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An Adaptive Searching Kriging Surrogate Model for Aerodynamic Optimization

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Abstract. The Kriging-based genetic algorithm applied to aerodynamic optimization design encounters a problem of unexpected sample size. In this paper, an adaptive method is proposed that the search space moves with the local optimum. The automatic division and hierarchical approximation of search space are realized by taking the selection of refinement samples into account. The typical function optimization and transonic supercritical airfoil drag reduction design are performed using this method. Results show that the number of samples required is greatly reduced, and the aerodynamic performance of the airfoil is efficiently improved.

1. Introduction

CFD (computational fluid dynamics) method has been widely used in aerodynamic simulation and optimization design of modern aircraft. Although the computer science and technology has developed rapidly, it still takes a long computing time. At present, the optimization methods [1] that can be combined with CFD are genetic algorithm, ant colony algorithm, particle swarm algorithm, etc. In the design process of applying these algorithms, it is necessary to calculate the design model iteratively. For complex flows, especially in the face of multi-objective and multi-design variables, the computational cost has increased dramatically and has become a difficult bottleneck. To solve the problem of large amount of numerical simulation, kriging surrogate model is introduced into the optimization design [2]. This statistical technique is used to construct approximate functional relations between variables and response functions.

Many people have already done the aerodynamic optimization design through the Kriging method with different characteristics. H.T. Wang [3] using the niching micro genetic algorithm to optimize the expected improvement function, and proposed an adaptive sequential optimization algorithm based on the Kriging agent model. H. Chung [4] using gradients to construct Cokriging approximation models, and reduced the computational cost needed for the common Kriging model to accurately capture multiple local extrema within a relatively large design. S.J. Leary [5] treating the results of a cheap model as knowledge to be incorporated in the training process, and developed a new knowledge-based Kriging model in multi-fidelity optimization. Compared with the work of this paper, their methods are not suitable for solve the disaster problem of samples number which caused by constructing surrogate model in high-dimensional space, and the accuracy of the optimization result is not high enough. Finding

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an adaptive method to reduce the number of samples required is valuable and challenging in optimization design.

Most of the time, we are more concerned with the accuracy of the optimized results than the global precision of the surrogate model. In this paper, the qualitative relationship between the number of samples, the dimension of design variables and the accuracy of fitting is statistically analyzed. Then an adaptive searching Kriging surrogate model is proposed that the search space moves with the local optimum, and combine it with genetic algorithm to achieve transonic supercritical airfoil aerodynamic optimization design. Finally, the optimization results are compared with the original RAE2822 airfoil and other algorithms.

2. Kriging Surrogate Model

The Kriging surrogate model [6, 7] expresses the unknown function y(x) as:

$$y(\mathbf{x}) = \mu + z(\mathbf{x}) \tag{1}$$

Where **x** is an m-dimensional vector, μ is a constant global model and $z(\mathbf{x})$ represents a local deviation from the global model. By using the specially weighted distance and the Gaussian random function, the correlation between the point \mathbf{x}^i and \mathbf{x}^j is expressed as

$$R(\mathbf{x}^{i}, \mathbf{x}^{j}) = \prod_{k=1}^{m} \exp\left(-\theta_{k} |x_{k}^{i} - x_{k}^{j}|^{2}\right)$$
(2)

Where θ_k is the k_{th} element of the correlation vector parameter $\boldsymbol{\theta}$. The Kriging predictor is

$$\hat{y}(\mathbf{x}) = \hat{\mu} + \mathbf{r}^{\mathrm{T}}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{y} - \mathbf{1}\hat{\mu})$$
(3)

Where **R** denotes the n×n matrix whose (i, j) entry is $R(\mathbf{x}^i, \mathbf{x}^j)$, **r** is vector whose i_{th} element is $R(\mathbf{x}, \mathbf{x}^j)$. $\hat{\mu}$ is the estimated value of μ , expressed as

$$\hat{\boldsymbol{\mu}} = \left(\mathbf{1}^{\mathrm{T}} \mathbf{R}^{-1} \mathbf{1}\right)^{-1} \mathbf{1}^{\mathrm{T}} \mathbf{R}^{-1} \mathbf{y}$$
(4)

The parameter θ can be estimated by maximizing the following likelihood function

$$\ln(Likelihood) = -\frac{1}{2} (n \ln \hat{\sigma}^2 + \ln |\mathbf{R}|)$$
(5)

Where

$$\hat{\sigma}^2 = \frac{1}{n} (\mathbf{y} - \mathbf{1}\hat{\mu})^{\mathrm{T}} \mathbf{R}^{-1} (\mathbf{y} - \mathbf{1}\hat{\mu})$$
(6)

The surrogate model is established based on the sample point data, a reasonable sample selection method is of great significance to improve design efficiency. Latin Hypercube Sampling method is adopted in this paper due to its fast design speed and uniform distribution [8]. The number of samples N is usually related to the dimension of the design variables m and the accuracy of the errors ε' required. Take the following test function, for example, to get the statistical relationship between N and m, ε' , as shown in Fig. 1.

$$f(x) = \frac{1}{m} \sum_{i=1}^{m} \left(\sum_{j=1}^{i} \frac{x_j}{i}\right)^2, \qquad |x_i| \le 10, i = 1, 2, ..., m$$
(7)

 ε' is expressed as

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$$\mathcal{E}' = \left(\sqrt{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2 / n}\right) / (y_{\max} - y_{\min}) \times 100\%$$
(8)

Where extra *n* sample points for testing model accuracy (n=1000m). we can see that the higher the dimension and the higher the precision, the total number of samples required will increase exponentially.



Figure 1. The relationship between N and m, ε'

3. Adaptive Searching Method

Among the optimization methods for solving nonlinear problems, the trust region algorithm has become a popular choice due to its characteristics of region search and good overall convergence. The main idea of trust region is to use an approximate model to fit the objective function in a neighborhood centered on the current iteration point, and obtain the optimal point of the approximate model in the current neighborhood through sub-optimization, and use this optimum as the center of the next iteration, redetermine the search neighborhood, and iterate to convergence.

Based on the idea of trust region update, an adaptive searching Kriging surrogate model is proposed in this paper. This efficient optimization method is suitable for high-dimensional design space which combined with genetic algorithm. During the optimization process, the refinement samples are selected to build Kriging model at different stages, so the automatic division and hierarchical approximation of search space can be realized. This method aim to abandon the pursuit of high fitting accuracy of the global space covered by a large number of samples and turn to search for the optimal solution space. Samples are encrypted to improve the accuracy of the search neighborhood, and each iterative search is carried out in a small interval, so that the total size of the samples can be effectively controlled and the global optimum with higher accuracy can be found. As shown in Fig. 2, the specific process of this method is as follows:

(1) Latin hypercube sampling for global variables space, constructing the initial Kriging surrogate model, using the GA algorithm to seek the global optimum;

(2) Take the global optimum as the center, and draw a small local search area, sampling in this area and adding the optimum found in the previous step, re-constructing the Kriging surrogate model, using the GA algorithm to seek the local optimum;

(3) Take the local optimum found in the previous step as the center, and draw a small local search area, re-seeking the local optimum in the way using in step (2);

(4) Repeat step (3) until the local optimum is no longer moving;

(5) Expand the search space, re-seeking the optimum in the way using in step (2). If the local optimum has changed, repeat step (2-5); If not, turn to the next step;

(6) Reduce the search space and sampling in this area, then the more accurate optimum can be obtained finally.

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Figure 2. Adaptive searching method

For verifying accuracy and efficiency of the adaptive searching method, two kinds of Kriging surrogate model combined with genetic algorithm are established based on different number of samples. Let the dimension of the variables m = 10 in the above test function (equation 7 in section 2), then the global minimum is f(0, 0, ..., 0)=0. In the process of adaptive Kriging Surrogate model, the number of initial sample points is 500, and the number of samples added adaptively is 400. In order to illustrate the validity of this method, 1000-3000 samples are used to construct a common Kriging surrogate model additionally, and the comparison of the optimization results is shown in Table 1. The drift rate λ is defined as the percentage of the optimal value found by the surrogate model that deviates from the theoretical optimum, expressed as:

$$\lambda = \sqrt{\sum_{i=1}^{m} \left(\frac{x_i - x_i^{opt}}{L_i}\right) / m \times 100\%}$$
(9)

Where x^{opt} is theoretical optimum, L_i is the interval length of the *i* dimension variable.

	-	_		
Model	Sample size	Drift rate λ	Prediction	Real value
Theoretical value	_	0%		0.0
Kriging	1000	3.749%	-1.8e-2	3.6e-02
Kriging	2000	0.888%	-3.6e-3	2.1e-04
Kriging	3000	0.713%	-7.3e-3	1.2e-04
Adaptive Kriging	900	0.004%	0.0	1.2e-05

Table 1. Optimization results comparison of 10D test function.

We can see that the optimum of the adaptive Kriging surrogate model predicted is almost equal to the theoretical value, and its result is far more accurate than the result of the common Kriging surrogate model. More importantly, this adaptive method saves a great amount of samples, and the higher the variable dimension is, the more samples are saved. This proves the great superiority of the adaptive Kriging surrogate model in this paper.

4. Application in Aerodynamic Optimization Design

The adaptive Kriging surrogate model method is used to solve the airfoil aerodynamic optimization problem in this section, for the sake of study its application value.

4.1. CST parameterized method

A suitable airfoil parameterization method plays an important role in the search efficiency of the optimization algorithm. At present, the commonly used parameterized methods include CST, Bezier curve and Hicks-Henne parameterization [9]. The CST method used in this paper is easy to use, intuitive and controllable, it's suitable for solving the problem of aerodynamic optimization combined with optimization algorithm [10, 11].

As presented in Fig. 3, a round nose and a sharp trailing edge is adopted. The design parameters are taken as the leading edge radius R_{le} , the trailing edge coordinate $\Delta z_{te}/c$, the airfoil boat-tail angle of upper and lower surfaces $\beta(\beta')$, shape function control parameters of upper and lower surfaces $b_i(b_i')$ ($i=1,2,\dots,n-1$).



Figure 3. Airfoil geometry control parameters

The coordinates for an airfoil shape can be easily obtained from a known analytic function as:

$$\frac{z}{c} = C(\frac{x}{c})S(\frac{x}{c}) + \frac{x}{c} \cdot \frac{\Delta z_{te}}{c}$$
(10)

The term C(x/c) will be called the "class function", and is defined in the general form as:

$$C(\frac{x}{c}) = (\frac{x}{c})^{N_1} (1 - \frac{x}{c})^{N_2}, 0 \le \frac{x}{c} \le 1$$
(11)

Where $N_1 = 0.5$ and $N_2 = 1.0$. It will be subsequently shown that the exponents of the class function define basic general classes of geometric shapes. The shape function S(x/c) expressed as:

$$S(\frac{x}{c}) = \sum_{i=0}^{n} \left[b_i \cdot \frac{n!}{i!(n-i)!} \cdot (\frac{x}{c})^i \cdot (1-\frac{x}{c})^{n-i} \right]$$
(12)

Where $b_0 = S(0) = \sqrt{2R_{le} / c}, b_n = S(1) = \tan \beta + \Delta z_{te} / c$.

In summary, the general and necessary form of the mathematical expression that represents the airfoil geometry is:

$$\frac{z}{c} = \sqrt{\frac{x}{c}} \left(1 - \frac{x}{c}\right) \left\{ \sqrt{\frac{2R_{le}}{c}} \left(1 - \frac{x}{c}\right)^n + (\tan\beta + \frac{\Delta z_{le}}{c}) \left(\frac{x}{c}\right)^n + \sum_{i=1}^{n-1} \left[b_i \cdot \frac{n!}{i!(n-i)!} \cdot \left(\frac{x}{c}\right)^i \cdot \left(1 - \frac{x}{c}\right)^{n-i} \right] \right\} + \frac{x}{c} \cdot \frac{\Delta z_{le}}{c}$$
(13)

4.2. Transonic Supercritical Airfoil Optimization

Taking the RAE2822 supercritical airfoil as the initial airfoil, the optimization design of its drag reduction is carried out. The design state is:

Ma =
$$0.73, \alpha = 2.31^{\circ}, Re = 1.7 \times 10^{7}$$

Min C_D

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S.t.
$$C_L \approx C_{L0}, S \ge S_0$$

The lift coefficient of the reference airfoil is $C_{L0}=0.6966$, the drag coefficient is $C_{D0}=0.01368$, the area is $S_0=0.0778m^2$. The optimization objective is to minimize the airfoil drag coefficients, constraints for the lift coefficient remained essentially unchanged, and the airfoil area is not reduced. Reynolds averaged Navier-Stokes equations are used to calculate the flow field around airfoil. The turbulence model is S-A, the structured mesh is adopted (see Fig. 4), and the number of grids is 73644. The design parameters and the corresponding constraint ranges are shown in Table 2.



Figure 4. Airfoil grid

Table 2. The design parameters of airfoil

Variable	Range	Variable	Range
R_{le}	[0.004,0.012]	b_2	[0.10,0.30]
β (rad)	[0.14,0.28]	b_3	[0.10,0.30]
β' (rad)	[0.00,0.14]	b_1'	[0.05,0.30]
$\triangle z_{te}/c$	[-0.005,0.005]	b_{2}'	[0.05,0.30]
b_1	[0.05,0.20]	b_{3}'	[0.10,0.25]

In order to compare aerodynamic optimization results, three optimization methods are used to airfoil design combined with genetic algorithm: Kriging surrogate model, Adaptive Kriging surrogate model, and High Precision CFD model. The optimization results are shown in Fig. 5~7 and Table 3, and the optimized airfoils with smaller drag coefficients than the original airfoils can be obtained by three methods.

Table 3. Optimization results

Model	S	C_L	C_D
Initial airfoil	0.0778	0.6966	0.01368
Kriging	0.0779	0.6937	0.01251
Adaptive Kriging	0.0794	0.6874	0.01200
High Precision	0.0788	0.6966	0.01210

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Figure 6. Optimized airfoil pressure distribution



Figure 7. Comparison of pressure contours

As presented in Fig. 5, in geometry shape, the upper wing becomes flatter, the lower wing becomes thicker, and concave increases in the latter half. As presented in Fig. 6~7, on the aerodynamic characteristics, the pressure distribution of the airfoils optimized by adaptive Kriging surrogate model and high precision CFD model are more placid than the initial RAE2822 airfoil, the shock wave in upper surface almost disappear completely, which shows that the decrease of drag force of the airfoil mainly comes from the decrease of the shock wave drag. As presented in Table 3, the drag coefficient of the airfoil optimized by adaptive Kriging surrogate model is reduced by 8.53%, the drag coefficient of the airfoil optimized by adaptive Kriging surrogate model is reduced by 12.24%, and the drag coefficient of the airfoil optimized by high precision CFD model is reduced by 11.5%. The flow field of 3225 individuals was calculated by using a high precision model to optimize airfoils, meanwhile only 350 individuals were used in the surrogate model, which greatly reduced the time of aerodynamic optimization design.

The results above show that the adaptive Kriging surrogate model developed in this paper has better performance than the common Kriging surrogate model, and is close to the effect of High Precision CFD model.

5. Conclusion

In this paper, an adaptive searching Kriging surrogate model is studied in detail, and the typical function is used to test the established model. The results show that the accuracy of the adaptive surrogate model is better than that of the common surrogate model under the same number of samples. In ensuring the same result accuracy, the workload of establishing surrogate model can be effectively reduced by using this adaptive method. The results of airfoil optimization example show that the adaptive searching method can predict aerodynamic performance better and has certain practical significance for aerodynamic optimization design. Due to the complexity of the aircraft aerodynamic optimization problem, the generalization ability and prediction accuracy of adaptive Kriging surrogate model still need to be improved, and the development of more rapid algorithm, which will become the main direction to improve the efficiency of optimization design.

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