \phi_f,\ \psi_f\right]^T \tag{3}$$

Where $t_f$ indicates the moment of arrival over the landing site, $\left[x_f, y_f, z_f\right]^T$ indicates target landing platform location, $\left[\dot{x}_f, \dot{y}_f\right]^T$ indicates the horizontal speed of the landing platform.

In order to ensure the stability of the tilt-rotor during flight and to reduce the computation of trajectory optimization, the motion state constraints of the tilt-rotor can be expressed as:

$$\begin{cases} -x_f \le x(t) \le 2x_f \\ -y_f \le y(t) \le 2x_f \\ z_f - 50m \le z(t) \le z_f + 50m \\ 0m/s \le V_a(t) \le 50m/s \\ -5m/s \le \dot{z}(t) \le 5m/s \\ -15^\circ \le \theta(t) \le 15^\circ \\ -20^\circ \le \phi(t) \le 20^\circ \\ -\pi \le \psi(t) \le \pi \end{cases} \tag{4}$$

Where $V_a(t)=\sqrt{\dot{x}^2(t)+\dot{y}^2(t)+\dot{z}^2(t)}$ indicates airspeed.

In order to ensure the normal operation of the power mechanism of the tilt-rotor, it is necessary to limit the output of each power mechanism of the aircraft during the optimal trajectory design of the tilt-rotor, the limitations of the tilt-rotor power mechanism can be expressed as:

$$\begin{cases} 0kg \le T(t) \le 75kg \\ 0kg \le T_b(t) \le 8kg \\ 0^\circ \le \varphi(t) \le 90^\circ \\ -30^\circ \le \varphi_b(t), \delta_l(t), \delta_r(t) \le 30^\circ \end{cases} \tag{5}$$

Where $T_b$ is the tension of the tail rotor of the tilt-rotor , $\varphi_b$ is the angle of the tail rotor of the tilt-rotor in the OY direction, $\delta_l$ and $\delta_r$ are the deflection angles of the left and right ailerons.

In order to make the tilt-rotor aircraft take the least amount of time, shortest path and lowest energy consumption from the starting point to the landing point, in this paper, it will take the two parameters of flight energy consumption and time spent in flight as the optimization objective indexes of the flight trajectory. The optimization objective function can be expressed as:

$$J = w_1 t_f + w_2 \int_{t_0}^{t_f} P(t)\, dt \tag{6}$$

As time and energy consumption cannot be minimized at the same time, the weighting of the two in the optimization objective index is determined by the actual situation.

So far, a mathematical model of the landing trajectory optimization problem of a tilt-rotor aircraft is formed by the kinetic constraint, initial-terminal state constraint, motion state constraint, actuator constraint and optimization objective function. Then, the trajectory optimization problem will be transformed into an optimal control problem and the optimal control problem will be solved by Gauss pseudospectral method.

## 3. Gauss Pseudospectral Method

The basic principle of the Gauss pseudospectral method is to discretize the state variables and control variables of the optimal control problem at the Gaussian point, and use this as a node to construct a (N+1) degree Lagrange interpolation polynomial to approximate the state variable and construct an N degree Lagrange interpolation polynomials approximate the control variables. The derivative which is approximated by deviating the global interpolation polynomial is caused from the state variable, thereby turning the differential equation constraints into a series of algebraic constraints [19]. The Gauss pseudospectral method turns the trajectory optimization problem of this paper into a nonlinear programming problem, which can be described as seeking discrete state variables $X_i$ ( $i = 0,1,\cdots,N$ ), control variables $U_k$ ( $k = 1,\cdots,N$ ) and a final time $t_f$ that can minimize the optimization objective function and satisfies the following constraints:

(1)State constraint at Gauss point:

$$\sum_{i=0}^{N} D_{ki} X_i - \frac{t_f - t_0}{2} f\left(X_k, U_k, \tau_k; t_0, t_f\right) = 0 \tag{7}$$

(2)End state constraints:

$$X_f - X_0 - \frac{t_f - t_0}{2} w_k f\left(X_k, U_k, \tau_k; t_0, t_f\right) = 0 \tag{8}$$

(3)Boundary conditions and process constraints:

$$\phi\left(X_0, t_0, X_f, t_f\right) = 0 \tag{9}$$

$$C\left(X_f, U_k, \tau_k; t_0, t_f\right) \le 0 \tag{10}$$

Where N is the number of points for Gauss, $D \in R^{N\times(N+1)}$ is differential matrix, $w_k$ is Gauss weight, $\tau_k$ is Lagrange-Gauss point.

## 4. Simulation and Result Analysis

In the simulation of this paper, the equivalent area of the wing is $4m^2$, the mass of the tilt-rotor aircraft is $67kg$, the maximum motor pull is limited to 75*kg*, the flat trajectory of the tilt-rotor is planned from receiving flight instructions to reaching the flight path above the landing platform to minimize the time and energy consumption of this process.

Assume that the tilt-rotor receives flight instructions at the origin of the geographic coordinates, the initial altitude is 300m, and the direction of flight is along the positive direction of the X-axis of the geographical system with the velocity of 33 m/s, the initial pitch angle is 2°, the initial roll angle and heading angle are both 0°. The destination horizontal coordinate is (1000, 500), target travel speed is

31m/s, The angle between the speed direction and the X axis is 30 °, the tilt-rotor altitude at the desired target is constant, the vertical velocity is zero, and the three attitude angles are the same as the initial state.

**Table 1.** Flat trajectory optimization simulation parameters

| Initial parameters of flat | Terminal parameters of flat |
|---|---|
| $[0,0,300m,33m/s,0,0,2°,0,0]$ | $[1000m,500m,300m,26.85m/s,15.5m/s,0,2°,0,0]$ |

The time and energy consumptions of the tilt rotor during flight are used as optimization objective indexes. Since the two indexes cannot be minimized at the same time and the units of the two are different, the weight coefficients need to be added before the two index functions. The optimization indicator function used in this paper can be expressed as:

$$J = t_f + \frac{1}{t_f - t_0}\int_{t_0}^{t_f} P(t)\,dt \tag{11}$$

Where $P(t) = T(t)\cdot\vec{v}(t)$.

The simulation results of the route planning based on Gauss pseudospectral method by Matlab are as follows:

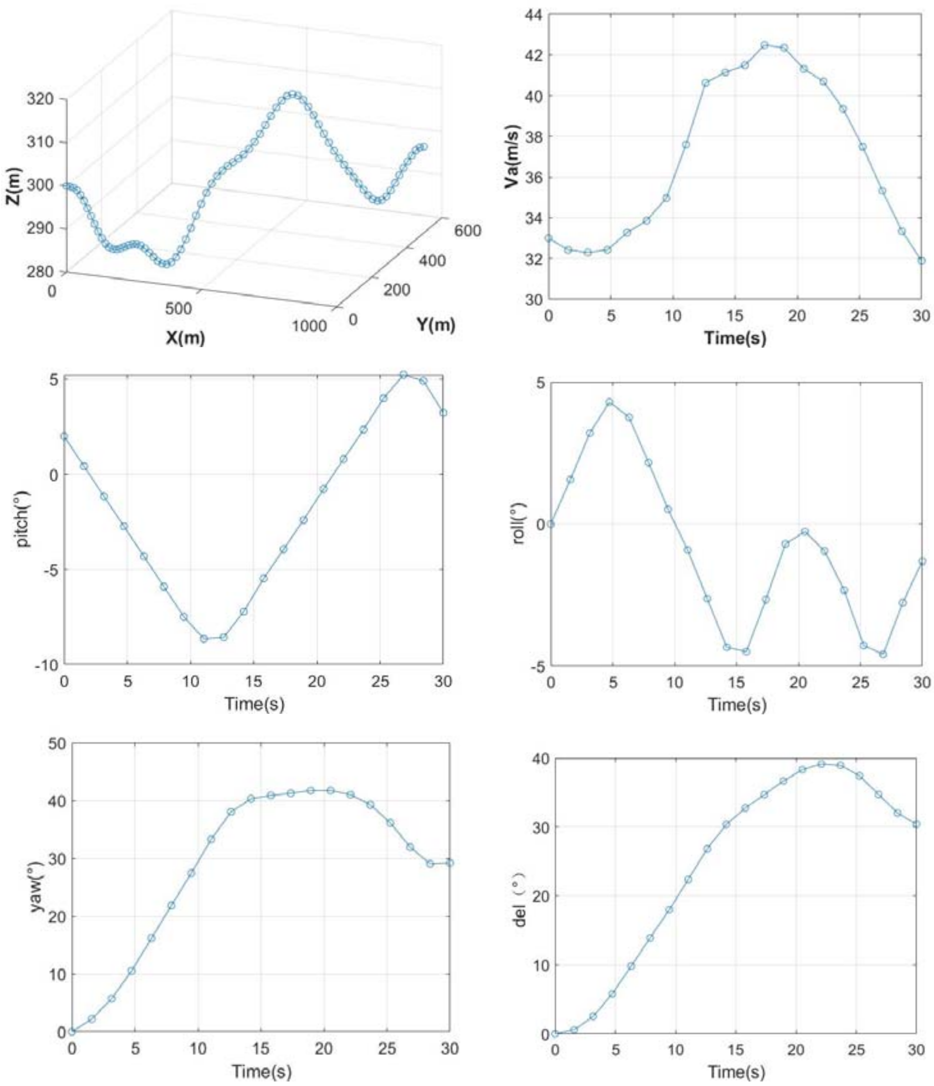


**Figure 2.** The preset trajectory

Where $\delta$ (del) is the angle between the terminal airspeed and the initial airspeed.
It can be seen from the diagrams that the resulting trajectory fits well with a set of constraints such as the initial state and terminal state.
After planning the course from the starting point to the landing point, the position and attitude angle obtained from the course planning simulation are input into the control program of the tilt-rotor aircraft as commands to control the tilt-rotor aircraft. The control simulation results are shown as follows:

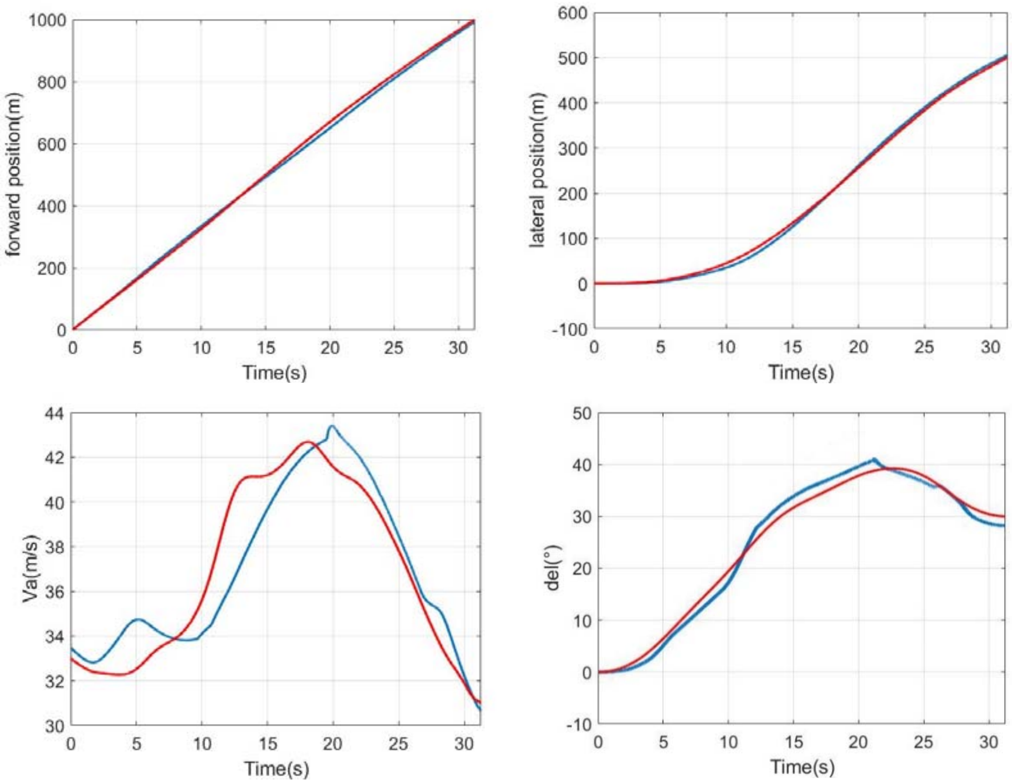


**Figure 3.** The control simulation result

Fig.3 shows that the forward position error is 0.74%, the lateral position error is 1.03%, and the velocity error is 1.09%, the heading error is 1.79°. It can be observed that the simulation results are good and the error is in line with the expectation. In order to verify the universality of this paper's flat trajectory, parameters are changed to perform multiple sets of simulation experiments, the simulation parameters and errors are shown as follows:

**Table 2.** Control simulation results

| Preset trajectory $[X_{pre}, Y_{pre}, Va_{pre}, \delta_{pre}]$ | Control trajectory $[X_c, Y_c, Va_c, \delta_c]$ | Error $[X_e, Y_e, Va_e, \delta_e]$ |
|---|---|---|
| $[1500m, 1500m, 33m/s, 45°]$ | $[1467m, 1497m, 33.31m/s, 44.47°]$ | $[2.20\%, 0.20\%, 0.93\%, 0.53°]$ |
| $[2000m, 2000m, 33m/s, 60°]$ | $[1966m, 1970m, 31.22m/s, 64.5°]$ | $[1.70\%, 1.50\%, 5.39\%, 4.50°]$ |
| $[1000m, 1500m, 30m/s, 65°]$ | $[983m, 1510m, 31.66m/s, 63.04°]$ | $[1.70\%, 0.67\%, 5.53\%, 1.94°]$ |

In Table.2, it can be seen that the trajectory optimization based on Gauss pseudospectral method is satisfactory and feasible.

## 5. Conclusion

In the study, Gauss pseudospectral method is used for reference, and we propose a method of trajectory optimization for the tilt-rotor aircraft. In this paper, the dynamic model during the tilt-rotor in the flight process is proposed, and relevant constraints are summed up, such as initial state constraints of tilt-rotor, terminal state constraints of tilt-rotor, the actuator constraints and the flight trajectory constraints during flight. Meanwhile, the objective function of trajectory optimization is set up by the time when the tilt-rotor arrived over the landing platform and total energy loss. Depending on the previous contents, a nonlinear programming problem is converted from the dynamic optimal control problem by using the Gauss pseudospectral method. In order to solving the nonlinear programming problem, MATLAB software is used to simulating the design trajectory, simulation results has proved the reliability and feasibility of tilt-rotor's flat trajectory optimization based on Gauss pseudospectral method.

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