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To cite this article: P Ravindra et al 2020 J. Phys.: Conf. Ser. 1597 012040

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Natural convection in an inclined parallelogrammic enclosure: Effect non-uniform heating

1597 (2020) 012040

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Abstract. In this paper, numerical simulations are presented to understand the influence of inclination angle of a tilted parallelogrammic enclosure. The upper and lower boundaries of the enclosure are treated as thermally insulated; right wall is uniform cooled while the left wall is non-uniformly heated with either sinusoidal or linear thermal profiles. Using Darcy law, the momentum equations are modeled and unsteady energy equation is considered. The model equations are numerically solved using finite difference method, in particular, using ADI and SLOR methods. Based on standard coordinate transformations, the governing equations are transformed to rectangular shaped enclosure computational domain. In this analysis, due to large number parameters, the inclination angle of the enclosure is fixed. The flow and thermal processes are depicted through streamlines and isotherms, while the thermal transport rates are illustrated through Nusselt number profiles. Numerical simulations capture the effects of side wall inclination angle and non-uniform thermal conditions on the flow and thermal patterns, heat transport rates for various parametric ranges of the problem.

1. Introduction

The phenomenon of buoyant convection is frequently occurred in many industrial applications, particularly for equipment kept in finite sized geometries. Among finite shaped geometries, the square or annular enclosure with differentially heated vertical walls and insulated horizontal walls is a standard and widely used model problem and hence many investigations are carried out in these enclosures [1, 2, 3, 4, 5, 6, 7, 8]. In such enclosures, porosity is used for thermal insulation and other important purposes. The detailed literature concerning the investigations on porous media are reported in the monographs or books by Nield and Bejan [9], Vafai [10] and Pop and Ingham [11].

In many heat transfer applications, the physical configuration of equipment may not be of regular shape and one or more geometrical parameters in such non-regular shaped enclosure significantly influence the convective flow and associated thermal transport processes. The buoyant flows in finite geometries are highly sensitive to the geometry of enclosure. Due to the requirement of variety of applications in different fields, investigations on natural convection heat transfer in finite non-rectangular geometries with and without porosity have increased to a large extent. In the midst of many finite enclosures, the parallelogram-shaped geometry is found to involve in many applications. As a result, buoyancy-driven convection in this enclosure has

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attracted many researchers by considering various boundary conditions and other constraints. Seki et al. [12] made a pioneering attempt to experimentally analyze the buoyancy-driven convection in a parallelogrammic enclosure by considering detailed set of parameters. Later Asako and Nakamura [13] extended the analysis by considering radiation and finite thickness of a bounding-wall. Hyun and Choi [14] performed numerical simulations of free convection in a parallelogrammic geometry for vast range of Ra numbers and inclination angles of side-walls of the geometry and predicted heat transfer correlations. Later, Baytas and Pop [15] considered an oblique shaped geometry (similar to parallelogram-shaped) and discussed the flow and thermal features using the Darcy model. In an upright arrangement of parallelogram-shaped partial enclosures, Costa et al. [16] numerically investigated free convection for various inclination angles, aspect ratios and Rayleigh numbers. Han and Hyun [17] developed a generalized model to predict the influence of porosity on buoyancy-driven convection in parallelogram-shaped geometry with insulated tilt walls and isothermal lateral boundaries.

Bairi and co-workers [18, 19, 20, 21] performed a detailed numerical and experimental analysis of buoyant convection in parallelogram-shaped geometries for different set of physical and geometrical conditions. They found a good agreement of their numerical and experimental predictions. Also, they found that the thermal transport for a discretely heated geometry is relatively higher as compared to uniformly heated wall. The predictions based on their investigations can be utilized for the design of thermal diodes and also developed a correlation to determine the heat transport rates for different set of parameters. The flow and thermal analysis of nanofluids in parallelogram-type of enclosure has also been examined in the literature [22, 23]. Recently, Jang et al. [24] performed full blown numerical simulations on convective flows and transport rates in a tilted parallelogrammic cavity by utilizing the Darcy model. They predicted an optimum tilt angle of the side walls and the enclosure as well at which either maximum or minimum heat transport can be achieved. Buoyant convection due to simultaneous presence of thermal and solutal buoyancies, known as double-diffusive or thermosolutal convection, in porous and non-porous parallelogrammic geometries is also received considerable attention among the researchers. Costa [25, 26] examined thermosolutal convection in a parallelogrammic geometry filled with clear fluid as well as fluid saturated porous materials. For porous case, the use of Darcy's law has been utilized. Jagadeesha et al. [27] numerically analyzed the impacts of inclination angles of enclosure and side walls on the thermosolutal convection in parallelogramtype geometry. Later, Jagadeesha et al. [27] extended their previous work to understand the impact of an internally generated thermal source on thermosolutal convection.

Many theoretical and experimental investigations are found to identify natural convection in tilted geometries mainly because of its direct relevance in many practical applications. The tilting angle of the geometry is a prime parameter to significantly control the flow structure and thermal performances in the enclosure [29, 30, 31, 32, 33]. Based on the wide literature survey, it is found that the influence of non-uniform thermal conditions on buoyant convection in a tilted parallelogrammic enclosure is not attempted in the literature. Hence, in this analysis, we performed numerical simulations to understand two different non-uniform thermal profiles along the bottom wall of the parallelogram-shaped geometry by considering vast range of physical and geometrical parameters.

2. Mathematical Formulation

Figure 1 portrays the physical geometry of the problem considered in the present analysis, a parallelogram-shaped enclosure containing fluid saturated porous material. The side walls are inclined to y- axis with an angle ϕ and are kept at different temperatures. The left tilted surface is non-uniformly heated with two different thermal profiles, while the right boundary is at a lower uniform temperature. The lower and upper surfaces of the geometry are taken as thermally insulated. Here, we consider two thermal profiles in the left inclined boundary; Case

1597 (2020) 012040 doi:10.1088/1742-6596/1597/1/012040

(I) sinusoidal heating and Case (II) linear heating. The Newtonian fluid with negligible viscous dissipation is chosen for the analysis, thermophysical characteristics of the fluid remain constant. However, density variation with temperature is considered through Boussinesq approximation. In the porous medium, the Darcy model is used with local thermal equilibrium condition. Using these assumptions, the governing equations in the dimensionless form are:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{gK\beta}{\nu} \left(\frac{\partial T}{\partial y} \sin\alpha - \frac{\partial T}{\partial x} \cos\alpha \right), \tag{1}$$

$$\sigma \frac{\partial T}{\partial t^*} + \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}.$$
 (2)

The stream function ψ is chosen as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$. In this study, the non-rectangular



Figure 1. Physical configuration, coordinate system and boundary conditions

physical geometry is transformed to rectangular computational geometry through the axis transformations used by [15]. The transformation used in this analysis is given below:

$$X = x - y tan\phi, \quad Y = y \tag{3}$$

The non-dimensional variables used in this study are

$$\xi = \frac{X}{L}, \quad \eta = \frac{Y}{H \cos \phi}, \quad t = \frac{t^* \alpha_m}{\sigma L H \cos \phi}, \quad \Psi = \frac{\psi}{\alpha_m}, \quad \theta = \frac{(T - T_r)}{T_h - T_c},$$
where $T_r = \frac{T_h + T_c}{2}$ is the reference temperature.

1597 (2020) 012040 doi:10.1088/1742-6596/1597/1/012040

Using the above non-dimensional variables and transformations (3), the model equations for the present study are:

$$\frac{\partial^2 \Psi}{\partial \xi^2} - 2 \frac{\sin\phi}{A} \frac{\partial^2 \Psi}{\partial \xi \partial \eta} + \frac{1}{A^2} \frac{\partial^2 \Psi}{\partial \eta^2} = Ra \cos\phi \left[\frac{\sin\alpha}{A} \frac{\partial\theta}{\partial \eta} - \cos(\phi - \alpha) \frac{\partial\theta}{\partial \xi} \right]$$
(4)

$$\frac{\partial\theta}{\partial t} + \frac{\partial\Psi}{\partial\eta}\frac{\partial\theta}{\partial\xi} - \frac{\partial\Psi}{\partial\xi}\frac{\partial\theta}{\partial\eta} = \frac{A}{\cos\phi}\left[\frac{\partial^2\theta}{\partial\xi^2} - 2\frac{\sin\phi}{A}\frac{\partial^2\theta}{\partial\xi\partial\eta} + \frac{1}{A^2}\frac{\partial^2\theta}{\partial\eta^2}\right].$$
 (5)

The non-dimensional boundary conditions are:

$$\begin{array}{rcl} t=0: & \theta & = & 0, \ \Psi=0; \ {\rm at} \ 0 \leq \xi \leq 1 \ {\rm and} \ 0 \leq \eta \leq A. \\ t>0: & \Psi & = & \frac{\partial \Psi}{\partial \xi}=0, \ \theta=\sin(\pi\eta) \ {\rm or} \ \ \theta=(1-\eta) \ {\rm at} \ \ \xi=0 \\ \\ \Psi & = & \frac{\partial \Psi}{\partial \xi}=0, \ \theta=0 \ {\rm at} \ \ \xi=1 \\ \\ \Psi & = & \frac{\partial \Psi}{\partial \eta}=0, \ \frac{\partial \theta}{\partial \eta}=0 \ {\rm at} \ \ \eta=0 \ {\rm and} \ \ \eta=A \end{array}$$

The thermal transport rate from the inclined wall is an important quantity in the design of heat transfer equipment. Thus, the local Nusselt number is defined as $Nu = \frac{hl}{k} = \frac{q_h L}{k(\theta_h - \theta_c)}$ along the left inclined boundary and this gives $Nu = -\frac{1}{\cos\phi} \left[\frac{\sin\phi}{A} \frac{\partial\theta}{\partial\eta} - \frac{\partial\theta}{\partial\xi} \right]$. The thermal transport rate across the complete het will be used.

The thermal transport rate across the complete hot wall is measured by the average Nusselt number and is given by

$$\overline{Nu} = \frac{1}{A} \int_0^A Nu \, d\eta. \tag{6}$$

3. Numerical Methodology

The model partial differential equations are non-linear and coupled, as a result analytical solutions for the present problem is not possible. Hence, the finite difference technique consisting of ADI and SLOR methods are used to solve the governing equations. The ADI method is used for unsteady energy equation while SLOR method is preferred to solve stream function form of momentum equation. The spatial and mixed derivatives, appearing due to coordinate transformations, are discretized using second order finite difference approximations. Thomas algorithm technique is adopted to solve the tridiagonal form of the discretized finite difference equations. A detailed grid independence tests are performed and after careful scrutiny of the results, we have chosen 101×101 grids for all simulations. Also, our results are validated with standard benchmark solutions for a parallelogrammic enclosure in the limiting case of $\alpha = 0$ and found an excellent agreement. The comparison of present streamlines and isotherms with Baytas and Pop [15] is portrayed in Fig.2.

4. Results and Discussion

This section illustrates the effects of various controlling parameters on the flow and thermal distributions in the parallelogram-shaped geometry. As there are more parameters involved in this analysis, the tilting angle of enclosure is fixed at $\alpha = 30^{\circ}$. Figure 3 shows the variation in flow and thermal pattern with respect to three different values of Ra. For $Ra = 10^2$, the thermal transport is due to conduction mechanism and a simple recirculating flow with smaller magnitude is observed in the enclosure. As the Rayleigh number value is raised, convective flow

0.8

0.6

0.4

0.2

0.0

0.8

0.6

0.4

0.2

0.0

-0.1

0.1

0.3

0.5

0.7

0.9



Figure 2. Comparison of streamlines and isotherms between the present study (left) and Baytas and Pop [15] (Right) for $Ra = 10^3$, $\phi = -15$, A = 1 and $\alpha = 0$

-0.2 0.0 0.2 0.4 0.6 0.8 1.0

over powers conduction and a strong convective flow is present for higher value of $Ra = 10^3$. Due to sinusoidal heating at left-side wall, bi-cellular flow pattern exists in the geometry.

An important geometrical parameter in parallelogrammic geometry to control the flow regimes and thermal transport is the inclination angle of side-wall and its impact on flow and thermal structure is exhibited in Fig. 4 for three different values of ϕ and for Case (I) (sinusoidal) heating. For negative and positive inclination angles ($\phi = -30^{\circ} \& \phi = 30^{\circ}$), the streamlines and isotherms occupy the entire enclosure and thermal stratification is also noticed inside the cavity. However, for $\phi = 0^{\circ}$, which represents a square geometry, a stagnant thermal region is observed from isotherms and slow flow circulation is noticed near bottom region. This is attributed due to the interaction two buoyancies when the thermally active walls are inclined. Figure 5 reveals the importance of two different non-uniform thermal heating on flow and thermal distributions for fixed values of Ra, α and ϕ . As regards to flow structure, sinusoidal heating produces bi-cellular flow (bottom) whereas linear heating causes a simple unicellular flow (top). Isothermal contour pattern also confirm the flow structure and thermal stratification as well as thermal boundary layer can be seen inside the geometry.

The quantified interest in all thermal transport analysis is the overall heat transport from heated wall to working fluid inside the geometry. Figure 6 illustrates the variation of global heat transport for different values of Ra and tilt angles of heated side-wall. To understand the impacts of Ra, three values are chosen that covers conduction to convection-dominated flow regimes. Also, the tilt angles of the thermally active walls are chosen that covers possible variation from negative to positive inclination angles. The simulation results show that thermal transport from heated side-wall enhances with the Rayleigh number for all tilt angles ϕ . However,

IOP Publishing **1597** (2020) 012040 doi:10.1088/1742-6596/1597/1/012040

interesting variations are observed with tilt angle. The average Nu has two different profiles depending on the magnitude of Ra and type of heating. For linear heating case and lower value of Ra, higher heat transfer is observed for $\phi = -45^{\circ}$ and decreases up to $\phi = 0^{\circ}$, but raises again with tilt angle. On contrast, for sinusoidal heating, thermal transport decreases as the tilt angle ϕ increases from -45° to $+45^{\circ}$. For higher Rayleigh number $Ra = 10^3$, variation of \overline{Nu} is different from lower value of Ra. It can be seen that overall Nu reveals a parabolic profile over the range of ϕ values considered in the analysis. From this, it can be predicted that it is possible to achieve either minimum or maximum thermal transport by properly choosing the tile angle ϕ .



Figure 3. Streamlines and isotherms for different Rayleigh numbers and $\alpha = 30^0$ and $\phi = -45^0$.

1597 (2020) 012040 doi:10.1088/1742-6596/1597/1/012040



Figure 4. Streamlines and isotherms for different wall inclinations (ϕ) and $\alpha = 30^0$ and $Ra = 10^3$.

5. Conclusions

This investigation aims to understand the impacts of two different non-uniform thermal boundary conditions on the buoyant flow and associated thermal characteristics in a

1597 (2020) 012040 doi:10.1088/1742-6596/1597/1/012040



Figure 5. Influence of thermal boundary conditions on streamlines and isotherms at $\alpha = 30^{0}$, $Ra = 10^{3}$ and $\phi = 45^{0}$.

parallelogram-shaped geometry. Based on the detailed simulations performed over considerable ranges of Rayleigh number and tilt angles of side-wall, the following conclusions are predicted:

- 1 Sinusoidal heating initiates bi-cellular flow pattern as compared to linear heating condition.
- 2 Flow and thermal structure is strongly influenced by the tilt angles.
- 3 Choice of heating has greater impact on the variation of global thermal transport from the hot wall.
- 4 Higher thermal transfer rate is found for linear heating rather than sinusoidal heating and hence linear heating is suggested for the purpose of better heat removal from the hot wall.

Acknowledgments

The authors are grateful to the respective Managements for their support and encouragement. Also, M. Sankar acknowledges the financial support from the Vision Group of Science and Technology, Government of Karnataka under Grant Number KSTePS/VGST-KFIST (L1)/2017.



Figure 6. Influence of wall inclination on average \overline{Nu} for different Rayleigh numbers. Case (I) heating (Top) and Case (II) heating (Bottom)

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ICNTMMA-2019

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