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Comparative Analysis of Innovation Diffusion Models: Empirical Results and Predictive Performance on Russian Mobile Phone Propagation Data

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Abstract. This article introduces a new model that describes the innovation diffusion and is an extension of the well-known logistic model to the case when a diffusion process has a more complex structure. Time series data of mobile phone subscribers for Russian Federation during 2000-2018 are examined to compare the performance of the proposed model with the wellknown innovation diffusion models (the Gompertz, Logistic, Bass models) and the time-series autoregressive moving average (ARMA) model, one of the most popular forecasting models. Empirical results show that the extended logistic model outperforms the other models and the proposed model has the best characteristics on real data for the Russian mobile communications market.

1. Introduction

In recent years, a lot of research papers have examined the topic of innovation diffusion. Since the diffusion can be observed in different economic sectors with corresponding peculiarities, some researchers have focused on the study and analysis of these specific areas in which the innovation diffusion takes place [1–3]. In our study, the diffusion of innovations is considered on the example of the Russian mobile phone market.

One of the most studied areas of research is the modeling of the diffusion of mobile telephony, namely the modelling of quantitative characteristics for the dynamics of the dissemination of mobile communication devices in society. On the one hand, the mobile telephony diffusion models are sufficiently general to be applicable for simulating the diffusion for other types of innovations. On the other hand, researchers have a large amount of real quantitative data on the propagation of mobile communication devices provided by state statistics agencies and/or mobile phone operators. The paper [4] analyzed mobile telephone subscriber data for Taiwan based on the Gompertz, logistic, Bass models and time-series autoregressive moving average (ARMA) model, while the paper [5] identify the Fisher-Pry model as one of the most widely used in the analysis of mobile diffusion data. The research presented in the paper [6] concluded that the demand for mobile services was caused not only by lower prices but also by network effects.

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The dynamics of adopters of mobile devices (as well as innovation adopters) can be visualized by S-shaped curve [7]. This curve has three stages in which it behaves differently. At the first stage (the exponential regime), the curve slowly grows up to a certain point; at the second stage, the growth rate increases significantly. The curve slows down its growth and reaches the saturation moment in the third stage (the saturation regime), after which it practically stops growing. Many researchers predict mobile technology diffusion using the logistic model, e.g. this model is better suited for the study of S-shaped diffusion of mobile network in different countries [8–12].

Many recent research papers have developed and examined new diffusion models that can be employed to study the spread of innovations. For example, the Bass model have been extended in [13,14]. A non-linear mathematical model have been proposed in [15]. The paper [16] provides a comparative analysis of Monte Carlo simulation methods for innovation diffusion. The review of agent-based models for innovation diffusion can be found in [17]. In recent years, one of the main topics has been the study of innovation diffusion on social network structures [1–3,18–20]. Some relevant studies can be found in [21,22].

Our study analyzed the growth of mobile subscribers in the Russian Federation during 2000-2018. In Russia, the mobile telecommunications market is actively developing. One of the highest cellular distribution indicators was recorded in Russia. This is almost 2000 subscribers per 1000 people, which is several times higher than the indicators of developed and developing countries. Thus, it would be interesting to examine if the Russian telecommunications market has exhausted the possibilities of intensive development or not, falling into the last stage of the S-shaped curve.

Due to the fact that mobile operators offer their services at a relatively low price, the typical Russian consumer can afford several SIM cards. The Russian market has some features that distinguish it from European or Asian ones. Therefore, it is interesting to study the Russian market more thoroughly and to develop new models that can take into account its particular qualities. Note that statistical models have been used to investigate mobile diffusion in Russian regions [12]. It is very interesting to compare the simulation results on different data sets.

One of the most widespread innovation diffusion models is the well-known logistic model which has proved its effectiveness in the analysis of real-world data, as well as in predicting important quantitative characteristics of the corresponding processes. However, if an observed process is a result of the interaction of several unobserved processes, this model may lead to inaccurate prediction. In this regard, in this paper we pursue two goals. The first of them consists of a comparative analysis of various models of innovation diffusion (the the Gompertz, Bass, Logistic models and time-series autoregressive moving average (ARMA)) using the data representing the dynamics of a number of mobile communication devices in the Russian mobile phone market. However, in order to more accurately describe, model and predict this dynamics, in this paper we will present *the extended logistic model* (Subsection 2.5) that allows us to model the behavior of unobserved processes based on an observed aggregate time series.

There are three sections in this paper. Diffusion models are described in section 2. Section 3 includes research methodology and data description as well as empirical results obtained by the logistic, Bass, Gompertz and time-series autoregressive moving average (ARMA) models. The empirical results presented in Section 3 show that the presented model has better predictive abilities than standard models for modeling diffusion of innovations.

2. Model review

For diffusion analysis, such models as the Bass model [23], models of logistic families [24], as well as the Gompertz model [25], and ARMA are most often used. All these models help describe the *S*-curve which reflects the spread of innovation among specific population groups.

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2.1. The Gompertz model

The Gompertz model is expressed as follows

$$\frac{dN}{dt} = rN\ln\frac{K}{N},\tag{1}$$

where N is the number of adopters at time t, r is the intrinsic growth rate, and K is the maximum (equilibrium) number of adopters. The solution for this first-order differential equation with an initial condition is

$$N(t) = K e^{-e^{-r(t-m)}}.$$
(2)

The Gompertz model has been used in many studies. For example, [8] studied factors determining the diffusion of mobile telephony across developed and developing countries with the aid of a Gompertz model. Another examples of uses for Gompertz curves include:

- Mobile phone uptake, where costs were initially high (so uptake was slow), followed by a period of rapid growth, followed by a slowing of uptake as saturation was reached.
- Population in a confined space, as birth rates first increase and then slow as resource limits are reached.
- Modelling of growth of tumors.
- Modelling market impact in finance.
- Detailing population growth in animals of prey, with regard to predator-prey relationships.
- Modelling bacterial cells within a population.
- Examining disease spread.

2.2. The Logistic model

The Logistic model is expressed as

$$\frac{dN}{dt} = \rho N \left(1 - \frac{N}{K} \right),\tag{3}$$

where N is the number of adopters at time t, r is the intrinsic growth rate, and K is the maximum (equilibrium) number of adopters. The solution of this first-order differential equation with an initial condition is

$$N(t) = \frac{K}{1 + e^{-r(t-m)}}$$
(4)

The model was first used by [26], who applied a logistic model to explain the widespread use of hybrid corn in the United States. Some researchers have taken this study as a model for using the logistic model in the papers [27–30], among many others.

2.3. The Bass model

The Bass model was proposed in [23] and classifies adopters into two categories, innovators and imitators. The Bass model is expressed as

$$\frac{f(t)}{1-F(t)} = p + qF(t) \tag{5}$$

where f(t) is the likelihood of adoption at time t; F(t) is the fraction of the ultimate potential adopted by time t, p is the innovation coefficient, and q is the imitation coefficient. Equation of Bass model can be rewritten as

$$\frac{\frac{dN}{dt}}{K-N} = p + q\frac{N}{K} \tag{6}$$

where K is maximum (equilibrium) adoption potential, and N, which equals KF(t), is the total number of adopters in the interval (0, t). The solution for N with an initial condition is

$$N(t) = K \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p}e^{-(p+q)t}}$$
(7)

2.4. The ARMA model

The ARMA model is a forecasting technique that utilizes previous values of a dependent variable rather than independent variables to forecast a dependent variable. The ARMA(p,q) process, where p is the order of the autoregressive process and q is the order of the moving average process, can be expressed as

$$Y_t = \beta_0 + \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + \ldots + \theta_p Y_{t-p} + \varepsilon_t + \phi_q \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + \ldots + \phi_p \varepsilon_{t-q}$$
(8)

2.5. The extended logistic model

In this section, we will present an expanded logistic model describing the diffusion of an innovation. The model assumes that the dynamics of the number of innovation adopters follows the following first-order differential equation:

$$\frac{dN}{dt} = \alpha N \left(1 - \frac{N}{K} \right),\tag{9}$$

where N(t) is the number of innovation adopters at time t; α is the diffusion coefficient that determines the rate of innovation penetration through population; K is the level of saturation. It should be noted that K in this model is a parameter (as well as the rate level α).

Further, the model assumes that each of the adopters can use the innovation more than once. For example, when buying a mobile phone, the adopter can install not one but two SIM cards; or the adopter can purchase another technical device with a SIM card (in addition to an existing mobile device).

For simplicity, we assume that the adopter can apply the innovation no more than two times. Let the dynamics of the number of S adopters using the innovation twice also follow the logistic equation:

$$\frac{dS}{dt} = \beta S \left(1 - \frac{S}{N} \right),\tag{10}$$

where β is the diffusion coefficient. Note that the saturation level in the model (10) is equal to the number of innovation adopters defined by the equation (9).

The model described by the equations (9)-(10) can be applied when N, the number of innovation adopters, and S, the number S of those that used it the second time, are unobservable, but the total number of the innovation's uses

$$(N-S) + 2S = N + S$$

is observable at some moments of time.

For example, for the Russian mobile phone market, state statistics agencies provide information on the number of SIM cards, while the number of users who have mobile phones, as well as the number of those who have two SIM cards, is not known.

It is easy to obtain that equations (9)-(10) with some initial conditions have solutions which yield the following dynamics of the total number of the innovation's uses:

$$N + S = \frac{K}{1 + \exp(-\alpha(t - m_1))} \left(1 + \frac{1}{1 + \exp(-\beta(t - m_2))} \right),$$
(11)

where m_1, m_2 are model parameters (inflection points of logistic curves (9) and (10)).

Thus, the model described by (9)-(10) allows

- to describe the (observed) dynamics of the number of uses of the innovation N + S;
- to evaluate and describe the (unobservable) dynamics of the number of adopters N and the number of S adopters of innovation who used it twice.
- to forecast the processes N(t), S(t) and (N+S)(t) after setting model parameters.

We should remark that we can extend the model (9)-(10) to the case when innovation adopters use it two or more times by adding equations to the model:

$$\frac{dS_i}{dt} = \beta_i S_i \left(1 - \frac{S_i}{N} \right), \ i = 2, 3, \dots, p,$$
(12)

where S_i is the number of adopters that are using the innovation *i* times; β_i is the diffusion coefficient, and the saturation level N is equal to the number of innovation adopters defined by the equation (9); *p* is the maximum possible number of usages of the innovation by one adopter.

In this case, the number of uses of innovation will be equal

$$(N - S_2 - S_3 - \dots - S_p) + 2S_2 + 3S_3 + \dots + pS_p = N + S_2 + 2S_3 + \dots + (p-1)S_p = = \frac{K}{1 + \exp(-\alpha(t - m_1))} \left(1 + \sum_{i=2}^p \frac{i - 1}{1 + \exp(-\beta_i(t - m_i))} \right), \quad (13)$$

where $\alpha > \beta_2 > \beta_3 > \ldots > \beta_p$ and $m_1 < m_2 < \ldots < m_p$.

The model (9)-(10), (12) could be potentially more accurate in describing the dynamics of the usages of the innovation. For example, SIM cards can be used not only in mobile phones, but also in other devices (such as refrigerators, video surveillance cameras and other smart home objects). Therefore, while the simple model (9) gives a "highest point" on the number of usages which is equal to the saturation level of the model K, the extended model (9)-(10), (12) suggests that saturation of the market has not yet been achieved.

Note that the model (9)-(10), (12) has a significantly larger number of customizable parameters, and therefore it may be difficult to estimate them. In this regard, in this paper we consider the model (9)-(10), which limits the number of configurable parameters to five items.

3. Empirical results

3.1. Data

Initially, mobile telephony in the USSR was regulated by the Ministry of Communications which changed its name several times after the dissolution of the Soviet Union. Today the Ministry of Digital Development, Communications and Mass Media of the Russian Federation (until 2018 was the Ministry of Telecom and Mass Communications), is the governmental agency in Russia established in May 2008 that is responsible for telecommunications, media and the post. The Ministry carries out the functions of developing state policy and legal regulation in the field of information technology, telecommunications, postal services, mass communications and the mass media since 2008. In Russia, mobile telephony began operating in 1990, and commercial use began on September 9, 1991, when Delta Telecom launched the first cellular network in the USSR (operated in the NMT-450 standard) and made the first symbolic call over cellular to St. Petersburg Mayor Anatoly Sobchak.

By July 1997, the total number of subscribers in Russia amounted to about 300,000. Operators who provided communication services of the first generation standards NMT-450 and AMPS in the 1990s gradually switched to the provision of services, respectively, in the CDMA-2000 and DAMPS standards, in 2000s.

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By 2007, the main cellular communication protocols used in Russia were GSM-900 and GSM-1800. In addition, CDMA networks worked in the CDMA-2000 standard, also known as MT-MC-450.

According to the data of the British research company Informa Telecoms & Media for 2006, the average cost per minute of cellular communication for a consumer in Russia was 0.05 - this is the lowest figure from the G8 countries.

In December 2007, the number of mobile phone users in Russia grew to 172.87 million. As of December 2007, the market share of the largest mobile operators was: 30.9% (MTS), 29.2% (VimpelCom), 19.9% (MegaFon), 20% (other operators).

According to a study by J'son & Partners, the number of SIM cards registered in Russia at the end of November 2008 reached 183.8 million. This figure is due to the lack of a monthly fee on popular tariff plans of Russian mobile operators and the low cost of connecting to the network. In some cases, subscribers have SIM cards of different operators, while they may not be used for a long time, or use one SIM card in an office mobile phone, and another for personal conversations.

The market share of the largest mobile operators as of December 2008 amounted to: 34.4% at MTS, 25.4% at Vimpelcom and 23.0% at MegaFon.

As of April 2010, the market share in Russia for subscribers is: MTS - 32.9%, MegaFon - 24.6%, Vimpelcom - 24.0%, Tele2 - 7.5%, other operators - 11.0%.

In subsequent years, the structure of the cellular services market in Russia continued to change, and by the end of 2015, according to the international consulting company J'son & Partners Consolting, it looked as follows: MTS - 31%, MegaFon - 29%, Vimpelcom - 24%, Tele2 - 14%, other operators - 2% [31].

At the end of 2017, few operators remained in Russia, independent of the Big Four operators (MTS, Beeline, MegaFon and Tele2). This was facilitated by the purchase of the latest Smarts assets by MegaFon. Also, Mobile TeleSystems PJSC (MTS) acquired 100% of the shares of the regional mobile communications operator Cellular Communications of Bashkortostan.

As of 2018, 7 regional mobile operators remained, 4 of them working in the Crimea and Sevastopol and 3 in the rest of Russia.

In December 2013, the State Duma of the Russian Federation adopted a law according to which it became possible to transfer a subscriber number from one operator to another. As of September 2019, according to the Central Research Institute for Scientific Information, 27.4 million applications were received from subscribers to switch to another mobile operator while maintaining their number. Demand for the service is growing every year. If in 2016 3.9 million applications were submitted, in 2018 there are already about 7.2 million applications. In the 8 months of 2019, it reached the 7.39 million mark.

There has been a decline in SIM card sales in the past few years. However, mobile operators report that the outflow of customers is reduced, which means that the customer base is becoming better. In 2018, sales of SIM cards decreased by 12% to approximately 97 million units, according to a report from AC & M-Consulting. In 2017, 110 million SIM cards were sold. Thus, for the first time since 2009 (91 million SIM cards) less than 100 million SIM cards were sold in Russia in a year.

According to all of the above, the Russian market has some features that distinguish it from European or Asian ones. It is necessary to study the Russian market more thoroughly and to develop new models that can take into account its particular qualities.

The data from this study are taken from the database of the Federal State Statistics Service, available on the official website. To compare the suitability and forecasting accuracy of the five proposed models, the data were divided into two periods: training data set (from 2000 to 2014, annual data) and test data set (from the first quarter of 2014 to the second quarter of 2019, quarterly data). Data set for 2000-2014 contains the number of subscribers of mobile

communication devices by years. From the beginning of 2014 to the middle of 2018, the data include information for each quarter.

3.2. Results

The parameters for Gompertz, logistic, and Bass models, represented by Eqs. (2), (4) and (7) respectively, are obtained using the nonlinear leastsquares procedure. The ARMA(2,0) model is proved suitable for this study by autocorrelation function (ACF) and partial autocorrelation function (PACF) tests. The ARMA(2,0) model is represented by Eq. (8) with p = 2 and q = 0. We used data for 2000-2014 as training data. The rest of the data set was taken for forecasting.

To compare the predictive abilities of the proposed innovation diffusion models the measure of root mean square error was used:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{n}}.$$

This section will present the empirical results of modeling the diffusion of mobile communications in Russia using appropriate diffusion models. The results of the evaluation of the parameters on training set of all models are shown in Table 1. Parameters of Bass, Gommertz, logistic and extended logistic models are estimated by nonlinear least squares (NLS) regression. The parameters of the ARMA model were estimated using the maximum likelihood method (ML). The evaluation of parameters was performed in R. The p-value for all coefficients are less than the level of 0.01, which indicates that they are statistically significant. As we see from the K coefficient for the first three models, the saturation with mobile phones (SIM cards) is approaching 200%. This can be explained by the fact that one person can have two or more phones (for example, for working and personal purposes). In addition, since 2005, mobile networks are increasingly used to access the Internet using various devices (modems, tablets, etc.). Unlike the first three models, the K coefficient for the example, for the K coefficient for the extended logistic models are using various devices (modems, tablets, etc.). Unlike the first three models, the K coefficient for the extended logistic model means that 97.8% of the Russian population will use mobile services.

Root mean square error (RMSE) was chosen as an indicator of the accuracy of the models and was calculated for both training set and for test data (Table 2). The Gompertz model is the most effective for studying the data of mobile diffusion. This model was as close as possible to the real data and made a quite accurate forecast on the test data set, which can be seen in Fig. 1. It should be noted that our extended logistic model has the least error 27.44 on training set in comparison with other models. However, this model is not so accurate in forecasting compared to the Gompertz model. At the same time, the main advantage of the proposed extended logistic model is the evaluation of the (unobserved) amount of mobile users. Fig. 4 shows the growth of mobile users and the number of SIM cards per 1000 person obtained by the extended logistic model. According to this model, starting with 2004 there has been a significant increase in the number of mobile devices compared to the actual amount of mobile users. Moreover, it is assumed that in 2004 about 50% of the Russian population did not use the services of mobile operators, while some active adopters of this innovation process had already bought their second mobile devices (or their second SIM cards). At the same time, the simple logistic model and the ARMA model showed the worst results. Moreover, the ARMA model began to deviate from the real data more and more in different directions starting with 2005, which is confirmed by the values of errors, e.g. the error on the training data set is 92.25, and the error is 222.62 on the test data set.

Fig. 2 shows the increase in the number of mobile devices per 1000 people. It can be seen that after 2013 the increase was negative or close to zero. This figure illustrates more clearly how the extended logistic model describes data better than other models.

To verify the reliability of the results of an empirical study, we used the Durbin-Watson statistic (DW), which is a measure of the autocorrelation in the residuals of a regression analysis.

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	Coefficients	Std. Error	<i>t</i> -value	<i>p</i> -value			
Logistic model $y \sim \frac{K}{1 + \exp(-r(t-m))}, R_{adj}^2 = 0.990$							
K	1893	43.84	43.17	1.55E-14			
r	0.57	0.05	12.51	3.05 E-08			
m	5.82	0.17	33.94	2.72E-13			
Bass model: $y \sim K \frac{1 - \exp(-(p+q)t)}{1 + q/p \exp(-(p+q)t)}, R_{adj}^2 = 0.994$							
K	1934	44.56	43.40	1.46E-14			
p	0.03	0.005	6.20	4.58E-05			
q	0.45	0.05	9.32	7.66 E-07			
I	The Gompertz	z model: $y \sim$	$K\exp(-e$	$xp(-r(t-m))), R_{adj}^2 = 0.990$			
K	1994	38.83	51.37	1.95E-15			
r	0.35	0.02	17.45	6.84E-10			
m	4.83	0.10	46.87	5.81E-15			
ARMA(2,0) with non-zero mean, $R_{adj}^2 = 0.987$							
β_0	1014.5	393.59	2.58	0.0100			
θ_1	1.80	0.11	16.80	2.9E-63			
θ_2	-0.85	0.12	-7.12	1.08E-12			
Ext. Log. model: $y \sim \frac{K}{1 + exp(-\alpha(t-m_1))} \left(1 + \frac{1}{1 + \exp(-\beta(t-m_2))}\right), R_{adj}^2 = 0.998$							
K	977.58	18.90	51.71	1.77E-13			
α	1.12	0.12	9.47	2.61E-06			
m_1	3.99	0.12	34.67	9.43E-12			
β	0.59	0.08	7.19	2.96E-05			
m_2	8.31	0.24	34.21	1.08E-11			

Table 1. Estimation results for all models

Table 2. Root mean square error (RMSE) for training set and for test data

	logistic model	Bass model	Gompertz model	ARMA(2,0)	Ext. log. model
RMSE Train RMSE Test	$63.4661 \\ 74.2299$	$51.2034 \\ 45.0059$	$38.7160 \\ 27.1477$	92.2448 222.6153	$27.4392 \\ 33.4793$

This test was first proposed in paper [32] and used to estimate the quality of diffusion models of mobile telephony in Greece [5]. It can be seen from Table 3 that the value of the DW statistic for the Gompertz model and extended logistic model is close to 2, which indicates the absence of autocorrelation (independence of model residuals). This means that these models are adequate and can be used for forecasting. For the other three models, there is a positive autocorrelation in the residuals, which may indicate negative results of the estimation of unknown model coefficients. The residuals plots for all models against time are shown in Fig. 3.

The results of the extended model show that the process of growth in the number of adopters

Table 3. Durbin-Watson	(DW) statistic	for residuals
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	logistic model	Bass model	Gompertz model	ARMA(2,0)	Ext. log. model
DW	0.7175	1.1604	1.9403	0.8388	2.0427



Figure 1. Mobile telephony diffusion in Russian Federation: penetration



Figure 2. Increase the number of mobile users: SIM cards per 1000 person / period

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Figure 3. Model evaluation results - residuals plots against time



Figure 4. Growth of adopters and SIM cards per 1000 person based on the extended logistic model

was quite fast and reached its saturation in 2008. However, the process (10) took place with a time shift of $m_2 - m_1 = 4$ years. In addition, the rate of diffusion β of the second process was half that of the main process described by Equation (9). Note that a too high value of the parameter K = 977.58, close to the saturation level of 1000, indicates that the model (12) deserves attention.

4. Conclusion

The Russian cellular market has experienced rapid growth in the 2000s and recently the number of connected mobile devices has achieved around two devices per user. We propose a model that takes into account the dynamics of two phenomena. The first of these is an increase in the number of adopters, and the second is multiple acquisition of SIM card. In this regard, the proposed model extends the logistic model and under the assumption that the evolutionary process is described by a system of differential equations. However, a similar approach can be applied to other diffusion models.

We estimate the model parameters using time series data of mobile telephone subscribers for Russian Federation during 2000-2018. We compare the performance of the proposed model with the Gompertz, logistic, Bass models and the ARMA model. Empirical results show that the extended logistic model outperforms the other models and the proposed model has the best characteristics on real data for the Russian mobile communications market. One of the main features of the extended logistic model consists in extracting characteristics of the unobserved processes such as the time shift and diffusion rates.

5. Future Research

Further research will include work in two directions. The first of these consists in the development of Bass and Gompertz models for the case when the diffusion process has a more complex structure. The second direction may include an empirical estimation of the parameters of the extended logistic model (13).

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