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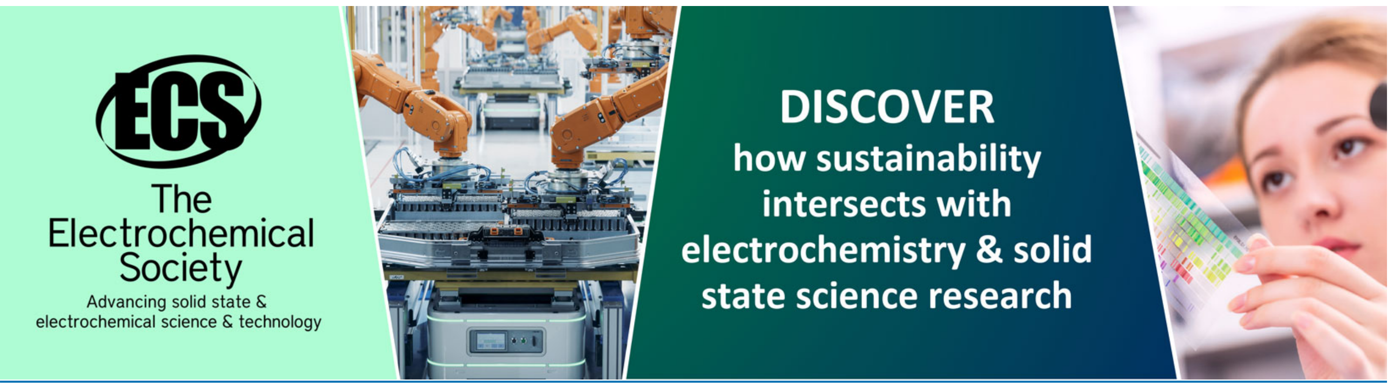

Application of Fractional Order PI Control in Single-phase Active Power Factor Corrector

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Abstract. Single-phase active power factor corrector (APFC) can eliminate harmonic current pollution to the grid harmonic current. APFC uses a double closed-loop control structure with a voltage outer loop and a current inner loop. Although this control method is very simple, the dynamic response characteristics need to be improved. After analyzing the definition of fractional PID and the design process of fractional PI controller parameter, the fractional-order operator s is calculated by using the Oustaloup approximation algorithm, so that a double closed-loop variable coefficient PI controller with a voltage outer loop and a current inner loop are designed in APFC to adopt fractional PI control. The pure resistive load is taken as an example for simulation analysis and compared with the three conditions of the traditional PI controller. The results demonstrate that the APFC can obtain good dynamic and static characteristics by adopting double closed-loop fractional PI control with proper order.

Key words. active power factor corrector; fractional PID; dynamic characteristics; static characteristics

1. Introduction

Single-phase Active Power Factor Correction (APFC) has been widely used in many fields, such as frequency conversion air conditioning, communication power supply and wireless transmission, because it can obtain input unit power factor, eliminate load nonlinearity and prevent harmonic pollution of power grid[1-2]. APFC adopts double closed-loop control structure of voltage outer loop and current inner loop. Voltage outer loop is used to control output voltage and voltage error amplifier is commonly used. The inner current loop is used to control the inductance current. Traditional PI, PR or quasi PR regulators are often used. For power electronic converters, the control object is often time-varying and non-linear, so the traditional PID performance is not particularly satisfactory. Some scholars believe that "the actual system is usually fractional order"[3-10]. By combining fractional order control with PID control theory, better control effect can be obtained. A fractional order PI control structure is introduced and designed. Fractional-order PID controller was first proposed by Professor I. Podlubny in 1999 and was labelled as $PI^{\lambda}D^{\mu}$ [11]. In recent years, many scholars have turned to the relevant characteristics of fractional order to select appropriate λ, μ and other parameters, so that to make the fractional-order PID control applicable to nonlinear systems, including power electronic converters. A lot of related literatures have supported this point. For examples, a PI^{λ} controller was designed in [12], a method for designing a fractional-order PID system was proposed in [13], an optimization algorithm was proposed in the literature [14] to adjust the $PI^{\lambda}D^{\mu}$ controller, a fractional-order PID control method for LCL-type grid-connected inverter systems was proposed in the literature [15], and so on. The significance of fractional-order control is just to establish a more accurate fractional-order model for the actual system than the integer-order model, and to improve the robustness of the control system while obtaining better dynamic response. The theoretical analysis and simulation verification of this novel single-phase AC-DC converter are carried out in this paper. The compensator is designed using fractional-order PI controller to obtain satisfactory control effect.

2. Theory of Fractional Calculus

2.1. Fractional Calculus Definition

Fractional calculus is an arbitrary order calculus and is an extension of integer-order calculus operations. Although fractional calculus has a history of more than 300 years, it is mainly limited to theoretical analysis. With the rapid development of computer science in recent decades, fractional calculus has been gradually expanded from theoretical research to practical application.
The basic operator of fractional calculus is ${}_aD_t^{\alpha}$, where a and t are the upper and lower limits of the operator, respectively, and α is the calculus order [15], ${}_aD_t^{\alpha}$ has the following plural form.

$$ {}_aD_t^{\alpha} = \begin{cases} \dfrac{\mathrm{d}^{\alpha}}{\mathrm{d}t^{\alpha}}, & R(\alpha)>0 \\ 1, & R(\alpha)=0 \\ \int_a^t (\mathrm{d}\tau)^{(-\alpha)}, & R(\alpha)<0 \end{cases} \tag{1} $$

The existing several common fractional calculus definitions include the Grünwald-Letnikov (GL) definition, the Riemann-Liouville (RL) definition, and the Caputo definition.

GL definition is

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{\Gamma(\alpha)h^{\alpha}} \sum_{k=0}^{(t-a)/h} \frac{\Gamma(k+\alpha)}{\Gamma(k+1)} f(t-kh) \tag{2} $$

where $\Gamma(.)$ is Euler Gamma function.
RL definition is an improved version of GL definition and is the most commonly used fractional calculus definition.

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} (\mathrm{d}\tau) \tag{3} $$

where m-1<α<m.
Caputo definition is another improved form of GL definition, which simplifies the Laplace transformation and is conducive to the solution of fractional differential equations

$$ {}_a^CD_t^{v} s(t) = \frac{1}{\Gamma(n-v)} \int_a^t (t-\tau)^{n-v-1} s^{(n)}(\tau)(\mathrm{d}\tau) \tag{4} $$

where $0 \le n-1 < v < n$, $n \in R$.
Because of the introduction of the ${}_aD_t^{a}$, the three definitions can unify the integral and the differential.

2.2. Design of Fractional Calculus Controller

The integer-order PID controller is a special situation of the fractional-order PID controller, where λ=1 and μ=1. In Figure 1, the parametric planar graphs of $0 \le \lambda \le 1$ and $0 \le \mu \le 1$ are given. Four vertices represent integer-order PID and shaded areas represent fractional-order PID. That means, fractional order PID controller is an extension of integer order PID controller from point to surface in real field.

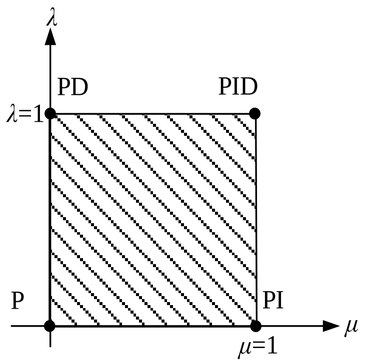

Figure 1. P-I-D parameter plane.

From the perspective of root trajectory, the existence of the D element is equivalent to the introduction of a new zero point, which makes the pole of the system move to the left half plane. And the stability margin is larger, which can overcome the phenomenon of capacity lag. For some self-unstable systems, it is necessary to introduce D element to cooperate with the PI element, and shift the closed-loop Characteristic root to the left half-plane by introducing a new zero. From the perspective of time domain response, D element can limit overshoot, improve dynamic performance, and restrain external disturbances.

However, there are some problems in the D link: (1) From the perspective of frequency response, the gain of the differential element become larger due to the increase of the frequency, and the D element is very sensitive to the measurement noise; (2) The ideal differential element transfer function is not a rational function and is physically difficult to implement. If the differential element is implemented with a digital controller, high-precision sensors are required to perform high-frequency sampling on the controlled signal, otherwise the quantization error will have a serious negative impact. In the existing power electronic converter control, the differential element is very sensitive to the noise of the input controlled signal, it is easy to cause the whole system to be interfered and the controlled variable to be oscillated. Thus, the PI controller with fast response and without static error is adopted [16].

The transfer function of the integer-order PI controller is

$$G_c(s)=\frac{U(s)}{E(s)}=K_P+\frac{K_I}{s} \tag{5}$$

where $U(s)$ is the output of the controller; $E(s)$ is the input error of the controller.

The transfer function of the fractional-order PI controller is

$$G_{cf}(s)=\frac{U(s)}{E(s)}=K_P+\frac{K_I}{s^{\lambda}},\lambda\in R \tag{6}$$

For controlled objects in an actual system, the various parameters of the fractional-order PI controller are typically determined on the expected amplitude margin A_m and phase margin ϕ_m.

Considering the stability and robustness requirements, the dynamically controlled object $G_p(s)$, the controller $G_c(s)$, and the open-loop transfer function $G(s)$ should satisfy the following conditions [17]

$$Arg[G(j\omega_g)]=Arg[G_c(j\omega_g)G_p(j\omega_g)]=\phi_m-\pi \tag{7}$$

$$A_m=\frac{1}{\left|G(j\omega_p)\right|}=\frac{1}{\left|G_c(j\omega_p)G_p(j\omega_p)\right|} \tag{8}$$

where $G_p(s)$ is the transfer functions of the controlled object and $G_c(s)$ is the transfer functions of controller.

ω_g is the cut-off frequency and satisfies:

$$\left|G_c(j\omega_g)G_p(j\omega_g)\right|=1 \tag{9}$$

ω_p needs to satisfies:

$$Arg[G_c(j\omega_p)G_p(j\omega_p)]=-\pi \tag{10}$$

The following equations can be obtained[18],

$$-\frac{1}{A_m G_p(j\omega_p)}=G_c(j\omega_p) \tag{11}$$

$$-\frac{\cos\phi_m+j\sin\phi_m}{G_p(j\omega_g)}=G_c(j\omega_g) \tag{12}$$

When $G_c(s)$ is represented by the transfer function in formula of the fractional-order PI controller, and employing the frequency domain analysis method, the following equations can be obtained

$$\begin{cases} K_P+K_I\dfrac{\cos(\pi\lambda/2)}{\omega_p^{\lambda}}=R_p \\ -K_I\dfrac{\sin(\pi\lambda/2)}{\omega_p^{\lambda}}=I_p \end{cases} \tag{13}$$

$$\begin{cases} K_P+K_I\dfrac{\cos(\pi\lambda/2)}{\omega_g^{\lambda}}=R_g \\ -K_I\dfrac{\sin(\pi\lambda/2)}{\omega_g^{\lambda}}=I_g \end{cases} \tag{14}$$

In the design process of the controller, the controlled object $G_p(s)$ and the expected amplitude margin A_m, the phase margin ϕ_m are known in advance, so R_p, I_p, R_g and I_g can be obtained by equations (11) and (12). Because the λ can be determined by system characteristics and practical experience, in equations (13) and (14), four variables (K_P, K_I, ω_p and ω_g) can be solved by four equations.

After the above calculation and analysis, the relevant parameters of the fractional-order PI controller can be determined.

In this paper, the Oustaloup approximation algorithm is used to calculate the fractional order operator s^{λ} [19]. This algorithm can make the zero-pole of the transfer function as small as possible while ensuring the fitting effect of the fractional transfer function. Assuming that the preset fitting frequency interval is (ω_b, ω_h) and s^{λ} is regarded as a continuous filter, the approximate transfer function model can be derived as

$$G_f(s)=K\prod_{n=-N}^{N}\frac{s+\omega_k^{'}}{s+\omega_n} \tag{15}$$

where zero $\omega_b^{'}$; pole ω_h and the gain K are

$$\begin{cases} \omega_k^{'}=\omega_b\left(\dfrac{\omega_h}{\omega_b}\right)^{\frac{k+N+0.5(1-\alpha)}{2N+1}} \\ \omega_n=\omega_b\left(\dfrac{\omega_h}{\omega_b}\right)^{\frac{n+N+0.5(1+\alpha)}{2N+1}} \end{cases} \tag{16}$$

$$K=\omega_h^{\ \alpha} \tag{17}$$

3. Application of fractional-order PI control in single-phase APFC

3.1. Control Structure

The principle of a typical single-phase power factor corrector with double closed-loop control structure is shown in Fig. 2, including power circuit 1 and control circuit 2. In control circuit 2, voltage error filter amplifier is generally used in the voltage outer loop controller. The inertia of the controlled object is large and the output voltage is allowed to have a certain static error. PI, PR or quasi PR regulators are commonly used in current inner loop controllers. The inertia of the controlled object is small and large. It is hoped that the smaller the static error of the instantaneous mean value of the controlled inductance current, the better.

The starting process of single-phase APFC power supply system is described as follows: (1) soft power-on process of AC power supply; (2) soft start process of APFC; (3) soft start process of post load.

In practical application, the potential load of single-phase APFC circuit is generally as follows: (1) electrically isolated or non-isolated DC-DC converter, which is used as telecommunication power supply, charging post power supply, etc.; (2) three-phase DC-AC converter, which is used for three-phase motor drive; (3) pure resistive load, which is generally used for short-time test, which is most commonly used; (4) electronic load, which can program the load characteristics, which is generally used for long-term testing.

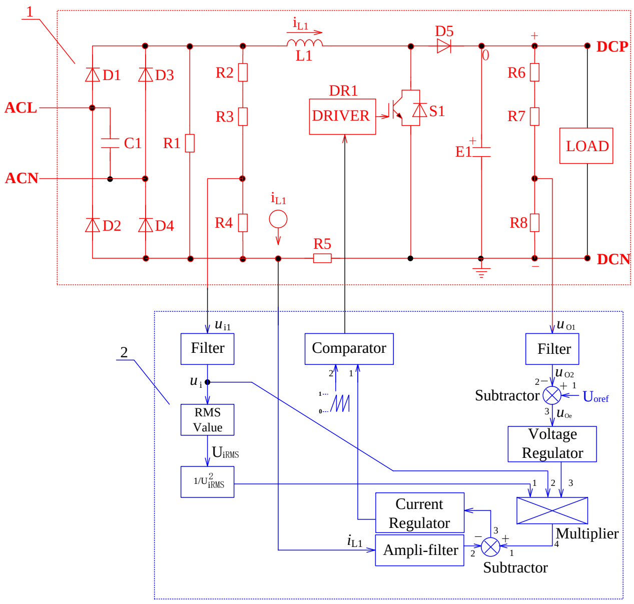

Figure 2. Functional block diagram of single-phase APFC.

For the first two load types, it has nonlinear characteristics, but the output power is constant in the steady state. The output voltage includes average voltage, double power frequency ripple voltage and high frequency ripple voltage. The output power depends on the load demand. When these two kinds of loads are started, soft start strategies are generally introduced, and the start-up phase has little effect on the dynamic characteristics of the output voltage. For the third type of load, it has linear characteristics. The output power is the ratio of the square of the output voltage to the resistance value in the steady state. When the APFC starts and the resistance value changes, it has a great influence on the dynamic characteristics of the output voltage. Considering the aforesaid, in this paper pure resistive load is chosen as an example for simulation analysis.

Single phase APFC adopts double fractional order PI controller, by setting the fitting frequency range as (0.000025 Hz, 40000 Hz) and the load as pure resistive load R_L. The main electrical parameters or control parameters under the rated conditions are as follows:

Table 1. Parameters of APFC.

Symbols	Parameters	Value
u_i	RMS value of input voltage	220V
f_i	Mains frequency	50Hz
$C1$	AC capacitance	0.22μF
$E1$	DC capacitance	2200μF
$L1$	Inductance	1mH
R_L	Load resistance	80Ω
u_o	Mean value of output voltage	400V
P_o	output power	2kW

3.2. Simulation results

In a single-phase APFC, the controlled quantity can be any power of the output voltage as well the inductance current. The first power and the second power of the controlled quantity have physical significance. The second power of the capacitance voltage is directly proportional to the capacitance energy storage, and the second power of the inductance current is directly proportional to the inductance energy storage. Therefore, to control the second power of the output voltage and the inductance current is equivalent to the direct energy storage control, which can accelerate the system response speed.

In order to show the advantage of fractional PI control, the regulator and controlled quantities in simulation is shown in Table 2.

Table 2. Controller selection and controlled quantities.

Method	Regulator	Controlled quantity
Method 1	First order PI controller	u_o^2 and $i_L^{0.5}$
Method 2	First order PI controller	u_o^1 and i_L^1
Method 3	First order PI controller	u_o^2 and i_L^2
Method 4	1/4 order PI controller	$u_o^{0.5}$ and $i_L^{0.5}$

In order to improve the performance of the controller, the fractional order PI controller with variable proportion coefficient and integral coefficient is adopted in the voltage outer loop in Figure 3. The K_P and K_i coefficients of the voltage outer loop are determined according to the output voltage and the given difference, so as to optimize the control performance of the system. The current internal loop coefficients K_P and K_i of the fractional order PI controller can be obtained by the previous calculation method. After several attempts, when the control order of the single-phase APFC system is 0.25, satisfactory regulation effect can be obtained.

After soft power on, the output DC voltage of single-phase APFC is 311V, and the output voltage given in the simulation is 400V, which is a step response. As a matter of fact, when the voltage of electrolytic capacitor rises to the rated value, the load will be put into operation. In this paper, the load resistance is put into use when the output voltage is rising. At this time, the storage energy of the electrolytic capacitor increases while providing energy to the load. Therefore, compared with the steady state, the effective value of the inductance current is higher.

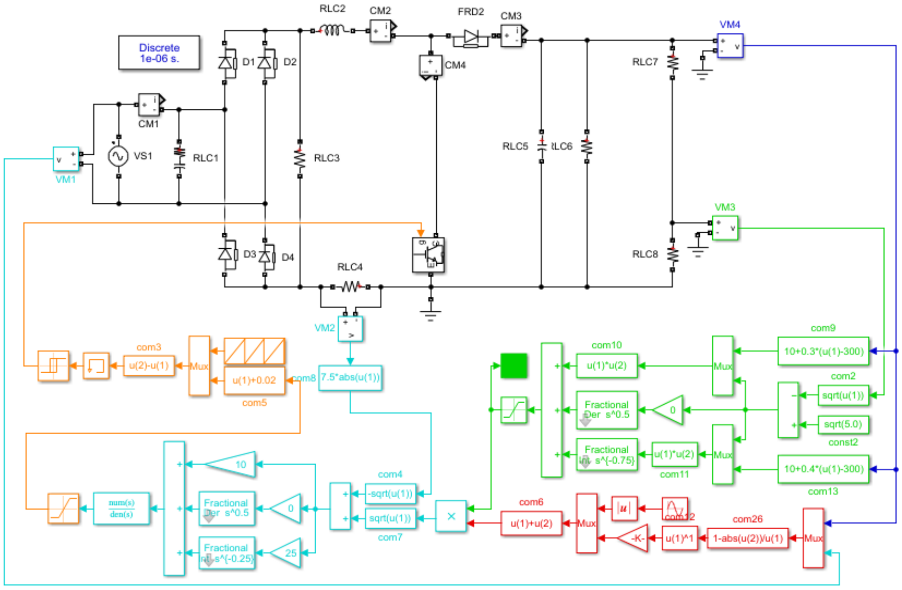

Figure 3. Variable Coefficients Fractional-order PI Control Based on MATLAB/Simulink Model.

In the four cases in Table 2, the output voltage response curve is as shown in Figure 4. It can be seen that only fractional order PI controller can obtain the dynamic voltage response process without overshoot and the shortest time, and the static difference is also the smallest.

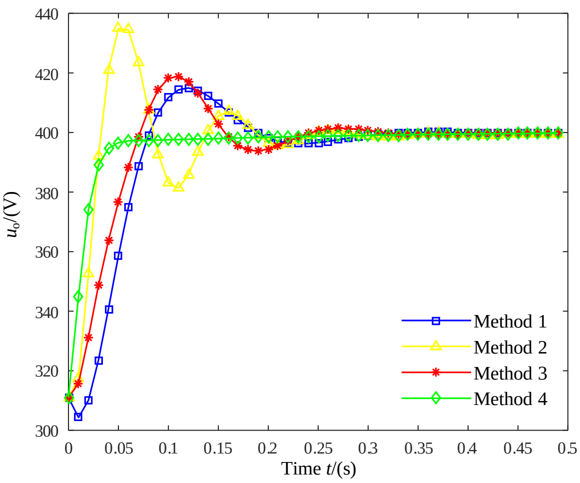

Figure 4. Output voltage response in four cases.

It can be seen from the Figure 4 that in the four cases of Table 2, the specific dynamic response performance indicators are shown in Table 3 below.

Table 3. Dynamic performance indexes in four cases.

Method	Overshoot	Rising Time	Time to Peak t_p(s)
Method 1	4.50%	0.048	0.117
Method 2	9.50%	0.014	0.057
Method 3	4.25%	0.050	0.107
Method 4	0%	0.012	0.037

It is obvious from the table that the variable coefficient fractional order PI controller designed in this paper can make the controlled system obtain smaller overshoot and faster dynamic response speed.

In four cases in Table 2, capacitance C1 filters out the high-frequency ripple of inductance current to obtain the grid side current. Figure 5 shows the waveform of input voltage u_i and input current i_i. For fractional order PI controller, the input current overshoot is small, the response time is the shortest, and the static difference is the smallest.

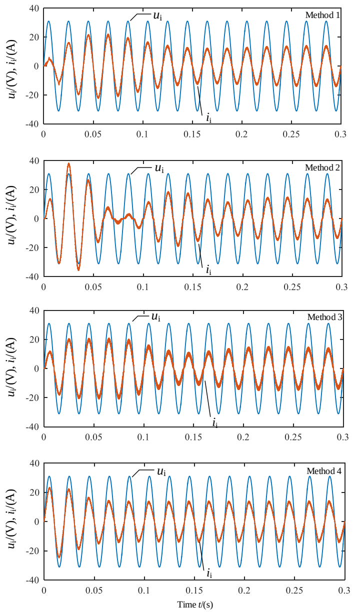

Figure 5. Input voltage and input current in four cases.

In order to further verify the response characteristics of fractional PI control system, the simulation analysis of variable load resistance is carried out in four cases in Table 2. Put two 200 Ω resistors into operation at 0.5s and 1s respectively, and disconnect two 200 Ω resistors at 1.5s and 2S respectively. The output voltage response curves in four cases are shown in Figure 6. The fractional order PI controller causes less voltage fluctuation and shorter voltage recovery time.

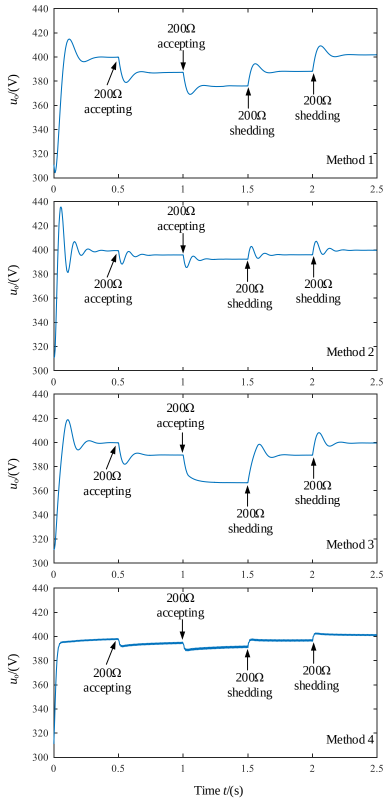

Figure 6. Stability of different control strategies.

4. Conclusions

After the analysis of fractional calculus and fractional PID, the fractional PI controller is introduced into single-phase APFC. Taking the pure resistance load as an example, four kinds of controllers and controlled variables are compared and analyzed, including the first order PI controller, the controlled variables are 0.5 power, 1 power and 2 power of output voltage and inductance current respectively; (2) the first fourth order PI controller, the controlled variables are 0.5 power of output voltage and inductance current respectively. From the simulation results, it can be seen that the controlled system can obtain fast dynamic response speed, low static error and good stability by setting a reasonable order fractional PI controller with variable coefficient.

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