The RME-based local instructional theory for teaching LCM and GCF in primary school

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Abstract. A learning trajectory (LT) for teaching the least common multiple (LCM) and the greatest common factor (GCF) using realistic mathematics education (RME) approach was developed to improve the students' mathematical reasoning abilities. The cycle of design research namely preparing for the experiment, conducting the experiment, and retrospective analysis were applied in the development processes. The subjects of the research were 26 fifth grade students at a primary school in Padang, Indonesia. The method of data collection were videotaping, interviews, observations, and analysing the students' worksheets. The validity and practicality of the LT were evaluated trough expert judgements, one-to-one and small group try out, and field test. The results show that the LT reached the criteria of validity as it fulfilled the state of the art and characteristics of RME. The LT also worked as intended in which the students could reinvent the concepts of the LCM and the GCF by themselves after solving a series of the contextual problems provided in the LT. It indicated that the LT satisfied the criteria of practicality. Finally, we discovered that the LT effective in improving the reasoning abilities of the students.

1. Introduction

People use mathematics to solve problem in daily life [1]. For example, we used mathematics (numbers) in trading (buying and selling), measuring land area, assigning house numbers, the license plate of cars, or telephone numbers. In computer science there were computer programs that used basic mathematical concepts to solve problems of daily life [2]. In contrast to this, it turned out that the pupils in Indonesia tended to have difficulty in applying mathematics in everyday life. According to [3,4], most of the students had difficulty applying mathematics to real life situations.

One of the topics that must be studied by elementary school students in Indonesia is the least common multiple (LCM) and the greatest common factor (GCF). As like as mathematics in general, students had difficulty learning LCM and GCF. LCM and GCF tended to use one method, namely the concept of factor trees (prime factorization) and tables, while the emergence of this concept was not reviewed so that the method for determining the LCM and GCF only followed the usual methods available in textbooks [5]. [6] stated the completion of LCM and GCF topics was still very procedural, namely by using a factor tree or prime factorization. Difficulties in studying the LCM and GCF were also found by Turkish instructors. [7] revealed that the LCM or GCF were the topics of mathematical studies that were difficult for students to understand in Turkey.

One of the tendencies that caused students that failed in mathematics the subjects was that students lacked understanding and used good reasoning in solving given problems. [8] founded the mathematics was a potential excellent vehicle to develop and improve the intellectual competence of a person in

spatial visualization, analysis, abstract thinking, and logical reasoning. [9] stated that the learning methods used by teachers has not varied. Meanwhile, teachers did not want to move away from conventional methods, and students tended not to like math lessons [10].

The tendency of teachers to teach mathematics mechanically in Indonesia has caused many problems when the students learning mathematics. First, the lack of interest and motivation of students towards mathematics [13,15,16]. Second, TIMSS study revealed that Indonesian students are still weak in the aspect of reasoning and problem solving [17]. The same condition was also found in the PISA studies which conquered that Indonesia's ranking was always at the bottom part (10%), and very few of Indonesian students that reached the two highest levels (levels 5 and 6) [13,18-20].

Observing the problems of learning mathematics as described above, schools should be the main actors in determining solutions to learning problems. Because according to [21] school meant a place to learn to use the mind well, think creatively facing important issues, and instill habits for thinking. The research team argued that RME approach has the potential to overcome these problems. RME approach, known in Indonesia as Indonesian Realistic Mathematics Education (PMRI) that aimed at changing the way mathematics is taught in the classrooms so that the students could solve math problems, develop mathematical knowledge and skills and enjoy learning mathematics [22]. [23] said that if we have a purpose that the students will discover mathematics through the activities of doing mathematics, then the teachers need to adjust to student reasoning and facilitate them to build mathematical concepts using their own ideas. For this purpose, the teacher needs to design a learning trajectory.

Gravemeijer defined a learning trajectory as a sequence of mathematical activities or tasks that could facilitate the growth of students' understanding of specific learning goals [24]. Learning trajectory was very helpful to bridge the work of researchers and practitioners [25]. Learning trajectory could be a vehicle for teachers to redesign their teaching plan. It also allowed the teachers to have a good understanding about the general vision of the class, when preparing mathematics lessons [26-27]. The previous studies revealed that the LT effective in helping the teachers to teach mathematics, not only in primary and secondary education [11,15,28], but also for teaching certain mathematics topics in universities [12,29-30]. The results of the studies indicated that LT was very useful to help the students to build understanding on mathematics concepts. The principles of RME for instructional design; didactic phenomenology, guided reinvention, and self-developed model [see 31-33] were used to design the LT for teaching LCM and GCM. When implementing the LT, we referred to some RME's characteristics [see 34-37]. Through this research we searched for the characteristics of a valid and practical RME-based LT for teaching LCM and GCF that effective to improve students' the reasoning of the students.

2. Method

Design research proposed by Gravemeijer & Cobb [23] was used as the research method. This design research aimed to develop a hypothetical learning trajectory (HLT) into the LT [38]. In this case, design research aimed to formulate, identify, and develop learning sequences that would facilitate the students' learning and thinking processes. The result of research was not design that works but the underlying principles explaining how and why this design works [39]. There were three stages in designing the HLT, namely preparing for the experiment, classroom experiment, and retrospective analysis. In the first stage, students' learning activities was developed in form of a HLT that was dynamic and could adapted to students' thinking strategies that occur during the classroom experiment. To develop HLT, various literatures related to LCM and GCF were studied. Then, the HLT was validated by three experts in mathematics education and RME. Furthermore, in the classroom experiment stage, the HLT was implemented by conducting one-to-one and small group try out. The focus in try out was to investigate the strategies, the thinking process and reasoning used by students when exploring the concepts of LCM and GCF. After small group try out, the HLT was implemented in a field test to the students at grade V in a primary school in Padang, Indonesia. Finally, collected data ware analyzed descriptively and the results were used as the basis for reflection and improvement conducted in the retrospective analysis

stage. Several data collection techniques were applied during three stages of design research, such as observations, interviews, videotaping, and analyzing the students' works.

3. Results and Discussions

Based on the literature reviews, we designed the HLT for teaching LCM and GCF. The HLT consisted of a series of activities that would give the experiences to the students to do mathematics from the informal stage to the formal stage. HLT implemented in class that used lesson plan and student work sheet. Three experts in RME and mathematics education validated the HLT, lesson plan, and student's worksheet. The things that validated include the aspect of content and the aspects of language, appearance, and graphics. After being revised in accordance with the validator's suggestion, then the product gave back to the validator to be assessed until all the products developed in valid category. The results of expert validation for the lesson plan revealed that the HLT reached the criteria of validity, as can be seen in Table 1.

Table 1. The Validity of HLT

No.	*Aspect*	*Mean of Validity*	*Category*
1.	Content	97,50	Valid
2.	Language	90,00	Valid
	All aspect	93,75	Valid

Table 2 below presents the general description of the validation results for students' worksheets.

Table 2. The Validity of STS

No.	*Aspect*	*Mean of Validity*	*Category*
1.	Didactical	90,00	Valid
2.	Content	80,00	Valid
3.	Language	93,33	Valid
4.	Graphical	95,00	Valid
	All aspect	89,58	Valid

In the HLT, the concept of LCM was reinvented by using the context of "swimming together". Using this contextual problems, the students could reinvent the concept of LCM (see Figure 1)

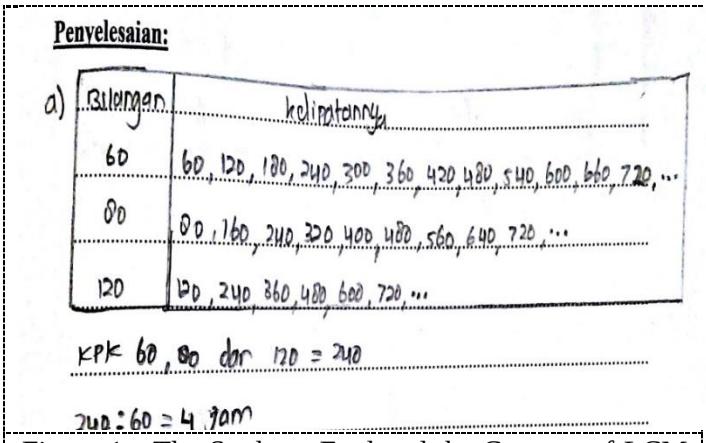

Figure 1a. The Students Explored the Concept of LCM as a first common multiple number in series

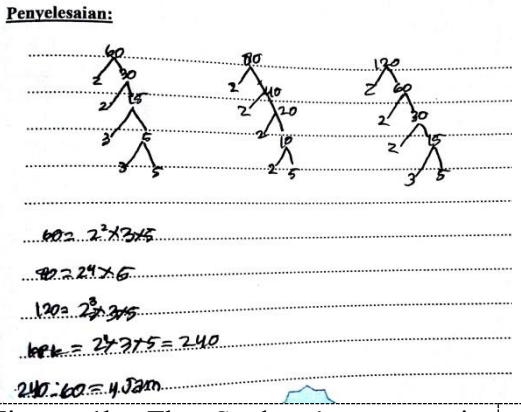

Figure 1b. The Students' strategy in reinventing the concept of LCM using tree factors.

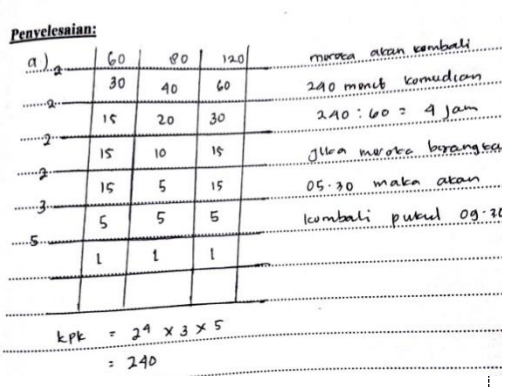

Figure 1c. The Students explored the LCM using common prime factor in table

Figure 1a showed the students reinvent the concept of LCM that explored the multiple number. Student founded LCM as a first common multiple number in series. Figure 1b showed that students could reinvent to observe the results above, it turned out to determine the LCM of two numbers done by multiplying all the different factors. If there are the same factors, then the highest rank is taken. And the figure 1c students explored the LCM with the division table, students founded the LCM by multiplying all the factors that could divided the three numbers.

The concept of GCF using the context "How do fair share of drink? The students' strategy in reinventing the concept of GCF can be seen in Figure 2.

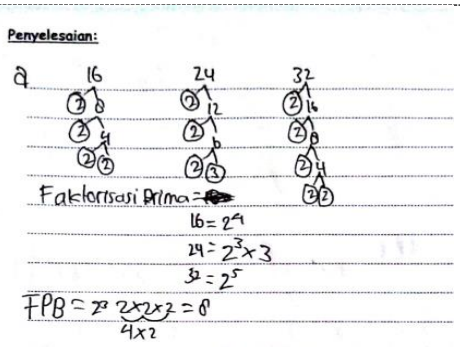

Figure 2a. The Students Explored the Concept GCF using tree factor

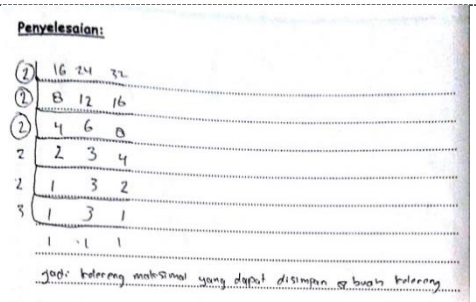

Figure 2b. The Students reinvent concept of GCF with table

From Figure 2a students found the concept of GCF using the alliance factor if a factor tree was the number contained in both of them and the number that is taken is the smallest rank. In figure 2b Students could found that GCF was a number that could divided the three numbers.

Some examples of the students' works presented before indicated that the LT for teaching LCM and GCF worked as intended. The students could reinvent the concepts by themselves. They also used their own ideas (students' free production) in reinventing mathematical concepts. These findings strengthen the results from previous researches which showed that RME and the LT help the students to learn mathematics in a meaningful way [see 40,41]. Based on the analysis of the students' works (on the worksheets and final test) we also found the improvement of the students' reasoning. This finding is in line with the results from [8, 42-44] which mentioned that RME and the LT could stimulate the students' ability in reasoning.

4. Conclusion

The results showed that the LT developed in this research satisfied the criteria of validity, with the characteristics: 1) it reflect the state of the art, design principles, and characteristics of RME, 2) the activities of solving contextual problems in the LT were well sequenced and could stimulate to reinvent the concepts in LCM and GCF; the activities were well sequenced, 3) the components in the LT are consistent between one and another. The LT reached the criteria of practicality as it worked as intended during the implementation. Finally, the RME-based learning trajectory gives positive impact on the students' mathematical reasoning ability.

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References

[1] Tobondo T, Y Victorina dan YVR Sinaga. (2014). Identifikasi dan analisis kesulitan siswa kelas IV dalam menyelesaikan soal cerita topik pecahan, KPK, dan FPB. Prosiding seminarnasional sains dan pendidikan sains IX, fakultas sains dan matematika. *UKSW. Salatiga, 21 juni 2014. Vol 5 no 1, issn; 2087-0922*

[2] Yuniati S. 2012. Menentukan Kelipatan Persekutuan Terkecil (Kpk) Dan Faktor Persekutuan Terbesar (Fpb) Dengan Menggunakan Metode "Pebi".http://jurnalbeta.ac.id . *p-ISSN: 2085-5893 / e-ISSN: 2541-0458 Vol. 5 No. 2 (Nopember) 2012, Hal. 149-165*

[3] Linguistika Y and Ikfan F. 2011. Permainan Dakonmatika Sebagai Media Pembelajaran Matematika Topik Faktor Persekutuan Terbesar (FPB) Dan Kelipatan Persekutuan Terkecil (KPK) Bagi Siswa Sekolah Dasar. Yogyakarta: *Prosiding Seminar Nasional Matematika dan Pendidikan Matematika.*

[4] Muniroh L. 2015. Pemahaman Konsep FPB Dengan Pendekatan RME. *Media Prestasi Vol. XV No.2 P-ISSN 1979 - 9225 e-ISSN 2356-2692.*

[5] Desriyati, Welly, Mashadi dan S Gemawati. 2015. Cara Lain Menentukan KPK dan FPB. *Jurnal Sains Matematika dan Statistika, Vol. I, No. 1, Januari 2015. ISSN 2460 – 4542*

[6] Marzuki. 2015. Prestasi Belajar Kelipatan Persekutuan Terkecil Dan Faktor Persekutuan Terbesar Dengan Mengunakan Alat Bantu Dekak-Dekak Pada Siswa Kelas Iv Sd Negeri 4 Bireuen. *JUPENDAS, ISSN 2355-3650, Vol. 2, No. 1*

[7] Camli, H and Bintars, J. 2009. Mathematical Problem Solving and computers: Investigation of the Effect of Computer aided instruction in solving lowest common multiple and greatest common factor problems. *International Journal of Human Sciences. Volume: 6 Issue: 2 Year: 2009 ISSN 1303-5134*.

[8] Adegoke, B Adesina. 2013. Modelling the Relationship between Mathematical Reasoning Ability and Mathematics Attainment. *Journal of Education and Practice www.iiste.org ISSN 2222-1735 (Paper) ISSN 2222-288X (Online) Vol.4, No.17, 2013*

[9] Yudhi P. 2017. Analisis Kebutuhan Pengembangan Lembar Kerja Siswa Berbasis Realistics Mathematics Education (Rme) Pada Materi KPK dan FPB Untuk Siswa Kelas Iv Sekolah Dasar. *Jurnal MENARA Ilmu Vol. XI Jilid 1 No.74 Januari 2017.*

[10] Marpaung. 2001. Prospek RME Untuk Pengemangan Matematika Di Indonesia. Makalah. *Seminar Nasional*. FMIPA UNESA Surabaya.

[11] Widjaja W, Dolk M. and Fauzan A. 2010. The role of contexts and teacher's questioning to enhance students' thinking. In *Journal of Science and Mathematics Education in Southeast Asia*, Vol 33(2), p 168-186.

[12] Webb DC, Kooij, Henk van der, and Geist MR. 2011. Design Research in the Netherlands: Introducing Lograritms Using Realistic Mathematics Education. *Journal of Mathematics Education at Teachers College*. Spring-Summer 2011. Volume 2.

[13] Fauzan A and Yerizon. 2013. Pengaruh Pendekatan RME dan Kemandirian Belajar Terhadap Kemamampuan Matematis Siswa. *Prosiding Semirata FMIPA di Lampung Tahun 2013*

[14] Rangkuti AN. 2015. *Pengembangan Alur Pembelajaran Topik Pecahan di Sekolah Dasar dengan Pendekatan Pendidikan Matematika Realistik* (Disertasi). Padang: PPs UNP.
[15] Fauzan A 2002 *Applying Realistic Mathematics Education (RME) in Teaching Geometry in Indonesian Primary Schools* (Enschede: PrintPartners Ipskamp)
[16] Fauzan A and Yezita E. 2016. Pengembangan Alur Belajar Topik Perbandingan dengan Pendekatan RME. *Prosiding Konaspi VIII di Jakarta Tahun 2016.*
[17] http://nces.ed.gov/timss/,
[18] Stacey K. 2011. The PISA View of Mathematical Literacy in Indonesia. *IndoMS Journal on Mathematics Education, Vol. 2(2), July 2010.* IndoMS: Palembang
[19] OECD. 2013. *PISA 2009 Results Vol. I - V.* OECD: Paris
[20] OECD. 2015. *PISA 2009 Results Vol. I - V.* OECD: Paris
[21] Johnson, E. B. (2011). *Contextual Teaching and Learning*. (Bandung: Kaifa)
[22] Sembiring R, Hoogland K, Dolk M. 2010. *A Decade of PMRI in Indonesia.* Utrecht: TenBrink.
[23] Gravemeijer K and Cobb P 2013 *Educational Design Research Part A: an Introduction* ed T Plomp and N Nieveen (Enschede: SLO) p 72-113
[24] Simon MA 1995 Reconstructing mathematics pedagogy from a constructivist perspective *J. Res.Math.Educ.* 26 114-145
[25] Larson CA, Wawro M and Zandieh M 2017 A hypothetical learning trajectory for conceptualizing matrices as linear transformations *Int.J.Math.Educ.Scie.Techno.* DOI: 10.1080/0020739X.2016.1276225
[26] Daro P, Mosher F and Corcoran T 2011 *Learning Trajectories in Mathematics: a Foundation for standards, curriculum, assessment, and instruction* (Philadelphia (USA): Consortium for Policy Research in Education)
[27] Bahamonde ADC, Aymemi JMF and Urgelles JVG 2016 Mathematical modelling and the learning trajectory: tools to support the teaching of linear algebra *Int.J.Math.Educ.Scie.Techno* DOI:10.1080/0020739X.2016.1241436
[28] Ramirez RE and Solis AH 2016 Hypothetical learning trajectories that use digital technology to tackle an optimization problem *Int.J.Techno.Math.Educ* 24 51-57
[29] Kwon ON 2002 *Proc.2nd Int.Con.on the Teaching of Mathematics* (Crete: EDRS)
[30] Prahmana R C I and Kusumah Y S 2016 The Hypothetical Learning Trajectory on Research in Mathematics Education Using Research-Based Learning *J Pedagogika* 123 42
[31] Gravemeijer K 1999 How emergent models may foster the constitution of formal mathematics *J.Math. Thinking and Learning* 1 155
[32] Gravemeijer K. 1994 *Developing Realistic Mathematics Education* (Utrecht: Freudenthal Institute)
[33] Gravemeijer K 2010 *A decade of PMRI in Indonesia* ed R Sembiring K Hoogland M Dolk (Utrecht: Tenbrink) p 41-50
[34] Streefland L 1991 *Realistic Mathematics Education in Primary Schools* (Utrecht: Freudenthal Institute)
[35] de Lange J 1987 *Mathematics, Insight, and Meaning* (Utrecht: OW & OC)
[36] Treffers A 1991 *Three Dimensions: A Model of Goal and Theory Description in Mathematics Education* (Dordrecht: Reidel)
[37] Freudenthal H 1991 *Revisiting Mathematics Education* (Dordrecht: Kluwer Academic)
[38] Baker A 2003 *Design Research on How IT May Support the Development of Symbols and Meaning in Mathematics Education*. Freudenthal Institute, Utrecht University
[39] Bardsley M E 2006 *Pre-Kindergarten Teachers' and Understanding of Hypothetical Learning Trajectories in Mathematics Education.* Utrecht: University of Utrecht
[40] Syariardi N *et al* 2019 The tools of mathematics learning based on realistic mathematics education approach in elementary school *Universal J.Ed.Res*. 7(7)
[41] Gee E *et al* 2018. *Designing learning trajectory for teaching sequence and series using RME* approach *to improve students' problem solving abilities*. IOP Conf. Series: Journal of Physics:

Conf. Series 1088 (2018). Doi :10.1088/1742-6596/1088/1/012096
[42] Afriadi J 2017 *Developing Instructional Design for Teaching Statistics Using RME Approach.* Master Theses: Universitas Negeri Padang
[43] Fauzan A *et al* 2018 A learning trajectory for teaching social arithmetic using RME approach *IOP Conf.Ser.:Mater.Sci.Eng.* 335012121 DOI 10.1088/1757-899X/335/1/012121
[44] Fauzan A *at al* 2018 Developing learning trajectory for teaching statistics at junior high school using RME approach *J.Phys.: Conf. Series* 1088 012040