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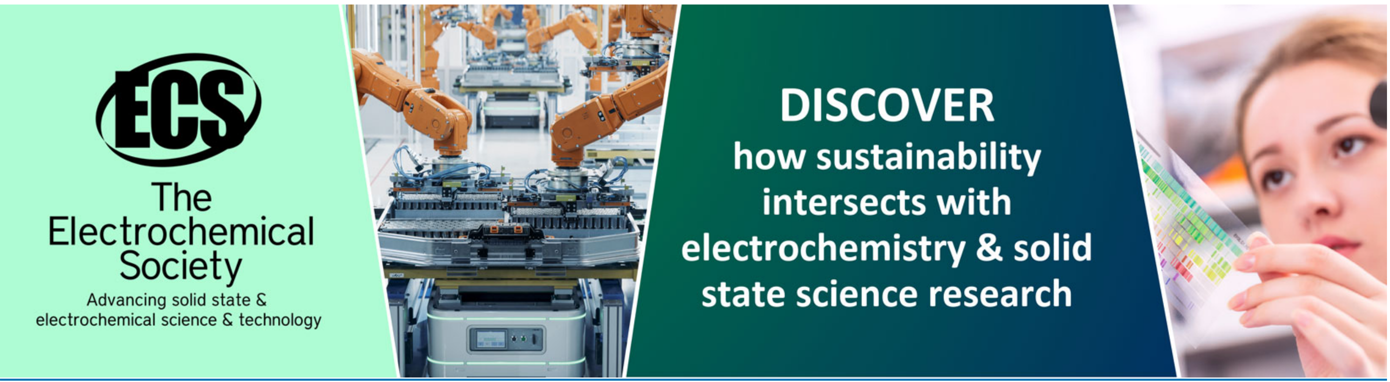

Unlocking neutrino mysteries via the inverse β-decay

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Abstract. We discuss some recent results on neutrino physics in the context of the inverse β-decay of uniformly accelerated protons. Specifically, we compute the decay rate of the process $p \to n + e^+ + \nu$ in both the laboratory frame (where the proton is non-inertial) and the comoving frame (where the proton is at rest and interacts with a thermal bath of electrons and neutrinos due to the Unruh effect). On the basis of the sole requirement of General Covariance of Quantum Field Theory, we manage to show that: *i*) the asymptotic (i.e. "in" and "out") neutrino states to be used for the evaluation of the S-matrix are Pontecorvo *flavor* eigenstates, *ii*) the Unruh thermal bath must be made up of *oscillating* neutrinos.

1. Introduction

Quantum vacuum is not vacuum at all. Notwithstanding the name, it exhibits a variety of features that underlie distinctive, though hardly detectable, physical phenomena. Among these, one of the most striking manifestations is the Casimir effect [1], which in recent decades has been getting increasing attention in a wide class of domains, ranging from physics beyond the Standard Model [2, 3] to gravity theories [4, 5, 6, 7] and quantum computing [8]. Notoriously, the Casimir force arises from the alteration of the zero-point electromagnetic energy by metallic plates. In the same way as conducting boundaries can affect the electromagnetic vacuum, gravity should in principle disturb all vacua, due to its coupling with all fields. In this context, the Hawking radiation provides the most emblematic example of the central rôle played by vacuum in regimes of extreme gravity.

Without straying too far from the Hawking prediction, in 1976 Unruh pointed out that, from the point of view of a uniformly accelerated observer, the Minkowski "inertial" vacuum appears as a thermal bath of particles at temperature $T_{\rm U} = a/2\pi$ (in natural units), where a is the magnitude of the proper acceleration [9]. By invoking the equivalence principle, this can be regarded, at least locally, as the non-gravitational counterpart of the Hawking radiation. In spite of such an analogy, some skepticism on the realness of the Unruh effect has been expressed during the years [10]. These concerns have thus stimulated the search for evidences that could definitively settle the controversy.

Along this direction, a clue to an answer was given in the context of the decay of accelerated protons in Refs. [11, 12], where it was shown that the Unruh effect is *mandatory* to obtain equal decay rates in the laboratory and comoving frames – a result which is expected since both of the cross-sections represent the probability of the same physical phenomenon and, thus, must be independent of the considered frame. Nevertheless, in the original analysis, the emitted neutrino was described as a particle with definite mass. The first attempt to embed *flavor mixing* in the above framework was made in Ref. [13], where a disagreement between the two rates was claimed to arise. Such an outcome was later revised in Refs. [14, 15, 16], showing that neutrino mixing is perfectly consistent with the General Covariance of Quantum Field Theory (QFT). Surprisingly, this was achieved by employing either flavor [14, 15] or mass [16] eigenstates, thereby adding more fuel to the fire of the debate on the physical nature of asymptotic neutrinos (see Refs. [17, 18, 19] and therein). In both the treatments, however, neutrino oscillations were overlooked.

Following Ref. [20], in this work we point out that, due to the fact that the emitted neutrinos undergo oscillations, the requirement of General Covariance of the formalism inevitably entails the use of flavor states for the computation of the S-matrix, as well as the occurrence of oscillations even of those virtual neutrinos which made up the Unruh bath.

The paper is organized as follows: in Sec. 2, we sketch the computation of the proton decay rate in the laboratory frame. The obtained result is then extended to the comoving frame in Sec. 3. Conclusions and outlook are finally discussed. Throughout the paper, we shall use natural units ($\hbar = c = 1$) and work in four dimensions. Furthermore, we consider a simplified two-flavor model: the generalization to three generations will be analyzed elsewhere.

2. Laboratory frame calculation

It is well-known that, according to the Standard Model, the proton is stable. To date, this prediction has found confirmations in all experiments, which set the lifetime of this particle to be longer than the current age of the universe. However, this is no longer true if we consider an accelerated (rather than inertial) proton. In that case, indeed, the process

$$\text{(a)} \quad p \to n + e^+ + \nu_e\,, \tag{1}$$

is not kinematically forbidden if the acceleration (which we assume for simplicity to be uniform) supplies the missing energy between the decay products and the proton itself (see Fig. 1a).

The decay rate of the process (1) can be easily calculated by treating the proton p and the neutron n as unexcited and excited states of a two-level system, the nucleon. Furthermore, we make use of the so-called *no-recoil condition* $a \ll m_p, m_n$, which ensures that the linear momenta of the emitted positron e^+ and neutrino ν are much smaller than the masses of hadrons. In this framework, it is reasonable to assume that the nucleon will move along a definite trajectory, the Rindler hyperbola, which indeed characterizes an eternally accelerated motion. As a consequence, the interaction between leptons and hadrons can be described by a Fermi-like effective action,

$$\hat{S}_I = \int d^4x \sqrt{-g}\, \hat{J}_{h,\lambda} \hat{J}^{\lambda}_{\ell}, \tag{2}$$

where $g \equiv \det(g_{\mu\nu})$ and γ^λ are the Dirac matrices. Here $\hat{J}^{\lambda}_{\ell} = \sum_{\alpha=e,\mu}(\hat{\overline{\Psi}}_{\nu_\alpha}\gamma^\lambda\hat{\Psi}_\alpha + \hat{\overline{\Psi}}_\alpha\gamma^\lambda\hat{\Psi}_{\nu_\alpha})$ denotes the leptonic current, with $\hat{\Psi}_\alpha$ ($\hat{\Psi}_{\nu_\alpha}$) being the charged lepton (neutrino) fields (see Ref. [14] for their explicit expansions). Note that the electron neutrino $\hat{\Psi}_{\nu_e}$ can be expressed as a mixture of neutrinos with definite masses $\hat{\Psi}_{\nu_i}$ ($i = 1, 2$) according to $\hat{\Psi}_{\nu_e} = \cos\theta\, \hat{\Psi}_{\nu_1} + \sin\theta\, \hat{\Psi}_{\nu_2}$ (and similarly for $\hat{\Psi}_{\nu_\mu}$), where θ is the Pontecorvo mixing angle. On the other hand, the accelerated hadronic current $\hat{J}_{h,\lambda}$ is given by $\hat{J}_{h,\lambda} = \hat{q}(\tau)u_\lambda\delta(x)\delta(y)\delta(u-1/a)$, where the coupling term $\hat{q}$ contains the effective Fermi constant G_F, v and $\tau = v/a$ are the Rindler time coordinate and the proper time of the nucleon, respectively, and a its proper acceleration[1]. The delta function fixes the spatial coordinate u to the value $1/a$, which corresponds to the Rindler

[1] In order to parameterize the trajectory of the uniformly accelerated nucleon, it comes in handy to introduce the set of Rindler coordinates (v, x', y', u). By assuming the acceleration to be directed along the z-axis, they are related to the corresponding Minkowski coordinates (t, x, y, z) by $t = u \sinh v$, $x' = x$, $y' = y$, $z = u \cosh v$.

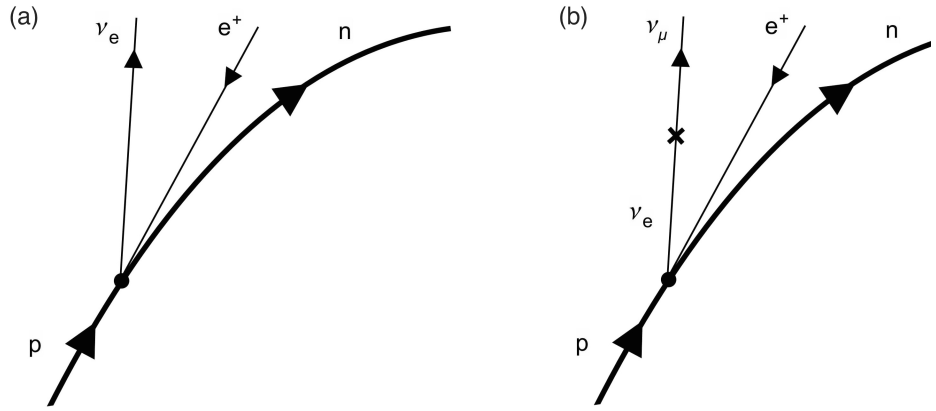

Figure 1. Inverse β-decay in the laboratory frame. On the left (right), the process in the absence (presence) of neutrino flavor oscillations (time flows in the vertical direction).

trajectory. Consequently, the nucleon four-velocity u^λ takes the forms $u^\lambda = (a, 0, 0, 0)$ and $u^\lambda = (\sqrt{a^2t^2 + 1}, 0, 0, at)$ in Rindler and Minkowski coordinates, respectively.

With these assumptions, the tree-level transition amplitude for the process (1) becomes [14]

$$\mathcal{A}^{(\mathrm{a})} \equiv \langle n| \otimes \langle e^+, \nu_e|\hat{S}_I|0\rangle \otimes |p\rangle = \frac{G_F}{2^4\pi^3}\Big[\cos^2\theta\, \mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1}, \omega_e) + \sin^2\theta\, \mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_2}, \omega_e)\Big], \qquad (3)$$

where σ_e (σ_ν) and ω_e (ω_ν) are the electron (neutrino) polarization and frequency, respectively (we have assumed equal polarizations and momenta for neutrinos with definite mass). The functions $\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_i}, \omega_e)$ $(i = 1, 2)$ are defined as in Eq. (14) of Ref. [14].

At this point, it is worth noting that the amplitude (3) has been evaluated by identifying the asymptotic neutrino state with the flavor eigenstate $|\nu_e\rangle$ in a very natural way. Such a state has then been expressed in terms of the corresponding mass eigenstates $|\nu_i\rangle$ via Pontecorvo transformations

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle, \qquad |\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \qquad (4)$$

In passing, we mention that the opposite choice of asymptotic mass neutrinos was pursued in Ref. [16]. In Ref. [15], however, it was argued that this scenario fails to correctly portray neutrino physics in the problem at hand. To strengthen such a result, in Sec. 3 we further show that the setting which truly allows to describe neutrino oscillations is the one based on flavor eigenstates.

From Eq. (3), the scalar decay rate of the process (1) reads

$$\Gamma^{(\mathrm{a})} \equiv \frac{1}{T}\int d^3k_\nu\, d^3k_e \sum_{\sigma_e,\sigma_\nu} \big|\mathcal{A}^{(\mathrm{a})}\big|^2 = \cos^4\theta\,\Gamma_1 + \sin^4\theta\,\Gamma_2 + \cos^2\theta\sin^2\theta\,\Gamma_{12}, \qquad (5)$$

where $T \equiv \int_{-\infty}^{+\infty} d\tau$ is the (infinite) total proper time of the nucleon, Γ_i $(i = 1, 2)$ is the decay rate we would obtain if we used $|\nu_i\rangle$ as asymptotic neutrino state and Γ_{12} arises from the interference between $|\nu_1\rangle$ and $|\nu_2\rangle$ (the explicit expressions of these terms are given in Ref. [14]). Thus, the decay probability is given by the coherent sum of the probabilities of production of different massive neutrinos.

Now, if we take into account that the emitted neutrino undergoes the oscillation $\nu_e \to \nu_\mu$ with a certain probability given by Eq. (4), the total decay rate in the laboratory frame also has a non-vanishing contribution from the flavor-violating process (see Fig. 1b)

$$(\mathrm{b}) \quad p \to n + e^+ + \nu_\mu\,. \qquad (6)$$

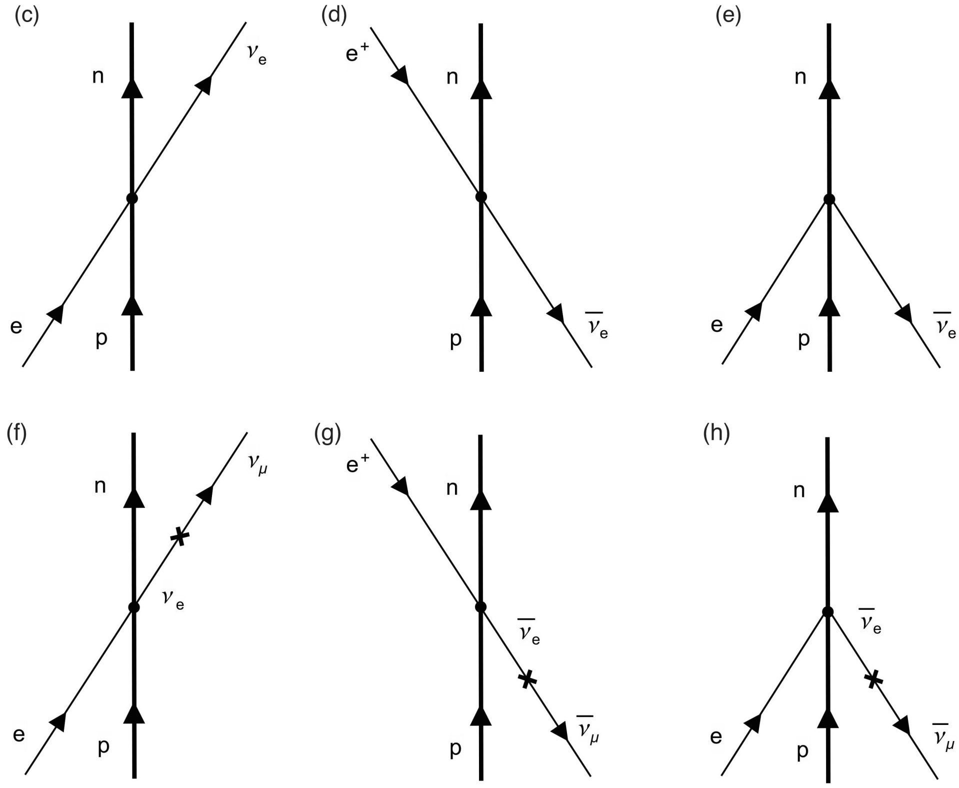

Figure 2. Inverse β-decay in the comoving frame. On the top (bottom), the processes in the absence (presence) of neutrino flavor oscillations (time flows in the vertical direction).

Therefore, by evaluating the transition amplitude (3) with respect to the state $|\nu_\mu\rangle$ in Eq. (4), we get the following expression for the decay rate of the above process:

$$\Gamma^{(b)} \;=\; \cos^2\theta \sin^2\theta \,(\Gamma_1 + \Gamma_2 - \Gamma_{12})\,. \tag{7}$$

Finally, the total decay rate in the laboratory frame is

$$\Gamma^{\rm lab} \;\equiv\; \Gamma^{(a)} \,+\, \Gamma^{(b)} \;=\; \cos^2\theta\,\Gamma_1 + \sin^2\theta\,\Gamma_2\,. \tag{8}$$

3. Comoving frame calculation

As discussed in the Introduction, in the comoving frame the proton at rest is allowed to decay due to the interaction with the Unruh thermal bath of electrons and (anti-)neutrinos (see Fig. 2). In Ref. [11], this was exhibited as a *theoretical check* of the Unruh effect, which is indeed necessary to maintain the consistency of the successfully tested QFT. In particular, the decay-channels to be considered in this case are

$$\text{(c)}\quad p^+ + e^- \to n + \nu_e\,, \quad \text{(d)}\quad p^+ + \overline{\nu}_e \to n + e^+\,, \quad \text{(e)}\quad p^+ + e^- + \overline{\nu}_e \to n\,. \tag{9}$$

The usual explanation of the Unruh effect relies on the existence of an event horizon which prevents the uniformly accelerated observer from having access to the entire Minkowski spacetime. Therefore, in order to compute the total decay rate in the comoving frame, we need to expand the fermionic fields in Eq. (2) according to the Rindler-Fulling scheme, which turns out to be the natural quantization for non-inertial observers (see Ref. [14] for details). After some laborious calculations, we get, for example, for the process (c) in Eq. (9)

$$\mathcal{A}^{(c)} \equiv \langle n| \otimes \langle \nu_e|\hat{S}_I|e^-\rangle \otimes |p\rangle = \frac{G_F}{(2\pi)^2}\left[\cos^2\theta\, \mathcal{J}^{(1)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) + \sin^2\theta\, \mathcal{J}^{(2)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\right], \tag{10}$$

where $\mathcal{J}^{(i)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)$ $(i=1,2)$ are defined as in Eq. (28) of Ref. [14].[2] The scalar decay rate $\Gamma^{(c)}$ is then obtained by resorting to the definition (5) and bearing in mind that the probability for the proton to draw a lepton from the Unruh thermal bath is given by the Fermi-Dirac distribution.

If we now perform similar calculations for the process (d) and (e) in Fig. 2, we have

$$\Gamma^{(c)} + \Gamma^{(d)} + \Gamma^{(e)} = \cos^4\theta\,\widetilde{\Gamma}_1 + \sin^4\theta\,\widetilde{\Gamma}_2 + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}_{12}, \tag{11}$$

where $\widetilde{\Gamma}_i$ $(i=1,2)$ and $\widetilde{\Gamma}_{12}$ have the same physical meaning as their inertial counterparts in Eq. (5) (we remand to Ref. [14] for their expressions).

In Ref. [14] we explicitly proved that the decay rates in the laboratory and comoving frames, Eqs. (5) and (11), respectively, do agree with each other, since $\Gamma_i = \widetilde{\Gamma}_i$ and $\Gamma_{12} = \widetilde{\Gamma}_{12}$ (the latter equality being valid at least up to the leading-order expansion around $\delta m/m_{\nu_i} \equiv (m_{\nu_2}-m_{\nu_1})/m_{\nu_i} \approx 0$). Hence, unlike the claims of Ref. [13], flavor mixing does not spoil the General Covariance of QFT at all, provided that asymptotic neutrinos are correctly described.

To put more flesh on the bones of the discussion, in the previous Section we have seen that, in addition to the flavor-conserving decay (1), the channel (6) also has a non-vanishing probability when neutrino oscillations are taken into account in the laboratory frame. In order to preserve the General Covariance of QFT, we thus search for the corresponding processes to be considered in the comoving frame. They are found to be (see the last three diagrams in Fig. 2) [20]

$$\text{(f)}\quad p^+ + e^- \to n + \nu_\mu\,,\quad \text{(g)}\quad p^+ + \overline{\nu}_\mu \to n + e^+\,,\quad \text{(h)}\quad p^+ + e^- + \overline{\nu}_\mu \to n\,. \tag{12}$$

We point out that, whilst the process (f) is of the same kind of (b), since it only provides for the oscillations of the emitted electron neutrino, the channels (g) and (h) bring new physics into play with respect to (d) and (e), as they require that an asymptotic muon antineutrino in the Unruh thermal bath undergoes oscillation *before* being absorbed by the proton (note that, at tree-level, the lepton charge must be conserved in the vertex).

The decay rate for the processes (12) can be calculated as in the previous case, leading to

$$\Gamma^{(f)} + \Gamma^{(g)} + \Gamma^{(h)} = \cos^2\theta\,\sin^2\theta\left(\widetilde{\Gamma}_1 + \widetilde{\Gamma}_2 - \widetilde{\Gamma}_{12}\right). \tag{13}$$

Therefore, the total decay rate in the comoving frame takes the form

$$\Gamma^{\text{com}} \equiv \Gamma^{(c)} + \Gamma^{(d)} + \Gamma^{(e)} + \Gamma^{(f)} + \Gamma^{(g)} + \Gamma^{(h)} = \cos^2\theta\,\widetilde{\Gamma}_1 + \sin^2\theta\,\widetilde{\Gamma}_2\,, \tag{14}$$

which is equal to the decay rate in the laboratory frame (Eq. (8)).

4. Discussion and conclusions

We have addressed the inverse β-decay of uniformly accelerated protons in connection with the phenomenon of neutrino flavor oscillations. Letting ourselves be guided by the lighthouse of General Covariance, we have shown that the Unruh thermal bath perceived by the proton in the

[2] Note that the Rindler frequencies in the comoving frame may assume arbitrary positive values since they do not obey any dispersion relation and, in particular, they do not depend on the mass (for this reason, we have omitted the subscript i to the neutrino frequency ω_ν).

comoving frame must be made up of oscillating neutrinos in order to recover the same expression of the decay rate as in the laboratory frame. We stress that this outcome has been achieved by using Pontecorvo flavor eigenstates as asymptotic neutrino states. A covariant, non-artificial description of the inverse β-decay with oscillating neutrinos by means of mass eigenstates is the *challenge* the present work poses for those who are still doubtful about the intimate nature of asymptotic neutrinos.

Despite providing such a net result, we emphasize that the above analysis may not represent the end of the story, since it has been developed in the limit of small difference between neutrino masses and to the leading order in $\delta m/m_\nu$. The question thus arises as to how to go beyond this approximation. In this regard, we envisage that some novel features of the formalism might come into play, as for example the necessity of a full-fledged QFT treatment of neutrino mixing instead of Pontecorvo quantum mechanical one [21], or the appearance of non-thermal corrections to the Unruh radiation [22, 23]. In passing, we mention that deviations of the Hawking-Unruh spectrum from the standard behavior are not uncommon in literature and have been recently pointed out also in other contexts (see for example Refs. [24, 25, 26, 27, 28]). Another interesting scenario to be investigated is how the neutrino oscillation probability gets modified for non-inertial observers. This has been preliminarily analyzed in the context of Quantum Mechanics in Refs. [29, 30], and, on the basis of the equivalence principle, in QFT on curved spacetime in Refs. [31, 32, 33]. More work is inevitably required to understand these aspects.

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